STATEMENT OF BOUNDARY CONDITIONS OF EQUATION FOR MAGNETIC FLUID INCLUDING NANOPARTICLES

K. Kotetishvili, E. Iordanishvili, G. Chikhladze

Georgian Technical University
Department of Engineering Physics
Tbilisi, Georgia
ketinooo@hotmail.com
gurchix@gmail.com

Accepted July 16, 2015

Abstract

The statement of boundary conditions of the closed system of equations for the magnetic fluid, which includes nanoparticles, has been carried out. It is shown that these conditions are received from Maxwell's equations and that the surface current at two extreme points may be directed oppositely. It is shown as well that the magnetic field outside the area of the magnetic fluid is zero, i.e. it may exist only within the definite area covered by the magnetic fluid.

After linearization of Langevin's function of the equation for magnetic fluids, the closed system of equations is received for complex amplitudes of magnetization $\hat{m}_x$, $\hat{m}_y$, spin-velocity ω_z and velocity of the magnetic flux v_x [1]:

$$\hat{m}_x = \frac{M_0}{\mu_0} \frac{\left(j\Omega\tau + 1 + \frac{M_0}{\mu_0}\right)\hat{h}_x - (\omega_z\tau)\frac{\hat{b}_y}{\mu_0}}{(j\Omega\tau + 1)\left(j\Omega\tau + 1 + \frac{M_0}{\mu_0}\right) + (\omega_z\tau)^2}, \tag{1}$$

$$\hat{m}_y = \frac{M_0}{\mu_0} \frac{(\omega_z\tau)\hat{h}_x + (j\Omega\tau + 1)\frac{\hat{b}_y}{\mu_0}}{(j\Omega\tau + 1)\left(j\Omega\tau + 1 + \frac{M_0}{\mu_0}\right) + (\omega_z\tau)^2}, \tag{2}$$

$$-2\xi\frac{\partial v_x}{\partial y} - 4\xi\omega_z + \langle T_{mag,z}\rangle = 0, \tag{3}$$

$$-2\frac{\partial p'}{\partial x} + 2\xi\frac{\partial \omega_z}{\partial y} + (\xi + \eta)\frac{\partial^2 v_x}{\partial y^2} = 0, \tag{4}$$

$\hat{b}_y$ and $\hat{h}_x$ being the fields of the small signal and are known quantities. Analytical solutions of this system of equations may be received, when the boundary conditions will be taken into account. From Maxwell's equations the corresponding boundary conditions for the components of the magnetic field are given as follows:

$$\vec{i}_n(\vec{B}_{int} - \vec{B}_{ext}) = 0, \tag{5}$$

$$\vec{i}_n \times (\vec{H}_{int} - \vec{H}_{ext}) = L_s, \tag{6}$$

where $\vec{i}_n$ is the unit vector (normal) at the surface between two mediums apart. L_s is the boundary surface current density in the area from $y = 0$ to $y = d$ (**Figure 1**), which is the source of the magnetic field in x and y directions. The density of the magnetic flux and the intensity of the magnetic field are given: $\vec{B}_{int}$ and $\vec{H}_{int}$ correspond to the internal part of the area of fields of the magnetic fluid, i.e. to $0 < y < d$ area, while $\vec{B}_{ext}$ and $\vec{H}_{ext}$ correspond to external part of the magnetic fluid, i.e. to $y < 0$ and $y > d$.

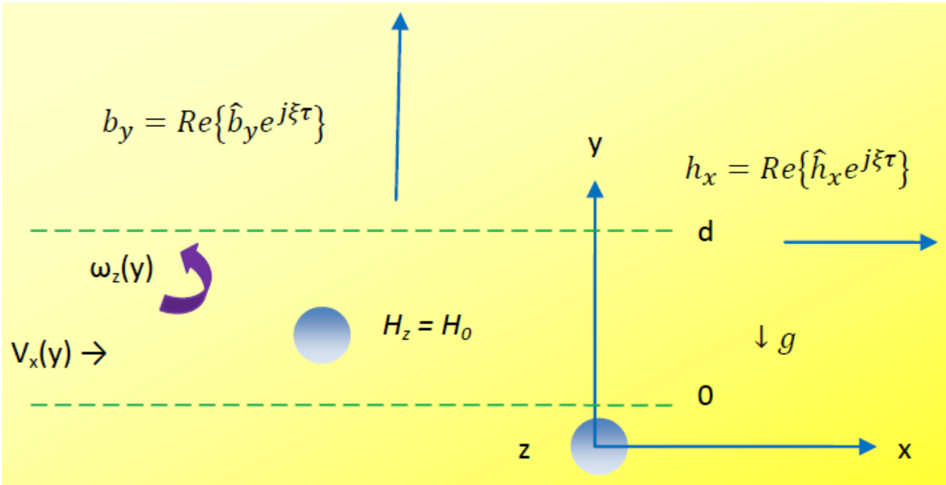

Figure 1. Magnetic fluid in the vessel
with solid, fixed walls ($y = 0$, and $y = d$).

The surface current at $y = 0$ and $y = d$ may be of opposite direction (e.g. generation of the surface current in x direction with uniformly distributed direct constant current in z direction and generation of varying in time and in z direction surface current in x direction). At that time the magnetic field outside the area of the magnetic fluid is zero. In y direction for $\vec{B}$ field the superposition of the field at the area takes place, when the magnetic field is located at $y = 0$ and $y = d$. For the spin-velocity ω_z the boundary conditions are absent as to the spin-velocity is not taken into account in the formula

$$\vec{T}_{mag} + 2\xi(\vec{\nabla} \times \vec{v} - 2\vec{\omega}) = 0, \quad (7)$$

while v_x velocity at the stationary border ($y = 0$ and $y = d$) is zero. Taking into account all these conditions, the system of equations for the magnetic fluid should be solved.

References

1. K. V. Kotetishvili, E. Iordanishvili, N. Kobalia, G. G. Chikhladze. Studying the equation of motion for magnetic fluids with nanoparticle inclusions. J. Appl. Electromagn., 2015 – in press.
2. K. V. Kotetishvili, G. G. Chikhladze. Linearization of the solution of Langevin's equation in magnetic liquids. Nano Studies, 2013, 7, 225-228.
3. R. Rosensweig. Ferrohydrodynamics. 1997, New York.
4. J. Popplewell, R. E. Rosensweig, R. J. Johnston. Magnetic field induced rotations in ferrofluids. IEEE Trans. Magn., 1990, 26, 5, 1852-1854.