MATHEMATICAL PREDICTION OF CRITICAL MULTIPHASE FLOW THROUGH RESTRICTION IN FLOW SYSTEM

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ABSTRACT
A method of predicting multiphase critical flow through restrictions is proposed. Hitherto, constant pressure ratios are assumed for critical flow conditions to be operational. This new method is dependent on the composition of the flowing fluid and the operating conditions of temperature and pressure.

The method is simple and employs dynamic conditions and is independent on the type of flow, the continuous phase and the position of restriction in the flow system. The procedure for the use of this method is outlined. An example illustrates its applicability in multiphase flow metering and evaluation of pressure drop across bit nozzles in foam and aerated drilling fluids.

KEYWORDS ;*Multiphase phase flow , Temperature , Pressure , Flows , fluids ,Restriction , Bit nozzles ,Pressure drop and Wellheads shokes .*

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INTRODUCTION

Multiphase flow through restrictions, is usually evaluated under critical or subcritical flow conditions. As a standard oil field practice, the evaluation of flowing well performance is usually based on the concept of critical flow, utilizing wellhead chokes. The choke or bean size is chosen in such a way that small variations in the downstream flowline or separator pressure do not affect the upstream wellhead pressure and thus the well's performance. Subcritical flow conditions are used to evaluate the well's performance at subsurface chokes and valves. Apart from the evaluation of the bean performance of flowing wells, a proper knowledge of multiphase critical and subcritical flow ranges is necessary, in the design of differential valves used in gas lifting , sizing of the minimum area in nozzle design^ and in some other industries .

For any multiphase critical flow correlation to be really effective, the prediction of when critical flow occurs must be accurate. However, there has been no agreement as to when multiphase critical flow actually occurs. Critical flow is a flow phenomenon in which the velocity of the fluid through the restriction^ is sonic or attains a Mach number of unity. It is known in fluid dynamics that when the velocity of propagation of a small pressure wave through the fluid reaches the level identical to the local velocity of sound in the fluid, the flow becomes independent of downstream disturbances of pressure or temperature because the disturbance cannot be transmitted in the upstream direction.

For single phase flow through restrictions, a generalized set of equations describing isentropic (adiabatic-frictionless) flow from upstream stagnation conditions (P,,p,,T,) to any other section (P, p,T) is given in references 4 and 5 and reproduced below, viz:

$$\frac{T_1}{T} = 1 + \frac{(K-1)}{2} M^2 \qquad (1)$$

$$\frac{P_1}{P} = \left[1 + \frac{(K-1)}{2} M^2\right]^{\frac{K}{K-1}} \qquad (2)$$

$$\frac{P_1}{P} = \left[1 + \frac{(K-1)}{2} M^2\right]^{\frac{1}{K-1}} \qquad (3)$$

where the polytropic constant
$K = C_P/C_V$, M = Mach Number,
T = temperature, °R,
P = pressure, psia (kPa),
p = density

For a diatomic gas, K is assumed to be equal to 1.4 for a Mach number of unity which represents critical flow condition. When these conditions are substituted into Equations (1) to (3) the downstream-to-upstream ratios become (see Figure 1)

$$\frac{T_2}{T_1} = \frac{2}{K+1} = 0.833 \qquad (4)$$

$$\frac{P_2}{P_1} = \left[\frac{2}{K+1}\right]^{\frac{K}{K-1}} = 0.833 \qquad (5)$$

$$\frac{P_2}{P_1} = \left[\frac{2}{K+1}\right]^{\frac{1}{K-1}} = 0.634 \qquad (6)$$

From the above, it obvious that for air and several other gases with k = 1.4, the absolute temperature drops byabout I7%, the pressure drops by about 47% and the density is reduced by about 37%.

At present, the general practice in predicting the condition of flow, is to use Equation (5) and draw inferences as follows:

That flow is critical if

$$\frac{P_2}{P_1} = \left[\frac{2}{K+1}\right]^{\frac{K}{K-1}} = 0.528$$

(ii)a That the critical flow region is represented by

$$\frac{P_2}{P_1} \lesssim \left[\frac{2}{K+1}\right]^{\frac{K}{K-1}}$$

(ii)b While the subcritical flow region is delineated by

$$\frac{P_2}{P_1} \lesssim \left[\frac{2}{K+1}\right]^{\frac{K}{K-1}}$$

Critical flow theory which was developed originally for gases has also been applied to the flow of liquids. However, since sonic flow (Mach No.= 1) for gases and liquids occurs at different velocities, the question of what is critical flow for a two-phase mixture has not been answered satisfactorily. The prediction of critical multiphase flow is a problem not only in the oil and gas field but also in such others as the nuclear industry. For gas-liquid mixtures, the acoustic or sonic velocity is known to be less than that for either phase alone(4). So far the criterion normally applied is that for two-phase flow, critical flow occurs if reduction of downstream pressure will not increase flowrates. Logical as this definition is the question as to how to predict when this happens still remains unanswered. Some authors(1,6,7)have stated that critical flow occurs if upstream pressure is approximately twice the downstream pressure and the fluid velocity through the choke is greater than that of sound.

Rogers and Mayhew(2) defined critical pressureratio, X as the downstream - to - upstream pressure ratio which will accelerate the flow to a velocity equal to the local velocity of sound in the fluid. It is the point in the flow when the area is minimum and the mass flowrate per unit area (or flowrate) is maximum. Critical pressure is the pressure in the restriction or throat when this situation occurs.

Available relationships describing critical multiphase flow have been categorized as follows:

1. Analytical models, applying mathematical analysis based on fundamental principles, to a simplified physical model with the development of equations. As an example of this category ofcorrelations, Ashford (8)implicitly assumed thatcritical flow occurs at a pressure ratio, X_C=0.544 fork = 1.04 as reported by Ros(9)
2. Empirical correlations using dimensional analysis to select and group the most important variables. An example of this category ofcorrelations is Omana et al correlation (10)Theyused $X_C = 0.546$, $R < 10$ or $V_{sg}<V_{sl}$to predict critical flow.
3. Empirical correlations from field or laboratory data. This category of correlations are the familiar been performance correlations by Gilbert, Baxendell, Ros and Achong (reviewed in References 5 and 6). They assumed that critical flow occurs at $X_c = 0.588$.

In their work on subcritical flow, Ashford andPierce(11) developed Equation (7) to predict critical multiphase flow.

$$\mathbf{X_C} = \left\{\frac{A\left[\frac{R_1}{b}\left(1-X_{est}^{b}\right)-x_{est}+1\right]x_{est}^{-e}-1}{R_1}\right\}^{-K} \quad \ldots\ldots\ldots\ldots \quad 7$$

Where

$$\mathbf{A} = \frac{2R_1}{K\left(1+R_1X_{est}^{\frac{-1}{k}}\right)}$$

B = (k-1)/k

C = (k+1)/k

Equation (7) is iteratated using estimated values of critical pressure ratio, X_{est} to predict X_cObserve

that Equation (7) has two roots, one less than X_c and one greater than X_C . Conditions can also occur where either one or no root can be found.
Fortunate[12] indicated that critical flow for two-phase flow can be as low as $X_c = 0.225$ and is highlydependent on the no-slip hold-up or input liquid content, λ_1He presented a graphical approach to predict critical flow assuming that mixture velocity is greater than 10 m/s, mixture Froude number is greater than 600 and flow is isothermal. The above references illustrate the lack of agreement and consistency in the prediction of critical multiphase flow.
Based on Mach number and sonic velocity, Fortunati[12]stated that "critical flow means a velocity of the fluid equal to the velocity of sound in that fluid through the choke". Thus the Mach number should be greater than or equal to unity in critical flow. He stated that sonic velocity occurs when:

$$\mathbf{V} = \left[\frac{\mathbf{144KPg}}{\lambda_{\mathbf{g}}\mathbf{p_n}}\right]^{\frac{1}{2}} \qquad (\mathbf{8})$$

Where

$$\mathbf{K} = \frac{(\mathbf{1}-\mathbf{x})\mathbf{C_{vL}} + \mathbf{xC_{pg}}}{(\mathbf{1}-\mathbf{x})\mathbf{C_{vL}} + \mathbf{xC_{vg}}} \qquad (\mathbf{9})$$

x = gas mass fraction or quality
and n = no-slip mixture density.

However, Wallis[13] defined sonic velocity of a homogenous mixture as:

$$\mathbf{V} = (\lambda_{\mathbf{g}}\mathbf{p_g} + \lambda_{\mathbf{1}}\mathbf{p_1})\left[\frac{\lambda_{\mathbf{g}}}{\mathbf{p_g V_G^2}} + \frac{\lambda_{\mathbf{1}}}{\mathbf{p_1 V_1^2}}\right]^{\frac{-1}{2}} \qquad (\mathbf{10})$$

with the minimum occurring at $\lambda_g = 0.5$

By a definition of Mach number as:

$$\mathbf{M} = \frac{\mathbf{V_{MB2}}}{\mathbf{V^*}} \qquad (\mathbf{11})$$

Where

$$\mathbf{V^*}\left[\frac{\mathbf{144KPg}}{\mathbf{p}}\right]^{\frac{1}{2}} \qquad (\mathbf{12})$$

And for M > 1.0, Brill et al[5] ' reported that they were able to isolate critical flow in the University of Tulsa API project.
From the foregoing, it can be observed that the use of sonic velocity to predict critical multiphase flow is based more on empirical deductions without any effort made to see that these relationships are reducible to single phase relationships known in fluid dynamics. It must be noted however, that at critical flow, the throat velocity is constant. Thus supersonic flow cannot occur in the throat and a further reduction in pressure such that P_2/P_1 is lessthan critical cannot cause an increase in volumetric flowrate.

PROPOSED METHOD FOR PREDICTING CRITICAL MULTIPHASE FLOW

This method is based on the definition that critical flow occurs when the flowrate through the restriction is maximum and the rate of change of flowratewith pressure ratio is zero. The word maximum is used with some reservation, for ideally, the maximumattainable velocity[2]' is the velocity when the fluid iscontinuously expanded to zero pressure. This is supersonic velocity which is not 'possible within the restriction.

Recently, a generalized multiphase orifice flowmoder[14,15] was presented. Grouped into factors, the multiphase flowrate irrespective of the continuous phase is given by references 14 and 15 as:

$$\mathbf{q_{TP} = F_b \beta R_{mp} F(P_2/P_1)} \qquad (13)$$

where the base factor F_b is defined as:

$$\mathbf{F_b = 1.970\ C_D d_c^2} \qquad (13a)$$

d_c = diameter of the chock in 64th of an inch
and

C_D = discharge coefficient

$$\mathbf{F_b = 7.89\ C_D d_c^2} \qquad (13b)$$

where d_c = 32nd of an inch as in bits. The beta factor is defined as:

$$\boldsymbol{\beta} = \{\mathbf{P_1}[\mathbf{p_{gl}} + \mathbf{p_1 LGR_1}]\} \qquad (13c)$$

$$\mathbf{R_{mp}} = \frac{(\mathbf{B_o}+\mathbf{WOR})+\frac{0.00504 Z_1 T_1 (R_P - R_S)}{P_1}}{62.4(\mathbf{Y_O B_O}+\mathbf{WOR})+0.01353 \mathbf{Y_g R_P}} \qquad (13d)$$

In Equation (13), the only factor incorporating pressure ratio P_2/P_1= X is the dimensionless pressure functionF(P_2/P_1), defined for polytropic flow as:
$F(P_2/P_1) = F(X)_{mp}$

$$= \frac{[\mathbf{LGR_1}(1-\mathbf{X})+\mathbf{B}]^{\frac{1}{2}}}{\mathbf{LGR_1}+\mathbf{X}-\frac{1}{\mathbf{K}}} \qquad (14)$$

Where

$$\mathbf{B} = \frac{\mathbf{K}}{\mathbf{K}-\mathbf{1}}(\mathbf{1}-\mathbf{X}^{\frac{\mathbf{K}-1}{\mathbf{K}}})$$

And

$$\mathbf{LGR_1} = \frac{5.615\ (\mathbf{B_O}+\mathbf{WOR})}{\mathbf{LGR_1}+\mathbf{X}-\frac{1}{\mathbf{K}}} \qquad (15)$$

is the dimensionless liquid-gas-ratio at upstream of choke conditions expressed in terms of the PVT characteristics of the mixture. LGR_1, can also be expressed as a function of
no-slip liquid hold-up, λ_1 or foam quality, Γ_1 such that the equations can be applied to foam and aerated drilling fluids. Expressed in terms of λ_L, or Γ_1:

$$\mathbf{LGR_1} = \frac{\lambda_L}{1-\lambda_1}\mathbf{1} = \frac{1-\Gamma_1}{\Gamma_1} \qquad (16)$$

For the special case or isothermal compressible flow, the dimensionless pressure function is derived [14,15] as:

$$\mathbf{F(X)_{mi}} = \frac{[\mathbf{LGR_1}(1-X)-\ln X]^{\frac{1}{2}}}{(\mathbf{LGR_1}+X^{-1})} \qquad (17)$$

By the above definition of critical flow, if we differentiate Equation (13) q_{TP} with respect to X, the other factors f_b,,R_{mp} will remain constant and at the maximum q_{TP}, F(X) is also maximum. Thus at

$$\frac{\mathbf{dq_{TP}(X)}}{\mathbf{dx}} = \mathbf{0}, \frac{\mathbf{dF(X)}}{\mathbf{dx}} = \mathbf{0} \qquad (18)$$

giving the critical pressure ratio X_C Differentiating.Equation (14) and equating to zero, gives

$$(\mathbf{X_c} - \mathbf{1})\mathbf{LGR_1} = \frac{\mathbf{k}}{\mathbf{k}-\mathbf{1}}\mathbf{A} - \frac{\mathbf{K}}{\mathbf{2}}\mathbf{B}\,\mathbf{X_C^{\frac{K-1}{K}}} \quad (19)$$

Where

$$\mathbf{A} = \mathbf{1} - \mathbf{X_C^{\frac{K-1}{K}}}, \quad \mathbf{B} = \left[\mathbf{LGR_1} + \mathbf{X_C^{\frac{-1}{K}}}\right]^2$$

Equation (19) is similar to Equation (7) by Ashford and Pierce[11]'. It is non-linear and iterative to obtain X_C .Ifthe, producing gas-oil-ratio, R(P,T), tends to infinity or the LGR, tends to zero, as the case may be, then flow
approaches single phase gas flow and Equation (19) reduces to the familiar critical pressure ratio for single phase real gas flow (Equation 5).

Limiting Conditions of the Dimensionless Pressure Functions

An evaluation .of the dimensionless pressure functions for limiting conditions satisfies known relationships, thus

(a) For polytropic dry gas flow

$\boldsymbol{\lambda_L = 0, \Gamma_1 = 1}$ **and** $\boldsymbol{LGR_1 \to 0}$

Equation (14) thus reduces to

$$\mathbf{F(X)_{gi}} = \left[\frac{\mathbf{K}}{\mathbf{K}-\mathbf{1}}\mathbf{x^{\frac{2}{K}}} - \mathbf{x^{\frac{K+1}{K}}}\right]^{\frac{1}{2}} \qquad \mathbf{20}$$

This is the dimensionless pressure function which Nind[1]' plotted against P_2/P_1. Equation[5]canalso be obtained from Equation (20) if we differentiate F(X) with respect to X.

(b) For isothermal gas flow, Equation (17) reduces to

$$\mathbf{F(X)_{gi}} = \mathbf{x}[-\mathbf{\ln x}]^{\frac{1}{2}} \qquad \mathbf{21}$$

(c) For only liquid flow

$\boldsymbol{LGR_1 \to \infty, \lambda_L = \Gamma_1 = 0}$

Therefore

$$\mathbf{F(X)_L} = [\mathbf{1} - \mathbf{x}]^{\frac{1}{2}} \qquad \mathbf{22}$$

This is the same for both polytropic and isothermal conditions of flow. Observe that mathematically $F(X)_L$ is maximum at x = 0,

For multiphase flow, the condition for critical flow is strongly dependent on LGR_1,. To avoid the

tedium
of an iterative procedure, a very simple graphical method of predicting critical multiphase flow is to plot the dimensionless pressure function, F(X) versus the downstream-to-upstream pressure ratio, X, as has beendemonstrated by Nind[(1)]' for dry gas flow through chokes.Critical multiphase flow will occur at a pressure ratio $X_c$where F(X) is maximum, and for the range of pressure ratio X >X_c, flow is subcritical and for X ≤X_c it is inthe critical flow range. A simple computer program can also be used to predict X_c, given the PVT and operatingconditions of the fluid at the point of flow restriction.
Note that critical flow is not seriously considered for isothermal flow, as practically, flow cannot be isothermal at sonic flow. Mathematically if we substitute K = 1 in Equation (5), X_c will be unity, which implies ano-flow condition. However, it is possible for practical interest to evaluate $F(X)_{mi}$ in Equation (17), to ascertainwhen this parameter is maximum.

Validation of Proposed Method

Using field data reported by Ashford[(8)]on an IBM-PC, the following critical pressure ratios were predicted.

Table 1:

Correlation	X predictedForK = 1.04
Category I (Ashford)	0.544 (constant)
Category El	0.588 (constant)
Proposed method	0.408 - 0.584(X_c = 0.518)

Table 3 gives details of the data and predicted X .

DISCUSSION OF RESULTS

For K = 1.04 and 1.25 the dimensionless pressure functions, $F(P_2/P_1)$ are plotted for various values of foam quality in Figures 2 to 5 for polytropic flow and Figures 6 and 7 for isothermal flow. These plots are also made for various values of LGR_1, for K =1.04 as functions of P_1 and T_1 in Figures 8 to 13. Generally, they are parabolas, with a unique maximum value of $F(P_2/P_1)$corresponding to a condition during which the fluid velocity through the choke is equal to the local velocity of sound. The value of the pressure ratio, P_2/P_1corresponding to this maximum value is the critical pressure ratio, X_c. For values of x less than $X_c$downstream pressure perturbations cannot be transmitted upstream and the flowrate through the choke becomes independent of the downstream pressure.
The left hand arch of the curve shown in Figures (2) to (13) as a broken line, is the so-called critical flow range, over which the value of the flowrate through the restriction is proportional to the upstream pressure p_1.
The right-hand arch beyond the maximum point is the subcritical flow range.
The values of K are insensitive to temperature variations as well as molecular weight within the range of gaseous hydrocarbons. For dry natural gases, K is taken to be 1.25 and the plot is symmetrical about the
pressure ratio of 0.5. In his text, Ikoku[(7)] reports an average value of X_c = 0.555. Plots for multiphase floware slightly skewed. The degree of skcwness depends on the value of λ_1 or Γ_1 and the polytropic expansion index, K. For K = 1.04, the maximum value of $F(P_2/P_1)$increases with increasing Γ_1 (Figures 2 and 3) and occur at X_c between 0.5 and 0.6. For K = 1.25, (Figures 4 and 5)it occurs between 0.5 and 0.556 for $\Gamma_1 \geq 0.70$. It is 0.54 at Γ_1= 0.90. This variation is also observed in Figures 8 to 13 as functions of P_1 and T_1. In Table

1 using field data, the mean X_c was found to be 0.518. $X_c$was as low as 0.408 and as high as 0.584. Therefore the use of X_c - 0.544 for K = 1.04 as reported by Ros[9]to evaluate critical multiphase flow may only be valid within some range of λ_L, or Γ_1 or LGR_1, particularly for highvalues of Γ_1, tending towards mist flow. However, it is now possible to analytically determine X_c accurately for varying values of K and LGR_1. For a particular producingfield, a single average value of X_c can be obtained to cover a wide range of operating conditions.
In the case of isothermal flow, Figures 6 and 7 are plotted as functions of Γ_1, the curves are moreasymmetrical than for polytropic flow and maximum points occur between X_c = 0.5 and 0.7 for $\Gamma_1 \geq$ 0.95. The degree of asymmetry about 0.5 increases with increasing Γ_1. Generally the effect of LGR_1 on the valueof X_c is that: as LGR_1 increases (or Γ_1 decreases), for a constant K, X_c decreases. Put differently, xc decreases with increase in P_1 and decrease in λ_L And for varying K, X_c increases with decrease in K. For dry gas, the trend as shown in Table 2.
For isothermal gas flow, X, is equal to 0.607. Itcan be observed that X_c decreases in the following order: liquid flow; isothermal dry real gas flow; polytropic dry real gas flow; two-phase isothermal flow and two-phase polytropic flow. This trend confirms the results ofOlson[4]The differences in isothermal and polytropicconsiderations are pronounced at lower pressure ratios.

Table 2: Values of K and corresponding Single Phase X_c

K	Xc (Equation 5)
1.04	0.598
1.25	0.555
1.4	0.528

APPLICATION

Procedure

Below arc steps on how to apply this method.
For each restriction in a field, use relevant PVT data at that point and the appropriate dimensionless pressure function, plot $F(P_2/P_1)$ versus P_2/P_1using an average value of LGR_1

For the empirical critical flow correlations, use X_c when F(X) is maximum.

For subcritical flow correlations, use $X > X_C$.
For the proposed model [14] or other analytical models, calculate P_2/P_1 and read off the corresponding F (P_2/P_1) using the appropriate flowrate equation. For critical flow, use the maximum $F(P_2/P_1)$ and for subcritical flow use $F(P_2/P_1)$ corresponding to the measured $P_2/P_1>X_c$,

Hitherto what is done in the evaluation of critical flow performance is: First, the critical pressure ratio is calculated, ifthe measured pressure ratio is less than thecalculated value, that is if

$$\frac{\mathbf{P_2}}{\mathbf{P_1}}\,\textbf{measured} < \left[\frac{\mathbf{2}}{\mathbf{K+1}}\right]^{\frac{\mathbf{K}}{\mathbf{K-1}}}$$

as if for single phase gas flow, then (P_2/P_l) is replaced in the correlation by

$$\mathbf{X_c} = \left[\frac{\mathbf{2}}{\mathbf{K+1}}\right]^{\frac{\mathbf{K}}{\mathbf{K-1}}}$$

If

$$\frac{P_2}{P_1}\textbf{measured} \geq \left[\frac{2}{K+1}\right]^{\frac{K}{K-1}}$$

the measured pressure ratio is used unchanged in the calculations.
However, by this method, for any measured pressure ratio, in the critical flow range, the exact value of F (P_2P_1) is read and used directly to avoid over prediction of flowrates. The choke size can then be adjusted to haveX_c

EXAMPLE

Given a wellhead choke with following fluid characteristics and operating conditions, determine the pressure ratio at which critical flow occurs and hence the subcritical flow region.

K	=	1.04	TI	=	560°R
B_0	1.01	WOR	=	0.0	
y_g	0.90	R_s	0.0		
Pi	=	500 psia			
R	1000 SCF/STB				

Solution

Using Figure 8 the maximum value of F(X) occurs at $X_c = 0.57$. Also using a simple computer program X_c corresponding to maximum F(X) can be obtained from Equation (14)
Critical flow region: $X \leq X_c$
Subcritical flow region: $X > X_c$
Reference 16 further illustrates the methods of application of the proposed approach to multiphase flow metering and the evaluation of pressure drop across bit nozzles in foam and aerated drilling fluids.

CONCLUSION

From this study, the following conclusions were reached:
A method of predicting when critical flow occurs in a multiphase flow system and thus accurately delineating the subcritical flow region is presented. This method is dependent on the dynamic state of the flowing fluid characteristics and operating conditions of temperature and pressure, and independent of the flowmeter, location of meter and the continuous phase.
Critical pressure ratio obtained analytically confirms the experimental results of R vsW, that for multiphase flow systems, where K = 1.04, $X_c = 0.544$ for a range of conditions. However, an average X_c can now be determined for eachtype of flowing condition and composition.
Critical pressure ratio for multiphase flow can be as low as 0.225 and as high as 0.60 depending on the composition of the mixture and operating conditions. This also confirms the observations of Olson^ and Fortunati^-
The polytropic and isothermal conditions define the maximum and minimum values of F(X) and thus flowrate respectively even though X_c for both of them may not coincide for the same set of conditions.

The proposed method is easy to incorporate into existing multiphase flow correlations.

NOMENCLATURE

B	=	Formation volume factor V (P,T) /V ($T_{,)}P_{,}$), bbl/STB (m^3/m^3)
Fb	=	Multiphase base factor, Equation (13a)

$F(P_2/P_1)$ = $F(X)$
= Dimensionless pressurefunction
K = Polytropic index of expansion Ratio of specific heat at constant pressure to specific heat at constant volume (C_p/C_v)
LGR_1 = Liquid-gas-ratio at upstream of choke conditions, dimensionless
M = Mach number
P = Pressure, psia (Kpa)
PVT = Pressure-volume-temperature
q = Flowrate, ft^3/sec or BPD (m^3/sec or m^3/D)
QM = Metered flowrate
R or R_P = Producing free gas-oil ratio, GOR,SCF/STB (m^3/m^3)
R_S = Solution GOR, SCF/STB (m^3/m^3)
RMP = Multiphase specific volume factor
T = Temperature, °R (°K)
T. NO = Test number
Vsg = Superficial gas velocity, ft/sec (m/sec)
VSL = Superficial liquid velocity, ft/sec(m/sec)
WOR = Water-oil ratio, dimensionless
W.NO = Well number
X = downstream-to-upstream pressure ratio

Greek Symbols

$\boldsymbol{\beta}$ = Beta factor
P = Density, Ibm/ft (kg/m3)
ƛ = no-slip hold-up
Γ = Foam quality
γ = Gas Gravity

Subscripts

1 = Upstream of choke conditions
2 = Downstream of choke conditions
C = Critical conditions
est = estimate
g = gas
gi = gas isothermal
gp = gas polytropic
L = liquid
mi = multiphase isothermal
mp = multiphase polytropic
n = no-slip
o = oil
TP = two-phase or total flow rate
w = water

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TABLE 3 FIELD DATA[8] AND PREDICTED CRITICAL PRESSURE RATIO, X_c 17

W NO	T NO	DC	RP	RS	P1	T1	SGO	SGG	BO	WOR	QM	X_c
1	1	32	1065.00	.00	485.0	530.0	.844	1.0000	1.0100	.00	1010.00	0.5750
1	2	16	1065.00	.00	505.0	580.0	.844	1.0000	1.0100	.00	230.00	0.5740
2	1	32	180.00	.00	325.0	580.0	.885	1.0000	1.0100	.00	1505.00	0.5270
2	2	21	180.00	.00	465.0	580.0	.885	1.0000	1.0100	.00	1190.00	0.5020
2	3	16	173.00	.00	665.0	580.0	.885	1.0000	1.0100	.00	720.00	0.4650
3	1	32	363.00	.00	425.0	580.0	.867	1.0000	1.0100	.00	1340.00	0.5470
3	2	21	337.00	.00	575.0	580.0	.867	1.0000	1.0100	.00	1055.00	0.5250
3	3	16	341.00	.07	775.0	580.0	.867	1.0000	1.0100	.00	390.00	0.5019
4	1	32	118.00	.00	375.0	580.0	.883	1.0000	1.0100	.00	2088.00	0.4900
4	2	21	107.00	.00	525.0	580.0	.883	1.0000	1.0100	.00	1752.00	0.4490
4	3	16	108.00	.00	740.0	580.0	.883	1.0000	1.0100	.00	1068.00	0.4080
5	1	32	127.00	.00	100.0	580.0	.882	1.0000	1.0100	.00	370.00	0.5650
5	2	21	120.00	.00	125.0	580.0	.882	1.0000	1.0100	.00	350.00	0.5360
5	3	16	102.00	.00	225.0	580.0	.882	1.0000	1.0100	.00	290.00	0.5180
6	1	32	197.00	.00	215.0	580.0	.901	1.0000	1.0100	.00	725.00	0.5530
6	2	21	185.00	.00	265.0	580.0	.901	1.0000	1.0100	.00	575.00	0.5400
6	3	16	160.00	.00	295.0	580.0	.901	1.0000	1.0100	.00	415.00	0.5270
7	1	32	377.00	.00	115.0	580.0	.883	1.0000	1.0100	.00	305.00	0.5840
7	2	21	391.00	.00	135.0	580.0	.883	1.0000	1.0100	.00	295.00	0.5820
7	3	16	290.00	.00	245.0	580.0	.883	1.0000	1.0100	.00	190.00	0.5610
8	1	36	407.00	92.00	1215.0	560.0	.863	.7200	1.0910	.00	3686.00	0.4840
9	1	36	392.00	92.00	1235.0	560.0	.863	.7200	1.0910	.00	3898.00	0.4800
10	1	40	410.00	92.00	1115.0	560.0	.863	.7200	1.0910	.00	4576.00	0.4930
11	1	40	410.00	92.00	1165.0	560.0	.863	.7200	1.0910	.00	4728.00	0.4890
12	1	30	561.00	92.00	1175.0	560.0	.863	.7200	1.0910	.00	2661.00	0.5100
13	1	30	463.00	92.00	1265.0	560.0	.863	.7200	1.0910	.00	2580.00	0.4900
14	1	30	457.00	92.00	1235.0	560.0	.863	.7200	1.0910	.00	2555.00	0.49

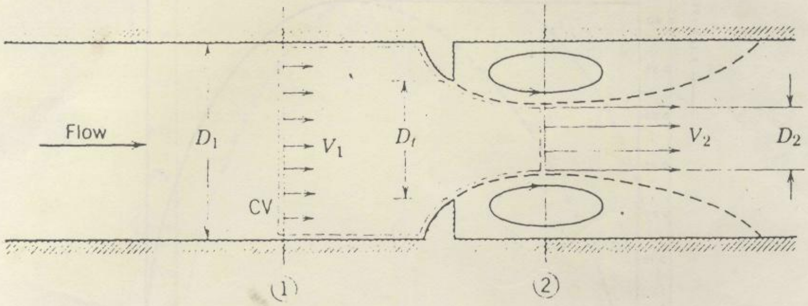

Fig. 1 Internal flow through a generalized nozzle, showing control volume used for analysis

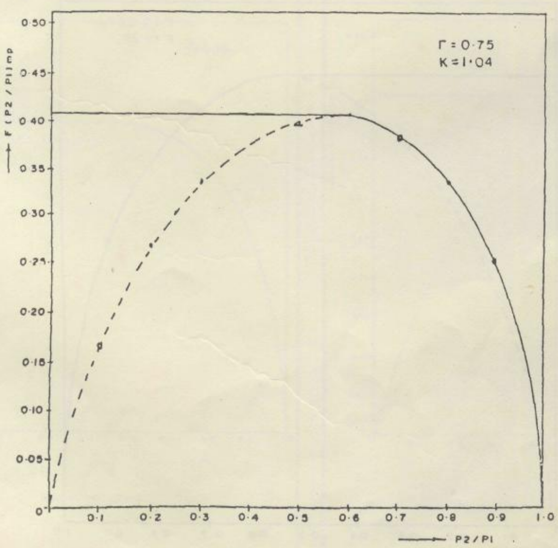

Figure 2 - Polytropic Flow

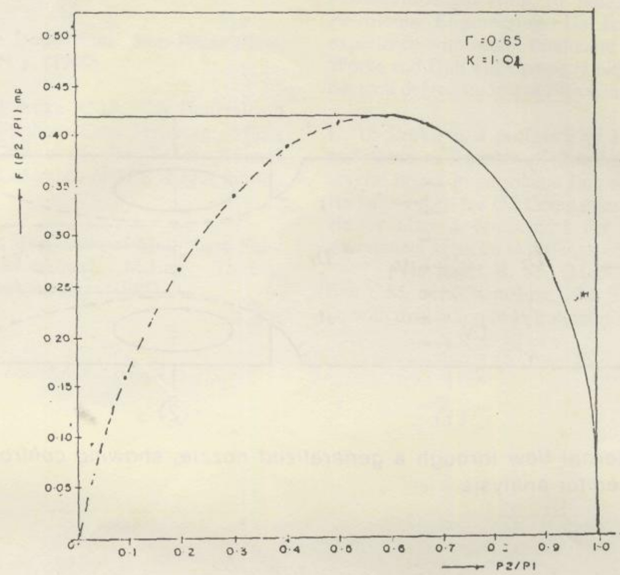

Figure 3 Polytropic Flow

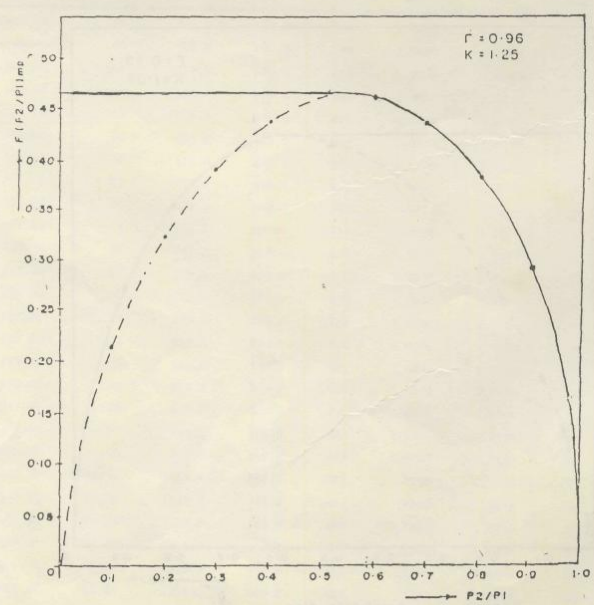

Figure 4 Polytropic Flow

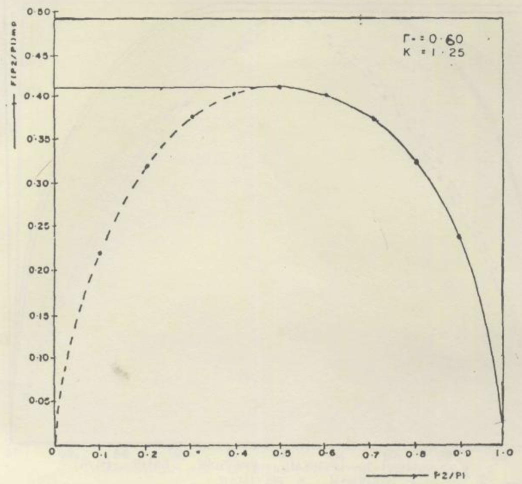

Figure 5 — Polytropic Flow

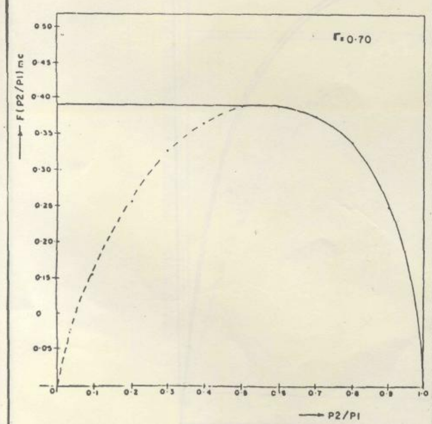

Figure 6 Isothermal Flow

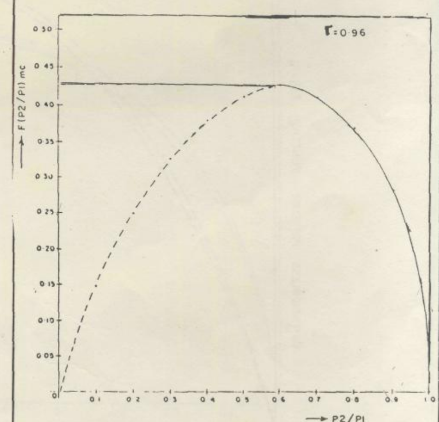

Figure 7 Isothermal Flow

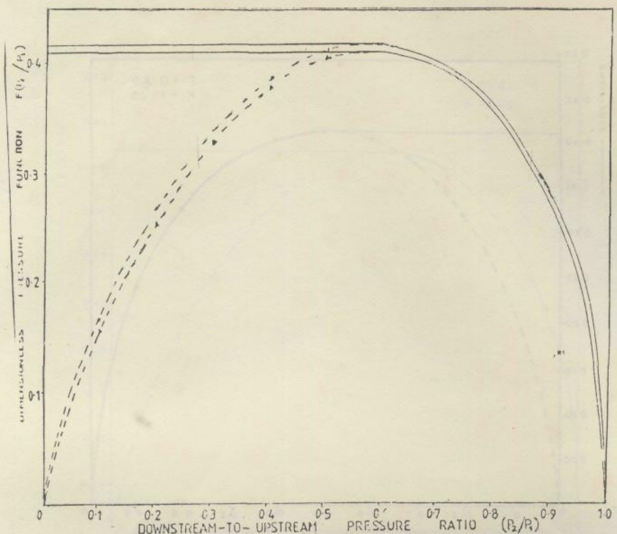

Figure 8 : ▲ ISOTHERMAL • POLYTROPIC
(P_1 = 560 psia, R_1 = 1000 SCF/STB , K = 1·04 , γg = 0·9)
(T_1 = 560 , B_o = 1·0 bbl/STB R_s = 0·0, WOR = 0 0)

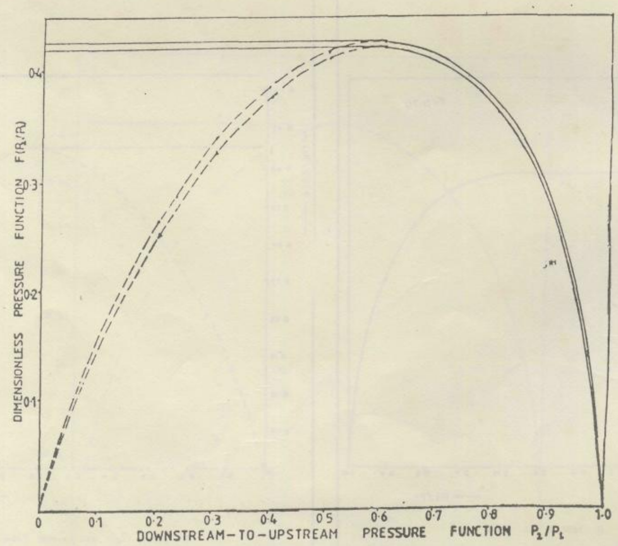

Figure 9 : ▲ –ISOTHERMAL . POLYTROPIC
(P_1 = 200 psia, R_1 = 1000 SCP/STB, K = 1·0'. γg = 0·9
T_1 = 560 °R, Bo = 1·01 bbl/STB, Rs = 0.0 WOR = 0·0)

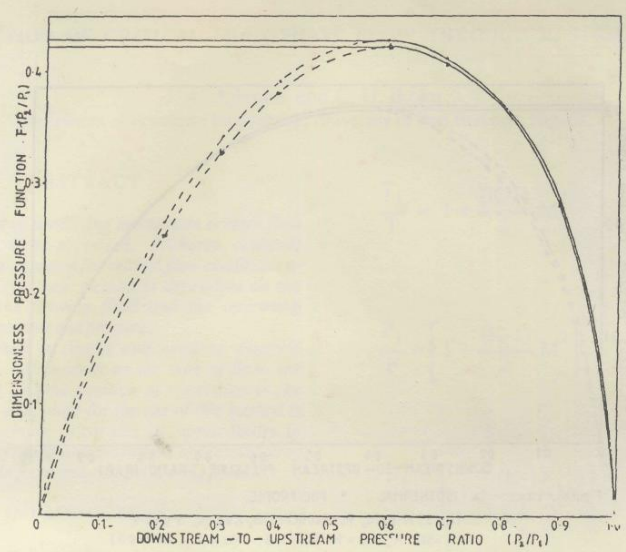

Figure 10 ▲ ISOTHERMAL, • POLYTROPIC
(P_1 = 100psia, R_1 = 500SCF/STB, K = 1·04, γg = 0·9
T_1 = 500 °R, Bo = 1·01 bbl/STB, Rs = 0·0, WOR = 0·0)

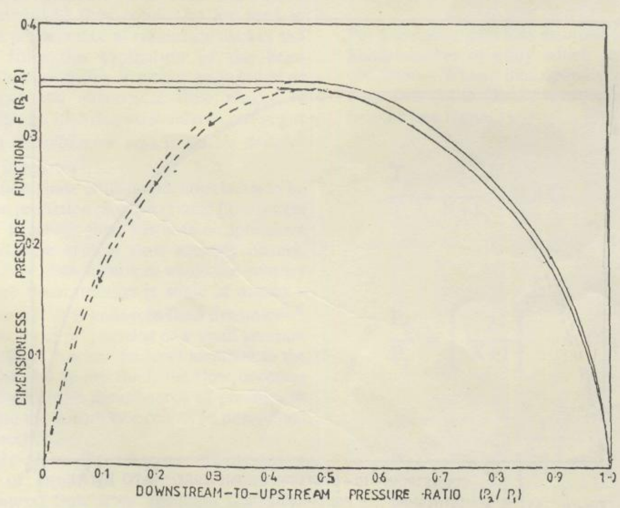

Figure 11 ▲—ISOTHERMAL, • POLYTROPIC
(P_1=2500psia, R_1 = 1000 SCF/STB, K=1·04, γ_g =0·9
T_1 =560 °R, Bo= 1·0 1 bbl/STB, Rs = 0·0, WOR = 0·0)

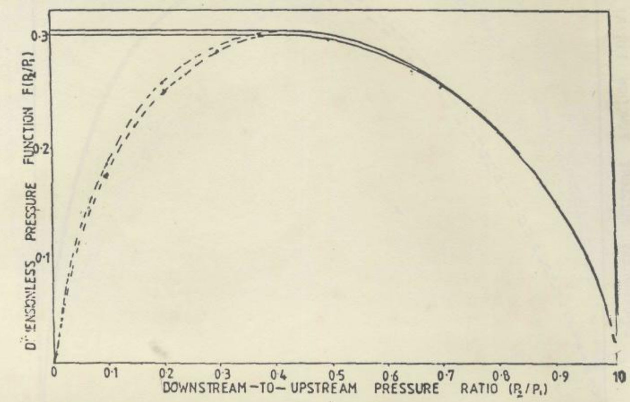

Figure - 12 ▲ ISOTHERMAL • POLYTROPIC
(P_1 = 2500 psia, R_1 = 500 SCP/STB, K = 1.04, γg = 0.9,
T_1 = 560 °R, Bo = 1.01 bbl/STB, Rs = 0.0, WOR = 0.0)

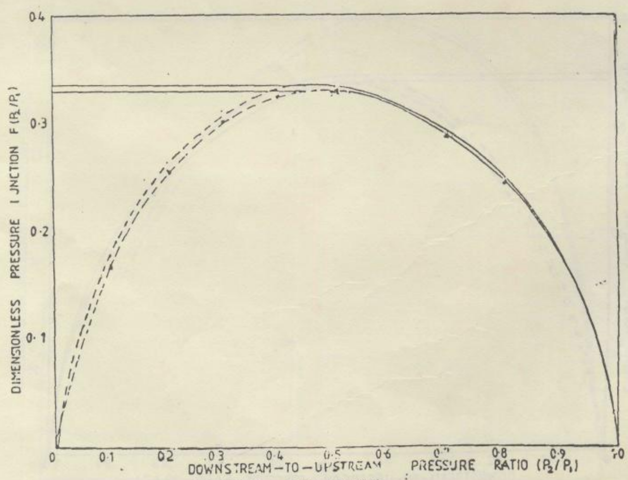

Figure 13 : ▲ ISOTHERMAL • POLYTROPIC
(P_1 = 1500 psia, R_1 = 500 SCP/STB, K = 1.04, γg = 0.9
T_1 = 560 °R, Bo = 1.01 bbl/STB, Rs = 0.0, WOR = 0.0)