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A STUDY ON NORMAL INTUITIONISTIC MULTI-FUZZY BG-IDEALS OF BG- ALGEBRA



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1.Introduction:

The notion of a fuzzy subset was initially introduced by Zadeh[13] in 1965, for representing uncertainty. The idea of Intuitionistic fuzzy set was first published by Atanassov [2], as a generalization of the notion of the fuzzy set. In 2000, S.Sabu and T.V.Ramakrishnan [10] proposed the theory of multi-fuzzy sets in terms of multi-dimensional membership function and investigated some properties of multi-level fuzziness. Theory of multi-fuzzy set is an extension of theory of fuzzy sets.

Y.Imai and K.Iseki introduced two classes of abstract algebras: BCK algebras and BCI-algebras[4]. It is shown that the BCK-algebras is a proper subclass of the class of BCI-algebras. C.B.Kim and

H.S.Kim[5] introduced the notion of a BG-algebra which is generalization of B-algebras. With these ideas, fuzzy subalgebras of BG-algebra were developed by S.S.Ahn and H.D.Lee[1]. R.Muthuraj and S.Devi[6,7,8] introduced the concept of multi-fuzzy subalgebra and intuitionistic multi-fuzzy subalgebras in BG-algebra in 2016. Also in 2017 they also introduced intuitionistic multi-fuzzy ideals in BG-algebra. In this paper, we define a new algebraic structure of normal intuitionistic multi-fuzzy ideals in BG-algebra and discuss some of their related properties. Also we investigate the properties of Cartesian product of intuitionistic multi fuzzy BG-ideals.

2.Preliminaries

Definition:2.1

A BG-algebra is a non-empty set X with a constant 0 and a binary operation $*$ satisfying the following axioms

- (i) $x * x = 0$
- (ii) $x * 0 = x$
- (iii) $(x * y) * (0 * y) = x \quad \forall x, y \in X$.

Definition:2.2

Let X be a non-empty set. An intuitionistic fuzzy set (IFS) A in X is a set of the form $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$, where $\mu_A : X \rightarrow [0, 1]$ and $\gamma_A : X \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively, with $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

Definition:2.3

Let X be a non-empty set. An intuitionistic multi fuzzy set A in X is a set of the form $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$, where $\mu_A(x) = (\mu_1(x), \mu_2(x), \dots, \mu_n(x))$, $\gamma_A(x) = (\gamma_1(x), \gamma_2(x), \dots, \gamma_n(x))$, and each $\mu_i : X \rightarrow [0, 1]$, $\gamma_i : X \rightarrow [0, 1]$ with $0 \leq \mu_i + \gamma_i \leq 1$, $x \in X$. Here $\mu_1(x) \geq \mu_2(x) \geq \dots \geq \mu_n(x)$, $x \in X$.

Definition:2.4

Let $A = \{(x, \mu_A(x), \gamma_A(x): x \in X\}$ and $B = \{(x, \mu_B(x), \gamma_B(x): x \in X\}$ be any two intuitionistic multi fuzzy sets having the same dimension of X . Then

(i) $A \subseteq B$ if and only if, $\mu_A(x) \leq \mu_B(x), \gamma_A(x) \leq \gamma_B(x)$, for all $x \in X$.

(ii) $A = B$ if and only if, $\mu_A(x) = \mu_B(x), \gamma_A(x) = \gamma_B(x)$, for all $x \in X$.

(iii) $A \cap B = \{(x, \mu_{A \cap B}(x), \gamma_{A \cap B}(x): x \in X\}$

$$\begin{aligned} \text{where } \mu_{A \cap B}(x) &= \min\{\mu_A(x), \mu_B(x)\} \\ &= (\min\{\mu_{iA}(x), \mu_{iB}(x)\})_{i=1}^n \end{aligned}$$

And

$$\begin{aligned} \text{where } \gamma_{A \cap B}(x) &= \max\{\gamma_A(x), \gamma_B(x)\} \\ &= (\max\{\gamma_{iA}(x), \gamma_{iB}(x)\})_{i=1}^n \end{aligned}$$

(iv) $A \cup B = \{(x, \mu_{A \cup B}(x), \gamma_{A \cup B}(x): x \in X\}$

$$\begin{aligned} \text{where } \mu_{A \cup B}(x) &= \max\{\mu_A(x), \mu_B(x)\} \\ &= (\max\{\mu_{iA}(x), \mu_{iB}(x)\})_{i=1}^n \end{aligned}$$

And

$$\begin{aligned} \text{where } \gamma_{A \cup B}(x) &= \min\{\gamma_A(x), \gamma_B(x)\} \\ &= (\min\{\gamma_{iA}(x), \gamma_{iB}(x)\})_{i=1}^n \end{aligned}$$

Definition:2.5

An intuitionistic multi fuzzy subset $A = \{(x, \mu_A(x), \gamma_A(x): x \in X\}$ in X is called an intuitionistic multi fuzzy subalgebra of X if it satisfies

(i) $\mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$

(ii) $\gamma_A(x * y) \leq \max\{\gamma_A(x), \gamma_A(y)\} \forall x, y \in X$.

Definition:2.6

Let A be a multi fuzzy set in X . Then A is called a multi fuzzy BG-Ideal in X if it satisfies the following condition

(i) $A(0) \geq A(x)$

(ii) $A(x) \geq \min\{A(x, y), A(y)\}$

(ii) $A(x * y) \geq \min\{A(x), A(y)\} \forall x, y \in X$

Definition:2.7

Let $A = (\mu_A, \gamma_A)$ be a multi fuzzy BG-Ideal in X . Then A is called an intuitionistic multi fuzzy BG-Ideal in X if it satisfies the following condition

(i) $\mu_A(0) \geq \mu_A(x)$

- (ii) $\mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\}$
- (iii) $\mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$
- (iv) $\gamma_A(0) \leq \gamma_A(x)$
- (v) $\gamma_A(x) \leq \max\{\gamma_A(x * y), \gamma_A(y)\}$
- (vi) $\gamma_A(x * y) \leq \max\{\mu_A(x), \mu_A(y)\} \forall x, y \in X$.

3.Normal Intuitionistic Multi-Fuzzy BG-Ideal of BG-algebra

Definition:3.1

An intuitionistic multi fuzzy BG-Ideal (μ_A, γ_A) in X is called normal, if there exist $x \in X$ such that $\mu_A(x) = 1_n = \gamma_A(x)$.where $1_n = (1, 1, \dots, n \text{ times})$

Lemma:3.2

An intuitionistic multi fuzzy BG-Ideal (μ_A, γ_A) in X is normal if and only if

$$\mu_A(0) = \gamma_A(0) = 1_n.$$

Theorem:3.3

Let $(\mu_A, \gamma_A) = A$ be a intuitionistic multi fuzzy set in a BG-algebra X. Then $A^+(x) = A(x) + 1_n - A(0) \forall x \in X$ is a intuitionistic normal multi fuzzy BG-Ideal of X which contain A.

Proof:

Let $x, y \in X$

$$\begin{aligned} \mu_A^+(x) &= \mu_A(x) + 1 - \mu_A(0) \\ &\leq \mu_A(0) + 1 - \mu_A(0) \\ &= \mu_A^+(0) \end{aligned}$$

$$\begin{aligned} \min\{\mu_A^+(x * y), \mu_A^+(y)\} &= \min\{\mu_A(x * y) + 1 - \mu_A(0), \mu_A(y) + 1 - \mu_A(0)\} \\ &= \min\{\mu_A(x * y), \mu_A(y)\} + 1 - \mu_A(0) \\ &\leq \mu_A(x) + 1 - \mu_A(0) \\ &= \mu_A^+(x) \end{aligned}$$

$$\begin{aligned} \min\{\mu_A^+(x), \mu_A^+(y)\} &= \min\{\mu_A(x) + 1 - \mu_A(0), \mu_A(y) + 1 - \mu_A(0)\} \\ &= \min\{\mu_A(x), \mu_A(y)\} + 1 - \mu_A(0) \\ &\leq \mu_A(x * y) + 1 - \mu_A(0) \\ &= \mu_A^+(x * y) \end{aligned}$$

Clearly this can be proved for maximum condition also

Therefore, $\gamma_A^+(x) \geq \gamma_A^+(0)$

$\max\{\gamma_A^+(x * y), \gamma_A^+(y)\} \geq \gamma_A^+(x)$

$\max\{\gamma_A^+(x), \gamma_A^+(y)\} \geq \gamma_A^+(x * y)$

Hence $(\mu_A, \gamma_A)^+ = A^+$ is a intuitionistic normal multi fuzzy BG-Ideal of X.

Clearly $A \subseteq A^+$

Thus (μ_A^+, γ_A^+) is a intuitionistic normal multi fuzzy BG-Ideal which contain (μ_A, γ_A) .

Corollary 3.4

If there is an element $x \in X$ such that $(\mu_A^+(x), \gamma_A^+(x)) = (0, 0)$ then $(\mu_A(x), \gamma_A(x)) = (0, 0)$

Proof:

Since $A \subseteq A^+$ then $A(x) = 0$

That is, $\mu_A(x) = 0$

Similarly, $\gamma_A(x) = 0$

Corollary 3.5

(i) If (μ_A, γ_A) itself is normal then $\mu_A = \mu_A^+, \gamma_A = \gamma_A^+$.

(ii) If μ_A is a intuitionistic multi fuzzy BG-Ideal in X then $(\mu_A^+)^+ = \mu_A^+$, Also γ_A is a intuitionistic multi fuzzy BG-Ideal in X then $(\gamma_A^+)^+ = \gamma_A^+$.

Theorem:3.6

Let μ_A be a intuitionistic multi fuzzy BG-Ideal in X. If there exist a intuitionistic multi fuzzy BG-Ideal γ_A of X such that $\gamma_A^+ \subseteq \mu_A$. then μ_A is normal.

Proof:

Since γ_A be a intuitionistic fuzzy BG-Ideal in X.

γ_A^+ is a normal intuitionistic multi fuzzy BG-Ideal of X.

Then

$$\gamma_A^+(0) = 1_n$$

This implies $\gamma_A^+(0) = 1$

$$\gamma_A^+(0) \subseteq \mu_A$$

Implies $\gamma_A(x) \leq \mu_A(x) \forall, x \in X$

$$1 = \gamma_A^+(0) \leq \mu_A(0)$$

Thus $1_n = \gamma_A^+(0) \leq \mu_A^+(0)$

Hence μ_A is normal.

Theorem 3.7

Let (μ_A, γ_A) be a intuitionistic multi fuzzy BG-Ideal and let $f: [0, A(0)] \rightarrow [0, 1]$ be an increasing function. Then a multi fuzzy set $\mu_A^f: X \rightarrow [0, 1]$ by $\mu_A^f(x) = f(\mu_A(x)) \forall x \in X$ is a intuitionistic fuzzy BG-Ideal in X. Also if $f(\mu_A(0)) = 1_n$ then (μ_A^f, γ_A^f) is normal.

Proof:

Since f is an increasing function

$$\begin{aligned}\mu_A^f(0) &= f(\mu_A(0)) \\ &\geq f(\mu_A(x)) \\ &= \mu_A^f(x)\end{aligned}$$

We have $\mu_A^f(x) = f(\mu_A(x))$

$$\begin{aligned}&\geq f(\min\{\mu_A(x * y), \mu_A(y)\}) \\ &= \min\{f(\mu_A(x * y)), f(\mu_A(y))\} \\ &= \min\{\mu_A^f(x * y), \mu_A^f(y)\}\end{aligned}$$

And $\mu_A^f(x * y) = f(\mu_A(x * y))$

$$\begin{aligned}&\geq f(\min\{\mu_A(x), \mu_A(y)\}) \\ &= \min\{f(\mu_A(x)), f(\mu_A(y))\} \\ &= \min\{\mu_A^f(x), \mu_A^f(y)\}\end{aligned}$$

Similarly,

$$\begin{aligned}\gamma_A^f(0) &\leq \gamma_A^f(x) \\ \gamma_A^f(x) &\leq \max\{\gamma_A^f(x * y), \gamma_A^f(y)\} \\ \gamma_A^f(x * y) &\leq \max\{\gamma_A^f(x), \gamma_A^f(y)\}\end{aligned}$$

This implies (μ_A^f, γ_A^f) is a intuitionistic multi fuzzy BG-Ideal in X.

$$\text{If } f(\mu_A(0)) = f(\gamma_A(0)) = 1_n$$

$$\mu_A^f(x) = \gamma_A^f(x) = 1_n$$

Therefore, (μ_A^f, γ_A^f) is normal.

4. Cartesian product of Intuitionistic Multi fuzzy BG-Ideals

In this section, the Cartesian product of intuitionistic multi fuzzy BG-Ideals of BG-algebra is defined and its properties are discussed.

Definition:4.1

Let μ_A and μ_B be two membership function and γ_A and γ_B be two nonmembership function of each $x \in X$. Then $\mu_{A \times B}(x, y)$ is membership function and $\gamma_{A \times B}(x, y)$ is non membership function of each element $(x, y) \in X \times X$ to the set $A \times B$ and defined by,

$$\mu_{A \times B}(x, y) = \min\{\mu_{A \times B}(x), \mu_{A \times B}(y)\}$$

$$\gamma_{A \times B}(x, y) = \max\{\gamma_{A \times B}(x), \gamma_{A \times B}(y)\}$$

Theorem 4.2

Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ are two intuitionistic multi fuzzy BG-Ideals of X , then $A \times B = (\mu_{A \times B}, \gamma_{A \times B})$ also a intuitionistic multi fuzzy BG-Ideals of $X \times X$.

Soln:

(i)

$$\mu_{A \times B}(0, 0) = \min\{\mu_A(0), \mu_B(0)\}$$

$$\geq \min\{\mu_A(x), \mu_B(x)\}$$

$$= \mu_{A \times B}(x, x)$$

$$\mu_{A \times B}(0, 0) \geq \mu_{A \times B}(x, x)$$

(ii) $\mu_{A \times B}(x, x) = \min\{\mu_A(x), \mu_B(x)\}$

$$\geq \min\{\min\{\mu_A(x * y), \mu_A(y)\}, \min\{\mu_B(x * y), \mu_B(y)\}\}$$

$$= \min\{\min\{\mu_A(x * y), \mu_B(x * y)\}, \min\{\mu_A(y), \mu_B(y)\}\}$$

$$\geq \min\{\mu_{A \times B}(x * y), \mu_{A \times B}(y)\}$$

(iii) $\mu_{A \times B}(x, y) = \min\{\mu_{A \times B}(x), \mu_{A \times B}(y)\}$

$$\geq \min\{\min\{\mu_A(x), \mu_B(x)\}, \min\{\mu_A(y), \mu_B(y)\}\}$$

$$= \min\{\mu_{A \times B}(x), \mu_{A \times B}(y)\}$$

Similarly,

(iv) $\gamma_{A \times B}(0, 0) \leq \gamma_{A \times B}(x, x)$

(v) $\gamma_{A \times B}(x, x) \leq \max\{\gamma_{A \times B}(x * y), \gamma_{A \times B}(y)\}$

(vi) $\gamma_{A \times B}(x, y) \leq \max\{\gamma_{A \times B}(x), \gamma_{A \times B}(y)\}$

Hence $A \times B = (\mu_{A \times B}, \gamma_{A \times B})$ is a intuitionistic multi fuzzy BG-Ideals of $X \times X$.

Theorem 4.3

Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ are two intuitionistic multi fuzzy set in a BG-algebra of X such that $A \times B = (\mu_{A \times B}, \gamma_{A \times B})$ is a intuitionistic multi fuzzy BG-Ideals of $X \times X$.

Then (i) Either $\mu_{A \times B}(0) \geq \mu_{A \times B}(x)$ or $\gamma_{A \times B}(0) \geq \gamma_{A \times B}(x)$

(ii) If $\mu_{A \times B}(0) \geq \mu_{A \times B}(x)$. Then either $\gamma_{A \times B}(0) \geq \gamma_{A \times B}(x)$ or $\gamma_{A \times B}(0) \geq \gamma_{A \times B}(x)$

(iii) $\gamma_{A \times B}(0) \geq \gamma_{A \times B}(x)$. Then either $\mu_{A \times B}(0) \geq \mu_{A \times B}(x)$ or $\mu_{A \times B}(0) \geq \gamma_{A \times B}(x)$

Proof:

$$\text{Let } \mu_{A \times B}(0) < \mu_{A \times B}(x) \text{ and } \gamma_A(0) < \gamma_A(x)$$

Then $\mu_{(A \times B)_i}(0) < \mu_{(A \times B)_i}(x)$ and $\gamma_{(A \times B)_i}(0) < \gamma_{(A \times B)_i}(x) \forall i = 1, 2, \dots, n$

$$\begin{aligned}\mu_{A \times B}(x, x) &= \min\{\mu_A(x), \mu_B(x)\} \\ &\geq \min\{\mu_A(0), \mu_B(0)\} \\ &\geq \mu_{A \times B}(0, 0)\end{aligned}$$

Which is a contradiction that $\mu_{A \times B}$ is a intuitionistic multi fuzzy BG-Ideals of $X \times X$.

Therefore either $\mu_{A \times B}(0) \geq \mu_{A \times B}(x)$ or $\gamma_{A \times B}(0) \geq \gamma_{A \times B}(x)$

Let $\gamma_{A \times B}(0) < \mu_{A \times B}(x)$ and $\gamma_{A \times B}(0) < \gamma_{A \times B}(x)$. then $\gamma_{(A \times B)_i}(0) < \mu_{(A \times B)_i}(x)$ and $\gamma_{(A \times B)_i}(0) < \gamma_{(A \times B)_i}(x) \forall i = 1, 2, \dots, n$

$$\begin{aligned}\mu_{A \times B}(0, 0) &= \min\{\mu_A(0), \mu_B(0)\} \\ &\geq \min\{\mu_A(x), \mu_B(x)\} \\ &\geq \min\{\gamma_A(0), \gamma_B(0)\} \\ &= \gamma_{A \times B}(0, 0) \\ \mu_{A \times B}(x, x) &= \min\{\mu_A(x), \mu_B(x)\} \\ &\geq \min\{\mu_A(0), \mu_B(0)\} \\ &\geq \min\{\gamma_A(0), \gamma_B(0)\} \\ &= \gamma_{A \times B}(0, 0)\end{aligned}$$

This implies that $\mu_{A \times B}(x, x) \geq \mu_{A \times B}(0, 0)$

Which is a contradiction that $A \times B$ is a intuitionistic multi fuzzy BG-Ideals of $X \times X$.

Therefore either $\mu_{A \times B}(x) \geq \mu_{A \times B}(x)$

Then either $\gamma_{A \times B}(0) \geq \mu_{A \times B}(x)$ or $\gamma_{A \times B}(0) \geq \gamma_{A \times B}(x)$

The Third proof is similar to second proof.

Theorem 4.4

If $A \times B = \mu_A \times \gamma_A$ is a intuitionistic multi fuzzy BG-Ideals of $X \times X$, then μ_A or γ_A is a intuitionistic multi fuzzy BG-Ideals of X .

Proof:

To Prove that γ_A is a intuitionistic multi fuzzy BG-Ideals of X .

By the above thrm

(i) Either $\mu_A(0) \geq \mu_A(x)$ or $\gamma_A(0) \geq \gamma_A(x)$

Assume that $\gamma_A(0) \geq \gamma_A(x)$

Also from the above thrm

(iii) Either $\mu_A(0) \geq \mu_A(x)$ or $\mu_A(0) \geq \gamma_A(x)$,

Then $\mu_A(0) \geq \gamma_A(x)$

$$\begin{aligned}(\mu_A \times \gamma_A)(0, x) &= \min\{\mu_A(0), \gamma_A(x)\} \\ &= \gamma_A(x) \\ &= (\mu_A \times \gamma_A)(0, x) \\ &\geq \min\{(\mu_A \times \gamma_A)((0, x) * (0, y)), (\mu_A \times \gamma_A)(0, y)\} \\ &= \min\{\mu_A \times \gamma_A((0, 0), (x * y)), \mu_A \times \gamma_A(0, y)\} \\ &= \min\{\mu_A(x * y), \gamma_A(y)\}\end{aligned}$$

$$\begin{aligned}\text{Also } \gamma_A(x * y) &= (\mu_A \times \gamma_A)(0, x * y) \\ &= (\mu_A \times \gamma_A)(0 * 0, x * y) \\ &= (\mu_A \times \gamma_A)(0 * x, 0 * y) \\ &= \min\{(\mu_A \times \gamma_A)(0 * x), (\mu_A \times \gamma_A)(0 * y)\} \\ &= \min\{\gamma_A(x), \gamma_A(y)\}\end{aligned}$$

Hence γ_A is a intuitionistic multi fuzzy BG-Ideals of X .

To Prove that μ_A is a intuitionistic multi-fuzzy BG-Ideals of X .

By the above thrm

(i) Either $\mu_A(0) \geq \mu_A(x)$ or $\gamma_A(0) \geq \gamma_A(x)$

Assume that $\mu_A(0) \geq \mu_A(x)$

Also from the above thrm

(iii) Either $\gamma_A(0) \geq \mu_A(x)$ or $\gamma_A(0) \geq \gamma_A(x)$,

If $\gamma_A(0) \geq \mu_A(x)$ then $\gamma_A(0) \geq \mu_A(x)$

$$\begin{aligned}(\mu_A \times \gamma_B)(0, 0) &= \min\{\mu_A(0), \gamma_B(0)\} \\ &= \mu_A(0) \\ \mu_A(x) &= (\mu_A \times \gamma_A)(x, 0) \\ &\geq \min\{(\mu_A \times \gamma_A)((x, 0) * (y, 0)), (\mu_A \times \gamma_A)(y, 0)\} \\ &= \min\{(\mu_A \times \gamma_A)((x * y), (0, 0)), (\mu_A \times \gamma_A)(y, 0)\} \\ &= \min\{\mu_A(x * y), \mu_A(y)\}\end{aligned}$$

$$\begin{aligned}\text{Also } \mu_A(x * y) &= (\mu_A \times \gamma_A)(x * y, 0) \\ &= (\mu_A \times \gamma_A)(x * y, 0) \\ &= (\mu_A \times \gamma_A)(x * y, 0 * 0) \\ &= \min\{(\mu_A \times \gamma_A)(x, 0), (\mu_A \times \gamma_A)(y, 0)\}\end{aligned}$$

$$= \min\{\mu_A(x), \mu_A(y)\}$$

Hence μ_A is a intuitionistic multi-fuzzy BG-Ideals of X .

Clearly, this can be proved for maximal condition also

Hence the result proved

Theorem 4.5 Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ are two normal intuitionistic multi fuzzy BG-Ideal of X such that $A \times B = (\mu_{A \times B}, \gamma_{A \times B})$ is a normal intuitionistic multi-fuzzy BG-Ideals of $X \times X$.

Proof:

Let (μ_A, γ_A) and (μ_B, γ_B) are two normal intuitionistic multi fuzzy BG-Ideal of X .

To Prove: $A \times B$ is a normal intuitionistic multi-fuzzy BG-Ideals of $X \times X$.

We Know that $\mu_A(0) = 1_n = 1, 1, \dots, n$ times

$$\mu_B(0) = \gamma_A(0) = \gamma_B(0) = 1_n = 1, 1, \dots, n \text{ times}$$

$$\mu_{A \times B}(0) = 1 = \gamma_{A \times B}$$

$$\mu_{A \times B}(0,0) = \min\{\mu_A(0), \mu_B(0)\}$$

$$= \min\{1,1\}$$

$$= 1_n(1,1, \dots, n \text{ times})$$

$$\gamma_{A \times B}(0,0) = \max\{\gamma_A(0), \gamma_B(0)\}$$

$$= \max\{1,1\}$$

$$= 1_n(1,1, \dots, n \text{ times})$$

Hence $A \times B$ is a normal intuitionistic multi-fuzzy BG-Ideals of $X \times X$.

5. Conclusion

In this paper the normal intuitionistic multi-fuzzy BG-ideal of BG-algebra and also the cartesian product of intuitionistic multi fuzzy BG-ideals of BG-algebra is defined and its properties are discussed.

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