A Novel Ranking Approach for Solving the Fuzzy LPP and Fuzzy DEA Model

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Abstract:

The ranking approach is one of the most used techniques in solving the fuzzy optimization problems. This article defines a novel ranking function to compare the trapezoidal fuzzy numbers (TrFNs).The fuzzy linear programming problem (FLPP) and fuzzy data envelopment analysis (FDEA) models are converted into its corresponding crisp LP problem and crisp DEA models. The ranking function associated with the risk factors that represent the decision maker's attitude towards taking the risk. The crisp LP problem solves with the different risk factors to obtain the optimum solution, and the crisp DEA model solves with the different risk factors to get the comparative efficiency scores of the DMUs. The DMUs are ranked and classified into efficient and inefficient groups according to the efficiency score obtained in crisp DEA model. Two different numerical examples are given to establish the existence and usability of the suggested methodologies for FLPP and FDEA models.

Keywords: Fuzzy Set, Linear Programming Problem, Data Envelopment Analysis, Ranking Function, Trapezoidal Fuzzy Numbers, Risk Factor.

1. Introduction

An essential tool for decision makers (DMs) is the linear programming (LP) problem to solve the practical optimization problems in the real world. The LP model is a technique for determining optimum solutions by maximizing or minimizing the linear objective function under various constraints. It has been used extensively to solve a variety of issues in business, economics, engineering, as well as in industries including “telecommunications, manufacturing, production, transportation, energy, finance, and marketing”. For LP problems, well-defined data with a higher information cost are needed but are not always possible to find. Optimization problems have some degree of imprecision and ambiguity. In modeling these problems, such phenomena have been addressed using fuzzy sets. However, the accuracy of data in real-world situations is generally false, which has an impact on the best solution to LP problems. Probability distributions were unable to deal with erroneous and ambiguous data. Since the actual world usually contains some degree of uncertainty, optimization under uncertainty is now one of the most intriguing topics. Fuzzy linear programming (FLP)problem [1] is a kind of LPP where the parameters or decision variables are expressed as fuzzy numbers. Many researchers investigated FLLPs and proposed various type of solution techniques [2].The trapezoidal FLPP (TrFLPP) is a kind of LPP where the parameters are expressed by TrFNs [3,4,5,6,7].The ranking technique is an essential tool for comparing fuzzy numbers and solving fuzzy optimization problems[8,9,10].

DEA is a nonparametric, data-driven, LP approach used to estimate the best practice frontier that defines the performance of decision-making units (DMUs). The DMU that lies on the frontier line is called an efficient DMU; otherwise, the DMU is called an inefficient DMU. Initially, Charnes et al. [11]developed this technique based on Farrell's concept. All the DEA models are developed based on two concepts (1) Minimize the inputs of the DMUs in such a way as to maintain the output levels and (2) Maximize the output using the available inputs. Most of the DEA models are developed based on the Constant return scale (CRS) [11] or Variable return scale (VRS)[12]. As a result of the significant growth of this topic and its application to numerous real-world situations, a numerous of research papers have been published and appeared in bibliographic database [13,14,15,16,17,18]. The data used in traditional DEA models must be in precise/crisp form. However, finding all the data in a crisp form in practical applications is never always possible. The combination of DEA and FS theory are used to calculate the efficiency score of DMUs in the context of imprecise and ambiguous data. The first time, Sengupta [19] 1992 used fuzzy input-output in the DEA model to measure the performance of the DMUs. Following that, a number of researchers developed various techniques to solve fuzzy DEA models, which grew significantly and appeared in bibliographical database [20,21]. Hatami-Marbini et al. [22] effectively reviewed and classified the fuzzy DEA approach into four categories. Then, Emrouznejad et al. [23] has added two more categories, which are "(1) tolerance technique, (2) fuzzy ranking approach, (3) α-level based approach, (4) possibility approach, (5) fuzzy arithmetic, and (6) fuzzy random/type-2 fuzzy set".

The following are the motivations for this research paper:

1. A novel ranking function is defined to compare the TrFNs.
2. The ranking technique is used to convert the fuzzy LPP into its corresponding crisp LPP.
3. The optimum solution of a crisp LPP is found with different risk factors and study the effect of risk factor in optimum solution.
4. The FDEA model converted into its corresponding crisp DEA model by using the proposed ranking technique.
5. The efficiency scores of the DMUs are obtained by solving the crisp model with different risk factors.
6. The risk factor represents attitude of decision maker whether he is a risk taker or risk averse.

The remaining portions of this study are organized as follows:

Section (2) introduces the essential concepts of fuzzy sets, the trapezoidal fuzzy numbers (TrFNs), and its arithmetic operation and defines a novel ranking function. Section (3) describes the structure of the general fuzzy LP problem. Section (4) discusses the general form of fuzzy

DEA model. Section (5) discusses the algorithm for solving FLPP and FDEA models. Two numerical examples are presented in section (6) to demonstrate the relevance and validity of the concept. Finally, Section (7) draws the conclusion and focus on the direction of future study.

2. Preliminary

Definition 1 [24] Let U be a universal set. A fuzzy set F in U is given by

$$F = \{\langle x, \mu_F(x)\rangle : x \in U\}, (1)$$

where $\mu_F : U \to [0,1]$ is the truth-membership grade. The falsity-membership grade of any $x \in F$ is defined as $\overline{\mu_F}(x) = 1 - \mu_F(x)$.

Definition 2 [25] A Trapezoidal fuzzy number (TrFNs) is denoted by $\hat{X} = \langle x^I, x^{II}, x^{III}, x^{IV}\rangle$ and its generalized membership grade of x is defined as

$$\mu_X(x) = \begin{cases} \dfrac{x - x^I}{x^{II} - x^I}\phi, if \quad x \in [x^I, x^{II}] \\ \phi, \quad if \quad x = [x^{II}, x^{III}] \\ \dfrac{x^{IV} - x}{x^{IV} - x^{III}}\phi, if \quad x \in [x^{III}, x^{IV}] \\ 0, \ otherwise \end{cases} \quad (2)$$

where $\phi \in [0,1]$.

Definition 3 Suppose $\widehat{X_1} = \langle x_1^I, x_1^{II}, x_1^{III}, x_1^{IV}\rangle$, and $\widehat{X_2} = \langle x_2^I, x_2^{II}, x_2^{III}, x_2^{IV}\rangle$, be two TrFNs. The arithmetic relationships are defined as follows:

1. $\widehat{X_1} \oplus \widehat{X_2} = \langle x_1^I + x_2^I, x_1^{II} + x_2^{II}, x_1^{III} + x_2^{III}, x_1^{IV} + x_2^{IV}\rangle$.
2. $\widehat{X_1} - \widehat{X_2} = \langle x_1^I - x_2^{IV}, x_1^{II} - x_2^{III}, x_1^{III} - x_2^{II}, x_1^{IV} - x_2^{I}\rangle$.
3. $\widehat{X_1} \otimes \widehat{X_2} = \langle x_1^I x_2^I, x_1^{II} x_2^{II}, x_1^{III} x_2^{III}, x_1^{IV} x_2^{IV}\rangle$
4. $\lambda\widehat{X_1} = \begin{cases} \langle \lambda x_1^{\mathrm{I}}, \lambda x_1^{II}, \lambda x_1^{III}, \lambda x_1^{IV}\rangle, & \lambda > 0. \\ \langle \lambda x_1^{IV}, \lambda x_1^{III}, \lambda x_1^{II}, \lambda x_1^{I}\rangle, & \lambda < 0. \end{cases}$
5. $\widehat{X_1}^{\lambda} = \left\langle {x^I}_1^{\lambda}, {x^{II}}_1^{\lambda}, {x^{III}}_1^{\lambda}, {x^{IV}}_1^{\lambda}\right\rangle$.

Definition 4 The $\alpha-$cut of a TrFN $\hat{X} = \langle x^I, x^{II}, x^{III}, x^{IV}\rangle$ is defined as

$$[L(\alpha), U(\alpha)] = \left[x^I + \frac{\alpha}{\phi}(x^{II} - x^I), x^{IV} - \frac{\alpha}{\phi}(x^{IV} - x^{III})\right] (3)$$

where $0 \leq \alpha \leq \phi$, $L(\alpha)$ and $U(\alpha)$ are the lower and upper bound.

Definition 5 The ranking function R for a TrFN $\hat{X}$ is defined as

$$R(\hat{X}) = \frac{\int_0^{\phi}[\lambda L(\alpha) + (1-\lambda)U(\alpha)]f(\alpha)d\alpha}{\int_0^{\phi} f(\alpha)d\alpha} \quad (4)$$

Where $\lambda \in [0,1]$ represent the risk taker's attitude.

1. When $\lambda < \frac{1}{2}$, the decision is optimistic.
2. When $\lambda = \frac{1}{2}$, the decision is neutral.
3. When $\lambda > \frac{1}{2}$, the decision is pessimistic.

Throughout the manuscript, we have taken $f(\alpha) = \alpha$ in equation (4). Then the ranking function will be

$$R(\hat{X}) = \lambda\left(\frac{x^{I} + 2x^{II}}{3}\right) + (1-\lambda)\left(\frac{x^{IV} + 2x^{III}}{3}\right) \quad (5)$$

Note: For any real crisp number $x \in R$ can be represented as a TrFN $\hat{x} = \langle x, x, x, x \rangle$ such that $R(\hat{x}) = \lambda\left(\frac{x+2x}{3}\right) + (1-\lambda)\left(\frac{x+2x}{3}\right) = x$, that is independent of the risk factor.

Theorem 1 Let $\widehat{K_t} = \langle k_t^{I}, k_t^{II}, k_t^{III}, k^{IV}{}_t \rangle$, for $t = 1,2,\cdots,p$ be the pTrFNs. The ranking function satisfies the following

1. $R(\widehat{K_1} + \widehat{K_2}) = R(\widehat{K_1}) + R(\widehat{K_2})$
2. $R(\widehat{K_1} - \widehat{K_2}) = R(\widehat{K_1}) - R(\widehat{K_2})$
3. $R(\widehat{K_1}) \leq R(\widehat{K_2})$ *iff* $\widehat{K_1} \preccurlyeq \widehat{K_2}$.
4. $R(\widehat{K_1}) \geq R(\widehat{K_2})$ iff $\widehat{K_1} \succcurlyeq \widehat{K_2}$.
5. $R(\widehat{K_1}) = R(\widehat{K_2})$ *iff* $\widehat{K_1} \approx \widehat{K_2}$.
6. $R(\sum_{t=1}^{p} \widehat{K_t}) = \sum_{t=1}^{p} R(\widehat{K_t})$
7. $R(\sum_{t=1}^{p} \psi_t \widehat{K_t}) = \sum_{t=1}^{p} \psi_t R(\widehat{K_t})$

Proof.: This can be proved using the ranking function defined in equation (5) which is satisfy linearity condition.

3. Fuzzy Linear Programming Problem (FLPP)

The general FLP problem (GFLPP) uses fuzzy numbers to represent each parameter.

$$\text{Min/ Max}Z = \sum_{j=1}^{n} \widehat{c_j} x_j$$

$$Subject\ to \sum_{j=1}^{n} \widehat{a_{ij}} x_j \leq = \geq \ \widehat{b_i}\,; i = 1,2,\cdots,m.$$

$$and\ x_j \geq 0. j = 1,2,3,\cdots,n.$$

Sometimes in the real world, some of the observed data are occasionally missing and confusing, not all data. In this situation, the LP model is called Mixed FLPP(Mix-FLPP), where the parameters are in crisp or fuzzy. n a real-world application, the decision maker made the export choice based on the incomplete, ambiguous, and uncertain knowledge of the LP problem parameters, which were expressed in terms of trapezoidal fuzzy numbers (TrFNs).The optimum solution to the problem is expected to be crisp optimum solution. Therefore, we need to develop a technique to solve the FLP problem or to covert the FLP problem to corresponding crisp LP problem which can be solve using any existing technique.

4. Fuzzy Data Envelopment Analysis (FDEA)

Various models have been presented in DEA to evaluate the performance of the DMUs in a different situation. The most popular and valuable models in DEA are the CCR model [11] and the BCC model [12]. The significant difference between the CCR and BCC models are based on CRS and VRS, respectively. Let us consider there are n DMUs having m inputs and s outputs. The $i^{th}(1 \leq i \leq m)$input and $r^{th}, (1 \leq r \leq s)$ output of the $j^{th}(1 \leq j \leq n)$ DMU is represented by x_{ij} and y_{rj} respectively. Following are the multiplier form and envelopment form of the CCR model.

Multiplier Form

$$\text{Max } \theta = \sum_{r=1}^{s} u_r y_{ro}$$

$$s.t. \sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \forall\, j$$

and $u_r, v_i \geq 0, \forall\, r, i.$

Envelopment Form

Min θ

$$s.t.\ \theta\, x_{io} - \sum_{j=1}^{n} \lambda_j x_{ij} \geq 0, \forall i$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \forall r$$

and $\lambda_j \geq 0.$

The Corresponding Fuzzy CCR model is defined as

Multiplier Form

$$\text{MNax } \theta = \sum_{r=1}^{s} u_r\, \widehat{y_{ro}}$$

$$s.t. \sum_{i=1}^{m} v_i \widehat{x_{io}} = 1,$$

$$\sum_{r=1}^{s} u_r \widehat{y_{rj}} - \sum_{i=1}^{m} v_i \widehat{x_{ij}} \leq 0, \forall\, j$$

Envelopment Form

Min θ

$$s.t.\ \theta\, \widehat{x_{io}} \geq \sum_{j=1}^{n} \lambda_j \widehat{x_{ij}}, \forall i$$

$$\sum_{j=1}^{n} \lambda_j \widehat{y_{rj}} \geq \widehat{y_{ro}}\ , \forall r$$

and $\lambda_j \geq 0.$

and $u_r, v_i \geq 0, \forall\, r, i.$

Here the inputs and outputs data are TrFNs, i.e., $\widehat{x_{ij}} = \langle x_{ij}^{I}, x_{ij}^{II}, x_{ij}^{III}, x_{ij}^{IV} \rangle$ and $\widehat{y_{rj}} = \langle y_{rj}^{I}, y_{rj}^{II}, y_{rj}^{III}, y_{rj}^{IV} \rangle$ for $i = 1,2,\cdots,m; j = 1,2,\cdots,n$ and $r = 1,2,\cdots,s$.In FDEA, The DMUs are categories into efficient and inefficient group according to their efficiency score θ^*.

Definition 6: A DMU is said to be efficient if its efficiency score(θ^*) is 1; Otherwise, it is considered as inefficient DMU.

5. Algorithms for Solving FLPP and FDEA Model

Two algorithms are given to solve the FLP problem and FDEA model.

5.1. Algorithm 1

The steps for solving the fuzzy LP problem are given below.

1. Formulate the Fuzzy LP problem based on expert decision.
2. If the FLP problem is mixed type, then convert it into general FLP (GFLP) problem form.
3. Covert the GFLP problem into the corresponding crisp LP problem using the Ranking function.
   The GFLPP becomes

$$\text{Min / Max } R(Z) = R\left(\sum_{j=1}^{n} \widehat{c_j} x_j\right)$$

$$Subject\ to\ R\left(\sum_{j=1}^{n} \widehat{a_{ij}} x_j\right) \leq = \geq \ R\left(\widehat{b_i}\right); i = 1,2,\cdots,m$$

$$and\ x_j \geq 0,\ j = 1,2,3,\cdots,n.$$

which implies that

$$\text{Min/ Max } Z = \sum_{j=1}^{n}\left[\lambda\left(\frac{c_j^{I} + 2c_j^{II}}{3}\right) + (1-\lambda)\left(\frac{c_j^{IV} + 2c_j^{III}}{3}\right)\right] x_j$$

$$Subject\ to \sum_{j=1}^{n}\left[\lambda\left(\frac{a_{ij}^{I} + 2a_{ij}^{II}}{3}\right) + (1-\lambda)\left(\frac{a_{ij}^{IV} + 2a_{ij}^{III}}{3}\right)\right] x_j \leq =$$

$$\geq \left[\lambda\left(\frac{b_i^{I} + 2b_i^{II}}{3}\right) + (1-\lambda)\left(\frac{b_i^{IV} + 2b_i^{III}}{3}\right)\right]; i = 1,2,\cdots,m$$

$$and\ x_j \geq 0.\ j = 1,2,3,\cdots,n.$$

which is a crisp LPP.

4. Solve the above crisp LP problem using any existing technique and obtain the optimum solution with the different risk levels.

5.2. Algorithm 2

The steps for solving the FDEA model are given.

1. The Fuzzification technique is used to convert the observed vague input-output data into trapezoidal fuzzy numbers.
2. Formulate the FDEA model based on the obtained trapezoidal fuzzy inputs and outputs.
3. Use the suggested Ranking function to transform the FDEA model into the corresponding crisp DEA model.

From Theorem 1, The Fuzzy CCR multiplier model is transformed to corresponding crisp CCR model.

$$\text{Max } \theta = \sum_{r=1}^{s}\left[\lambda\left(\frac{y_{ro}^{I} + 2y_{ro}^{II}}{3}\right) + (1-\lambda)\left(\frac{y_{ro}^{IV} + 2\, y_{ro}^{III}}{3}\right)\right]u_r$$

$$s.t. \sum_{i=1}^{m}\left[\lambda\left(\frac{x_{io}^{I} + 2x_{io}^{II}}{3}\right) + (1-\lambda)\left(\frac{x_{io}^{IV} + 2\, x_{io}^{III}}{3}\right)\right]v_i = 1,$$

$$\sum_{r=1}^{s}\left[\lambda\left(\frac{y_{rj}^{I} + 2y_{rj}^{II}}{3}\right) + (1-\lambda)\left(\frac{y_{rj}^{IV} + 2\, y_{rj}^{III}}{3}\right)\right]u_r - \sum_{i=1}^{m}\left[\lambda\left(\frac{x_{ij}^{I} + 2x_{ij}^{II}}{3}\right) + (1-\lambda)\left(\frac{x_{ij}^{IV} + 2\, x_{ij}^{III}}{3}\right)\right]v_i \le 0, \forall\, j$$

and $u_r, v_i \ge 0, \forall\, r, i.$

Also, from Theorem 1, The Fuzzy CCR Envelopment model is transformed to corresponding crisp CCR model.

$$\text{Min } \theta$$

$$s.t.\ \theta\left[\lambda\left(\frac{x_{io}^{I} + 2x_{io}^{II}}{3}\right) + (1-\lambda)\left(\frac{x_{io}^{IV} + 2\, x_{io}^{III}}{3}\right)\right] \ge \sum_{j=1}^{n}\left[\lambda\left(\frac{x_{ij}^{I} + 2x_{ij}^{II}}{3}\right) + (1-\lambda)\left(\frac{x_{ij}^{IV} + 2\, x_{ij}^{III}}{3}\right)\right]\lambda_j, \forall i$$

$$\sum_{j=1}^{n}\left[\lambda\left(\frac{y_{rj}^{I}+2y_{rj}^{II}}{3}\right)+(1-\lambda)\left(\frac{y_{rj}^{IV}+2\,y_{rj}^{III}}{3}\right)\right]\lambda_j \geq \left[\lambda\left(\frac{y_{ro}^{I}+2y_{ro}^{II}}{3}\right)+(1-\lambda)\left(\frac{y_{ro}^{IV}+2\,y_{ro}^{III}}{3}\right)\right],\forall r$$

and $\lambda_j \geq 0$.

4. Solve the crisp DEA model using any existing LP technique with different risk levels.
5. Evaluate the efficiency score of each DMUs with the different risk levels.

6 Numerical Example

Two examples are provided to show the validity and applicability of the suggested solution technique.

Example 1 [26] **(Fuzzy LP problem)**

Consider the example of FLPP

$$Max\ Z = \langle 5,6,7,8\rangle x_1 + \langle 4,5,6,7\rangle x_2$$

Subject to

$$\langle 6,7,8,9\rangle\, x_1 + \langle 4,5,6,7\rangle\, x_2 \leq \langle 24,35,48,63\rangle$$

$$\langle 1,1,1,1\rangle x_1 + \langle 2,3,4,5\rangle x_2 \leq \langle 6,10,14,19\rangle$$

$$\langle 1,1,1,1\rangle x_1 + \langle 1,1,1,1\rangle x_2 \leq \langle 1,5,9,12\rangle$$

$$\langle 1,1,1,1\rangle x_1 \leq \langle 2,6,10,15\rangle$$

and $x_1, x_2 \geq 0$.

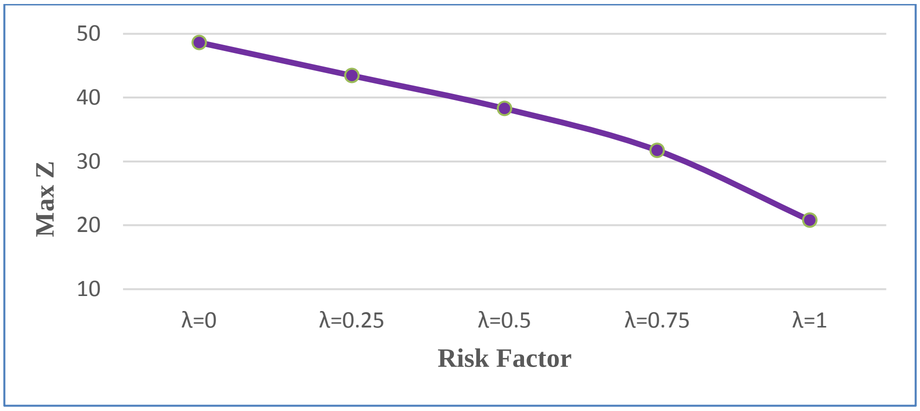
Applying algorithm 1, the optimum solution of the given Fuzzy LP problem with the different risk factors is obtained in Table 1.

Table 1: Optimum Solution of the Fuzzy LPP

	$\lambda = 0$	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$	$\lambda = 1$
x_1	4.3806	4.1461	3.8876	5.0313	3.6667
x_2	2.6045	2.4946	2.3655	0.2187	0.0000
$Max\ Z$	48.6194	43.4372	38.2791	31.7188	20.7778

The Optimum solution of the given FLLP is gradually decreases with optimistic to pessimistic decision as shown in Figure 1.

Figure 1: Optimum solution with different risk factor



Example 2 [27] **(Fuzzy DEA Model)**

The following example is given to determine the relative efficiency score of the DMUs using the suggested approach.

Table 2: Trapezoidal Fuzzy Input-Output data

DMUs	Input 1	Input 2	Output 1	Output 2
DMU1	⟨3.25, 3.75, 4.25, 4.75⟩	⟨2.25,2.5,2.75,3⟩	⟨1,2,3,4⟩	⟨4,4.5,5,5.5⟩
DMU2	⟨2,4,6,8⟩	⟨4,4.5,5,5.5⟩	⟨0.5,1,1.5,2⟩	⟨2,3,4,5⟩
DMU3	⟨2.25,2.75,3.25,3.75⟩	⟨2,3,4,5⟩	⟨1,3,5,7⟩	⟨1.5,2.5,3.5,4.5⟩
DMU4	⟨3.5,4.5,5.5,6.5⟩	⟨4.5,5,5.5,6⟩	⟨1.5,2,2.5,3⟩	⟨0.25,0.5,0.75,1⟩
DMU5	⟨0.5,1,1.5,2⟩	⟨1.5,1.75,2,2.25⟩	⟨3,4,5,6⟩	⟨2,2.75,3.5,4.25⟩

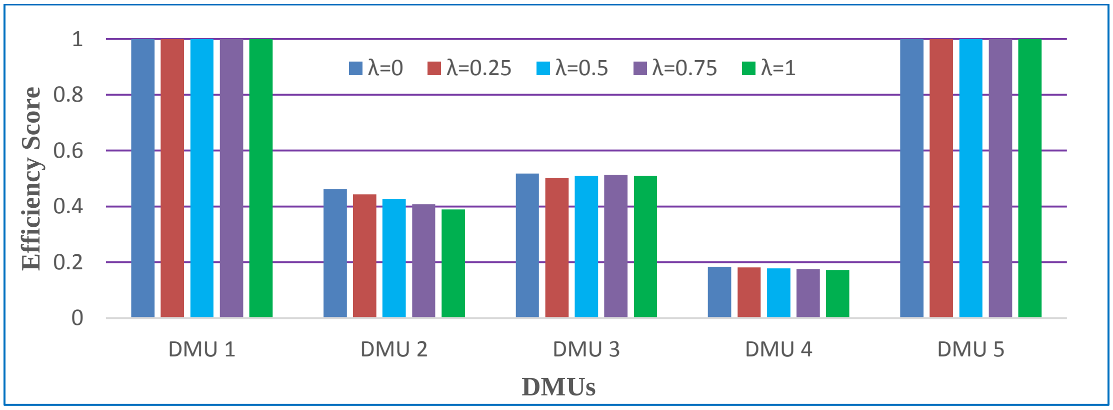
Applying Algorithm 2, the efficiency scores of the DMUs is obtain in Table 3. Using Definition 6 DMUs are categories into efficient and inefficient groups.

Table 3: Efficiency Scores of the DMUs with Different risk factor

DMU	λ=0	λ=0.25	λ=0.5	λ=0.75	λ=1	Type
DMU1	1	1	1	1	1	Efficient
DMU2	0.462	0.4436	0.4257	0.4078	0.3894	Inefficient
DMU3	0.5183	0.5018	0.5095	0.5133	0.5098	Inefficient
DMU4	0.1838	0.1813	0.1786	0.1756	0.1724	Inefficient
DMU5	1	1	1	1	1	Efficient

Here, Two DMUs (i.e., $DMU\ 1, DMU\ 5$) are efficient whereas three DMUs are inefficient. The DMUs are ranked according to the efficiency score i.e., $DMU\ 1 = DMU\ 5 > DMU\ 3 > DMU\ 2 > DMU\ 4$. The performance of the DMUs are compare in Figure 2.

Figure 2: Efficiency Score of the DMUs



7 Conclusion

This article developed a new type of ranking function to compare the TrFNs. This ranking function has the ability to convert directly the general FLPP, in which all the coefficients are TrFNs, into its corresponding crisp LP problem. The ranking function is associated with the risk factors, which represents the risk taker's attitude towards taking the risk, i.e., the decision is optimistic, neutral, or pessimistic. Example 1 is presented to show the applicability and validity of the suggested solution technique in which the optimum solution of the FLLP problem is determined with the different risk levels.

The Fuzzy DEA is a relatively new topic in organizations' performance assessment under uncertainty. Decision makers and researchers developed different concepts to handle the uncertainty and evaluate the relative efficiency score of the DMUs. This paper suggested a ranking function that converts the FDEA model into its corresponding crisp DEA model. The efficiency scores of the DMUs are obtained using different risk factors. Example 2 is given to show the applicability and validity of the proposed solution technique.

This ranking approach has the ability to solve other linear optimization problems like transportation problems, invest mate problems, and knapsack problems. It may be used to solve MCDM problems under uncertainty environment. Real-world application of the proposed approach is one of the fascinating areas for future studies which will have scopes in multiple fields. Other DEA models like BCC, SBM, Additive, Undesirable, Time series data model, etc., under uncertainty conditions, this approach may be helpful for solving these models.

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