A SHORT REVIEW ON VARIOUS RETRIAL QUEUEING SYSTEMS WITH WORKING VACATIONS

R. Jeyachandhiran[1] and P. Rajendran[1]*

[1] Department of Mathematics, School of advanced sciences, Vellore Institute of Technology, Vellore-632014, INDIA. jeyachandhiran.r@vit.ac.in

*** Correspondence:** E-mail: prajendran@vit.ac.in

Abstract: In this short review article, we quickly examine various retrial queueing systems with various working vacation model types. In computer communication systems, manufacturing/production systems, and inventory systems, in particular network service, web service, file transfer service, mail service, etc., the idea of working vacations has a wide variety of applications. Retrial queues are very helpful for studying stochastic computer and communications networks, among other systems. The goal of this study is to give various retrial queueing systems with variable service.

Keywords: Retrial queuing models, working vacation, Queueing systems.

Mathematics Subject Classification: 68M20, 60K25, 60K30.

1. Introduction

Congestion is explained via mathematical models that make queueing systems simpler. Every time "customers" want "service" from a facility, there is typically a queueing mechanism in place; frequently, both the customer arrival and the periods for giving service are meant to be random. If all of the'servers' are already full when new clients arrive, they will frequently line up for the next server that becomes available. A basic queueing system is made up of three components: the arrival pattern, the service mechanism, and queue discipline.

When congestion sets in, queues (and queueing networks) have developed into highly helpful instruments for assessing (at least roughly) the performance of intricately interconnected stochastic systems. Applications are readily apparent at check-out lanes at various counters, hospitals, and airline terminals. In the past, they have also been used in fields such as chemistry (polymerization, clustering), electrical engineering (flow models), and biology (migration, population models). Queueing systems and networks are most often employed in the manufacturing, computer networking, and telecommunications and broadcasting sectors. However, there are a lot of applications in the field of social sciences as well. Many systems in real life depend on queueing systems with breaks. There are several potential causes for the vacations. According to the traditional vacation policy, clients are not served while away.

Working vacation (WV) is a sort of vacation policy in which the server continues to provide service while on vacation but at a reduced rate. Numerous real-world systems, such as email, file transfers, networks, and online services, can benefit from this queueing strategy. It was initially employed in an M/M/1 queueing system by Servi and Finn [33]. They looked at an M/M/1 queue with working holidays and found the transform equations for the steady-state distribution of the system's client count and sojourn duration. They then examined the performance of the gateway router in fibre communication networks using their results. Using working vacation models prevents the server from returning to its typical busy period until the vacation has ended. Additionally, if there are customers at a service completion instant during the WV time, the server can terminate the vacation and resume its typical busy state. The vacation interruption policy is what it is called.

Arriving customers who discover that there are no more servers or waiting areas available may retry for service after waiting in retrial queues or queues with repeated orders. Retrial queues have frequently been used to model a variety of issues in phone switching networks, computer networks, and computer systems. Many academics are interested in retry queueing systems because they are widely used in real-world settings such cellular networks, contact centres, computers, inventory systems, and production management. These systems also allow for vacations and client feedback. In this review study, we explore three significant retrial queueing models and outline their key analytical findings as well as the methodologies they employ. We generally focus on single-server queueing models. Readers who are interested in learning more about multi-server queueing methods are directed to the source works. For example, see ([8], [13], [18], [34], [36], [38],[42]).

This review study provides a quick overview of the previous ten years' worth of working vacation model types and retrial queueing systems. The readers will find it helpful when it comes to managing vacation lines. The remainder of the essay is structured as follows: The most current models with M/M/1 and M/G/1 retrial queueing models with working vacations were covered in Section 2 of this article. The most recent changes to various working vacation models are presented in Section 4. Finally, section 5 presents the work's conclusion.

2. M/M/1 and M/G/1 Retrial Queueing Models with Working Vacations

The effectiveness of various approaches has been assessed using vacation queues and retrial queues. Numerous scholars have recently addressed the idea of retry queues with working vacation. When a customer is turned down during a trial, their name is placed to the orbit, which is a collection of unfinished projects such as star local area networks, collision avoidance, packet switching networks, telephone networks, and web access.

In Tien Van Do [10], the performance study of a Media Access Control function in wireless networks served as the impetus for the introduction of the new M/M/1 retrial queue with working vacations. In this study, the author provided a requirement for the model's stability, which has a significant influence on determining the retry rate for systems of this type. Also

shown that the conditional stochastic decomposition holds for the retrial M/M/1 queue with working vacations by deriving the closed form equilibrium solution for this model.

In [4], the authors demonstrated how queueing theory might aid in the performance assessment of various contemporary systems using the example of the M/G/1 queue with working vacations and obtained the system's circling customer count and server state's joint probability distribution. This distribution demonstrates how mean performance measurements may be produced using Laplace and z-transforms. M/M/1 retrial queue with collisions and working vacation interruption under N-policy was discussed in [35].

In [24] the authors analyzed an M/M/1 retrial queue with multiple working vacations under the classical retrial policy, and vacation interruption policy is considered. [15] examined an M/G/1 retrial queue with generic retrial times and included a policy for vacation interruptions and working vacations. The stationary probability distribution and other performance metrics were produced by the authors using the additional variable approach. The authors of [26] gave the M/G/1 Retrial Queue with Single Working Vacation some thought. The authors calculated the mean orbit size during the WV period and regular service period as well as the mean waiting time of the customer in the orbit during the WV period and regular service period using the supplementary variable technique. They also obtained the probability generating function for the number of customers in the system.

In [20] an M/M/1 retrial queue with working vacation, orbit search and balking are considered. Using the matrix-analytic method, the authors obtained the necessary and sufficient condition for system to be stable and also derived the stationary probability distribution and some performance measures. In [25], the authors studied an M/M/1 retrial queue with Bernoulli schedule-controlled vacation and vacation interruption. The M/M/1 retrial queue with working vacations and negative customers is introduced. The arrival processes of positive customers and negative customers is Poisson. Upon the arrival of a positive customer, if the server is busy the customer would enter an orbit of infinite size and the orbital customers send their requests for service with a constant retrial rate introduced in [12]. In [16] the authors treat an M/G/1 queue with retrial customers due to server vacation, which can be used to model a hospital service system. In [11] the authors the M/M/1 retrial queue with working vacations and a constant retrial rate. In the queue, customers decide about the entry based on the information upon their arrival instants. Scenarios regarding the availability of information (i.e., the server is occupied or not, and the server is on the vacation or not) for customers are compared.

Recently, [17] presented Analysis of an M/G/1 Retrial Queue with Delayed Repair and Feedback under Working Vacation Policy with Impatient Customers. A research on the M/G/1 feedback retrial queue that is prone to server downtime and maintenance is taken into account in [31]. In [30], the authors used a modified Bernoulli vacation to search for arrivals of an M/G/1 retrial queueing system with delayed repair and possible re-service. [28] examined the effects of catastrophe on an M/G/1 retrial queueing system with working vacations and working breakdowns. In [5], the author took into account a queueing system of the M/M/1 type with repeated claims in the scenario where the system loading, or the number of claims in the queue

awaiting service, determines the service rate. Additionally, the formulae and existence requirements for the ergodic distribution of the system's claim count with finite and unbounded queues of repeated claims were constructed.

An M/G/1 retrial queueing system with modified multiple vacations was investigated in [41], where a fresh external arrival may kick the customer who is now being serviced out of the system and start serving others right away or enter the retrial orbit. A repairable M/M/1 retrial queueing paradigm with setup times was studied in [37]. To lower operational expenses, the server will be shut down after the system has been completely emptied. Additionally, the system won't turn on until a fresh consumer shows up. A finite-source M/G/1 retrial queue with outbound calls was examined in [14]. In [2], the authors examined the priority services for the M[X1], M[X2]/G1, and G2/1 retrial queues, differentiating between breakdown, repair, synchronised reneging, and optional vacation.

3. Other Working Vacation Models

In this section we can study the other types of retrial queues with working vacation models.

The Waiting Time Reduction in an M/M/1/N Encouraged Arrival Queue with Feedback, Baulking, and Maintaining of Reneged Customers was researched in [19]. An M/G/1 retrial G-queue with two service phases, rapid feedback, and working vacations was examined by the authors in [40]. A study on an M/(G1, G2)/1 feedback trial queue with two phases of service and a variant vacation policy under the condition of delaying repair for impatient customers can be found in [32]. An investigation of a M[X]/G/1 unreliable retrial G-queue with orbital search and feedback under Bernoulli vacation schedule was looked at by the authors in [29]. The authors of [23] took into account an M/G/1 retrial G-queue with general retrial times in which the server is prone to malfunctions and repairs. [27] explored an M/G/1 retrial G-queue with preemptive resumption priority and collisions due to server outages and postponed maintenance.

A discrete-time MAP/PH/1 queue with finite system capacity and two-stage vacations was investigated in [43]. With priority services, working breakdown, start up/close down time, Bernoulli vacation, reneging, and baulking, [3] explored the transient analysis of the M[X1],M[X2]/G1,G2/1 retrial queueing system. The authors of [22] examined the Geo/Geo/1 retrial queue with working vacations and interruptions. Model of the Geo/Geo/1 retrial queue with working vacations and non-persistent clients was examined by [21]. A discrete-time GeoX/Geo/1 retrial queue system was investigated by [39] and its working vacation policy.

The system using Markov Arrival Process (MAP) is a new discovery in the study of working vacation models.[44] analyses the Markovian Flow of Breakdowns Repairable Server in the MAP/M/1 Retrial Queueing Model. A retrial queuing model with MAP arrivals and two customer categories was examined in [6]. Customer impatience, catastrophic failures with repairs, and a retry queueing paradigm were all explored in [7]. The MAP 1 and MAP 2 /M/c/b,

retrial queue with infinite retrial group, geometric loss, guard channels, and finite priority queue for hand-off class were taken into consideration by the authors in [9]. The author of [1] examined the MAP/PH/1 retrial queue with a constant retrial rate, working vacations, abandonment, flush out, customer search, breakdown, and repair.

4. Conclusion:

Over the past thirty years, the domain of vacation models has seen a number of comprehensive developments. In this study, we provided a brief overview of the research on working vacation queueing models' various arrival procedures and service operations. A summary of the concept mentioned in numerous works has been produced. Researchers, engineers, managers, operations analysts, statisticians, and engineers can all benefit from adopting these models. The appropriate sources have been mentioned, and a wide spectrum of literature has been covered.

References

[1] G. Ayyappan and R. Gowthami, Analysis of MAP/PH/1 retrial queue with constant retrial rate, working vacations, abandonment, flush out, search of customers, breakdown and repair, International Journal of Operational Research, 42 (3) (2021), 310-331.

[2] G. Ayyappan and S. Nithya, Analysis of M[X1], M[X2]/G1, G2/1 retrial queue with priority services, differentiate breakdown, repair, synchronized reneging and optional vacation, Reliability: Theory & Applications, 2 (73) (2023), 24-38. https://doi.org/10.24412/1932-2321-2023-273-376-391.

[3] G. Ayyappan and J. Udayageetha, Transient Analysis of M[X1], M[X2]/G1, G2/1 retrial queueing system with priority services, working breakdown, start up/close down time, Bernoulli vacation, reneging and balking, Pakistan Journal of Statistics and Operation Research, 16 (1) (2020), 203-216. https://doi.org/10.18187/pjsor.v16i1.2181.

[4] A. Aissani, S. Taleb, T. Kernane, G. Saidi and D. Hamadouche, An M/G/1 Retrial Queue with Working Vacation, In: J. Swiątek, A. Grzech, P. Swiątek, J. Tomczak (eds) Advances in Systems Science, Advances in Intelligent Systems and Computing, Springer, 240 (2014), 443-452. Cham. https://doi.org/10.1007/978-3-319-01857-7_43.

[5] M. S. Bratiichuk, A. A. Chechelnitsky and I. Y. Usar, M/M/1 Retrial Queueing System with Variable Service Rate, Ukrainian Mathematical Journal, 72 (2020), 403-415. https://doi.org/10.1007/s11253-020-01790-1.

[6] S. R. Chakravarthy and A. Dudin, Analysis of a retrial queuing model with MAP arrivals and two types of customers, Mathematical and Computer Modelling, 37 (3-4) (2003), 343-363.

[7] S. Chakravarthy, A. Dudin and V. Klimenok, A retrial queueing model with map arrivals, catastrophic failures with repairs, and customer impatience, Asia-Pacific Journal of Operational

Research, 27 (6) (2010), 727-752.

[8] V. Chandrasekaran, K. Indhira, M. Saravanarajan and P. Rajadurai, A survey on working vacation queueing models, International Journal of Pure and Applied Mathematics, 106 (6) (2016), 33-41.

[9] B. D. Choi, Y. Chang and B. Kim, MAP_1, MAP_2/M/C retrial queue with guard channels and its application to cellular networks, 7 (1999), 231-248. https://doi.org/10.1007/BF02564724.

[10] T. V. Do, M/M/1 retrial queue with working vacations, Acta Informatica, 47 (2010), 67-75. https://doi.org/10.1007/s00236-009-0110-y.

[11] N. H. Do, T. V. Do and A. Melikov, Equilibrium customer behavior in the M/M/1 retrial queue with working vacations and a constant retrial rate, Operational Research, 20 (2020), 627-646. https://doi.org/10.1007/s12351-017-0369-7.

[12] T. V. Do, D. Papp, R. Chakka, J. Sztrik and J. Wang, M/M/1 retrial queue with working vacations and negative customer arrivals, Special Issue on: Random Neural Networks: Advances and Applications, International Journal of Advanced Intelligence Paradigms, 6 (1) (2013), 1-13.

[13] B. T. Doshi, Queueing systems with vacations: a survey, Queueing System, 1 (1986), 29-66.

[14] V. I. Dragieva and T. Phung-Duc, A finite-source M/G/1 retrial queue with outgoing calls, Annals of Operations Research, 293 (2020), 101-21. Doi: https://doi.org/10.1007/s10479-019-03359-z.

[15] S. Gao, J. Wang and W. W. Li, An M/G/1 retrial queue with general retrial times, working vacations and vacation interruption, Asia-Pacific Journal of Operational Research, 31 (02) (2014), 1440006. https://doi.org/10.1142/S0217595914400065.

[16] S. Gao and D. Zhang, Performance and sensitivity analysis of an M/G/1 queue with retrial customers due to server vacation, Ain Shams Engineering Journal, 11 (3) (2020), 795-803.

[17] M. M. N. Gnanasekar, I. Kandaiyan, Analysis of an M/G/1 Retrial Queue with Delayed Repair and Feedback under Working Vacation policy with Impatient Customers, Symmetry, 14 (10) (2022), 2024. https://doi.org/10.3390/sym14102024.

[18] J. C. Ke, C. H. Wu and Z. G. Zhang, Recent developments in vacation queueing models: A short survey, International Journal of Operations Research, 7 (2010), 3-8.

[19] I. E. Khan and R. Paramasivam, Reduction in Waiting Time in an M/M/1/N Encouraged Arrival Queue with Feedback, Balking and Maintaining of Reneged Customers, Symmetry, 14 (8) (2022), 1743.

[20] J. Li and T. Li, An M/M/1 Retrial Queue with Working Vacation, Orbit Search and Balking, Engineering Letters, 27 (1) (2019).

[21] Z. Liu and Y. Song, Geo/Geo/1 retrial queue with non-persistent customers and working vacations, Journal of Applied Mathematics and Computing, 42 (2013), 103-115.

[22] T. Li, Z. Wang and Z. Liu, Geo/Geo/1 retrial queue with working vacations and vacation interruption, Journal of Applied Mathematics and Computing, 39 (2012), 131-143.

[23] T. Li and L. Zhang, An M/G/1 Retrial G-Queue with General Retrial Times and Working Breakdowns, Mathematical and Computational Applications, 22 (1) (2017), 15. https://doi.org/10.3390/mca22010015.

[24] T. Li, L. Zhang and S. Gao, Performance of an M/M/1 Retrial Queue with Working Vacation Interruption and Classical Retrial Policy, Advances in Operations Research, 2016 (2016), Article ID 4538031. http://dx.doi.org/10.1155/2016/4538031.

[25] T. Li, L. Zhang and S. Gao, An M/M/1 retrial queue with Bernoulli schedule-controlled vacation and vacation interruption, ICIC Express Letters, 9 (12) (2015), 3385-3392.

[26] S. P. B. Murugan and K. Santhi, An M/G/1 Retrial Queue with Single Working Vacation, Applications and Applied Mathematics, 12 (1) (2017), 1 – 22.

[27] Y. Peng, Z. Liu and J. Wu, An M/G/1 retrial G-queue with preemptive resume priority and collisions subject to the server breakdowns and delayed repairs, Journal of Applied Mathematics Computing, 44 (2014), 187-213.

[28] P. Rajadurai, Sensitivity analysis of an M/G/1 retrial queueing system with disaster under working vacations and working breakdowns, RAIRO-Operations Research, 52 (1) (2018), 35-54.

[29] P. Rajadurai, V. M. Chandrasekaran and M. C. Saravanarajan, Analysis of an $M^{[X]}$/G/1 unreliable retrial *G*-queue with orbital search and feedback under Bernoulli vacation schedule, OPSEARCH, 53 (2016), 197-223. https://doi.org/10.1007/s12597-015-0226-5.

[30] C. Revathi and L. Francis Raj, Search of arrivals of an M/G/1 retrial queueing system with delayed repair and optional re-service using modified Bernoulli vacation, Journal of Computational Mathematica, 6 (1) (2022), 200-209.

[31] P. Rajadurai, M. C. Saravanarajan and V. M. Chandrasekaran, A study on M/G/1 feedback retrial queue with subject to server breakdown and repair under multiple working vacation policy, Alexandria Engineering Journal, 57 (2) (2018), 947-962.

[32] P. Rajadurai, M. C. Saravanarajan, V. M. Chandrasekaran and K. Indhira, An M/(G1, G2)/1 Feedback Retrial Queue with Two Phase Service, Variant Vacation Policy Under Delaying Repair for Impatient Customer, International Journal of Fuzzy Mathematical Archive, 6 (1) (2015), 45-55.

[33] L. D. Servi and S. G. Finn, M/M/1 queues with working vacations, Performance Evaluation, 50 (2002), 41-52.

[34] H. Takagi, Queueing analysis: A foundation of performance evaluation (Vol. 1). North Holland, (1991).

[35] L. Tao, Z. Liu and Z. Wang, M/M/1 retrial queue with collisions and working vacation interruption under N-policy, RAIRO-Operations Research-Recherche Opérationnelle, 46 (4) (2012), 355-371.

[36] N. Tian, J. Li and Z. G. Zhang, Matrix analytic method and working vacation queues A survey, International Journal of Information and Management Sciences, 20 (2009), 603-633.

[37] R. Tian, X. Wu, L. He and Y. Han, Strategic Analysis of Retrial Queues with Setup Times, Breakdown and Repairs, Discrete Dynamics in Nature and Society, 2023 (2023), Article ID-4930414. https://doi.org/10.1155/2023/4930414.

[38] N. Tian, and Z. G. Zhang, Vacation queueing models: Theory and applications, New York: Springer-Verlag (2006).

[39] S. Upadhyaya, Working vacation policy for a discrete-time Geo^X/Geo/1 retrial queue, OPSEARCH, 52 (2015), 650-669.

[40] M. Varalakshmi, V. M. Chandrasekaran and M. C. Saravanarajan, A study on M/G/1 retrial G-queue with two phases of service, immediate feedback and working vacations, In Proceedings of the IOP Conference Series: Materials Science and Engineering, Birmingham, UK, 13–15 October 2017; IOP Publishing: 2017; Volume 263, p. 042156.

[41] J. Xu, L. Liu and K. Wu, Analysis of a retrial queueing system with priority service and modified multiple vacations, Communications in Statistics-Theory and Methods, 2022. DOI: 10.1080/03610926.2022.2027448.

[42] T. Yang and J. G. C. Templeton, A survey on retrial queues, Queueing Systems, 2 (1987) 201–233. https://doi.org/10.1007/BF01158899.

[43] Q. Ye, Algorithmic computation of MAP/PH/1 queue with finite system capacity and two-stage vacations, Journal Industrial Management and Optimization, 16 (5) (2020), 2459-2477. Doi: 10.3934/jimo.2019063.

[44] Z. Zhou and Z. Zhou, Analysis of the MAP/M/1 Retrial Queueing Model with Markovian Flow of Breakdowns Repairable Server, *2018 IEEE 4th Information Technology and Mechatronics Engineering Conference (ITOEC)*, Chongqing, China, 2018, pp. 1082-1085. Doi: 10.1109/ITOEC.2018.8740351.