VERTEX ODD ROOT SQUARE MEAN GRACEFUL LABELING ON 2-TUPLE GRAPHS

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Abstract: *LetG(V, E) be a graph. A one to one function M is defined on V to {1, 3, ..., 2q + 1} is termed as vertex odd root square mean graceful labeling of G if the function $M^*(e = cd)$ induces $\lceil \sqrt{(M(c)^2 + M(d)^2)/2} \rceil$ if $|M(c)–M(d)|$ is even for all cd ∈ E which is bijective.Through this research work, we examine vertex odd root square mean graceful labeling of some 2-tuple graphs.*

Keywords: *Vertex odd root square mean graceful labeling, Root square mean graceful labeling.*

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INTRODUCTION

A Graph G(V, E) where the elementsof |V| = p and |E| = q are respectively called vertices and edges. Rosa was the first to define "β-valuation" which was renamed by Golomb [8] as "Graceful labeling" which is now the prevalent phrase. A one to one function M is defined on V to {0, 1, 2, … q} is termed as Graceful labeling of G if the function M^* induces $M^*(e = cd) =|M(c) – M(d)|$ which is bijective. S.S Sandhya, S. Somasundaram, and S. Anusa were the first to introduce root square mean graph labeling [3]. A one to one function M is defined on V to{1, 2, ..., q + 2} is termed as Root square mean graceful (RSMG) of G if the function M^* inducesM$^*(e = cd) = \lceil \sqrt{(M(c)^2 + M(d)^2)/2} \rceil$ or $\lfloor \sqrt{(M(c)^2 + M(d)^2)/2} \rfloor$ for all cd ∈ E(G) which is bijective[1]. We extend root square mean graceful to vertex odd root square mean graceful (VORSMG) labeling by replacing the positive integer of the vertex label with an odd positive integer and investigating thislabeling on some 2-tuple graph.

PRELIMINARIES

For the basic concepts and definitions, we refer to [6]. Let G^*be the copy of simple graph G, each vertex 'c' of G is connected to the corresponding vertex 'd' of G^* by an edge to generate the 2-tuple graph. It is denoted by$T^2(G)$.A path(P_s)is a walk with distinct vertices. The partition (X_1, X_2)obtains the complete bipartite($K_{s,t}$), in which every line of G connects a point X_1 to the points of X_2. The graph $K_{1, s}$is called the star graph. From the path P_s, a coconut Tree(CT(s, t)) can be generated by joining 't' K_1vertices to the one vertex end of the path P_s. A slanting ladder(SL_s) is obtained by joining the vertex " f " of the path $P'_s$with(f+ 1)th vertex of the path P''_s, $1 \le f \le s – 1$. Ladder (L_s)is obtained from the cartesianproduct $P_s$andP_2.A cycle graph(C_s) is a non-empty trail in which the first and last vertices are equal.

MAIN RESULTS

Theorems

Theorem 1. The 2-tuple graph of path $P_s$is VORSMG.

Proof.The graph $T^2(P_s)$ has '2s' vertices and '3s–2' edges. Let { c_1 , c_2 , . . . c_s } be the vertices of $P_s$and {d_1 , d_2 , . . . d_s} be the vertices of P^*_swhich is another copy of P_s.

We define the vertex labeling M:V→{1, 3, 5, …, 2q +1} as follows

$M(c_1) = 6m - 3$;
$M(c_f) = M(c_{f-1}) - 4, 2 \le f \le s$;
$M(d_1) = 6m - 5$;
$M(d_f) = M(d_{f-1}) - 4, 2 \le f \le s$.
The corresponding edge labeling obtained as
$M^*(c_fc_{f+1}) = \lceil \sqrt{(M(c_f)^2 + M(c_{f+1})^2)/2} \rceil, 1 \le f \le s - 1$.
$M^*(d_fd_{f+1}) = \lceil \sqrt{(M(d_f)^2 + M(d_{f+1})^2)/2} \rceil, 1 \le f \le s - 1$.
$M^*(c_fd_f) = \lceil \sqrt{(M(c_f)^2 + M(d_f)^2)/2} \rceil, 1 \le f \le s$.
Thus $M^*(c_fc_{f+1}) \neq M^*(d_fd_{f+1}) \neq M^*(c_fd_f)$ are distinct for each f, $T^2(P_s)$ is VORSMG. Figure 1 shows VORSMG labeling of $T^2(P_4)$.

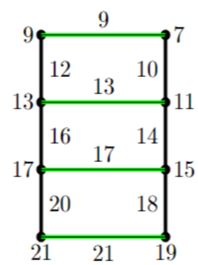

Figure 1

Theorem 2.2-tuple graph of star graph $K_{1,s}$ is VORSMG.
Proof. Let $T^2(K_{1,s})$ be the graph with "2s+2" vertices and "3s+1" edges.
Let "c" and "d" be the apex of $K_{1,s}$ and $K^*_{1,s}$.
Let $\{c_1, c_2, \ldots c_s\}$ be the vertices attached to 'c' of $K_{1,s}$ and $\{d_1, d_2, \ldots d_s\}$ be the vertices attached to 'd' of $K^*_{1,s}$.
We define vertex labeling $M: V \rightarrow \{1, 3, 5, \ldots, 2q + 1\}$ as follows:
$M(c) = 1$;
$M(c_s) = 2s+1$;
$M(d) = 6m + 3$;
$M(d_1) = 6m + 1$;
$M(d_f) = M(d_{f-1}) - 4, 2 \le f \le s$.
The corresponding edge labeling obtained as
$M^*(cd) = \lceil \sqrt{(M(c)^2 + M(d)^2)/2} \rceil$;
$M^*(dd_f) = \lceil \sqrt{(M(d)^2 + M(d_f)^2)/2} \rceil, 1 \le f \le s$;
$M^*(cc_f) = \lceil \sqrt{(M(c)^2 + M(c_f)^2)/2} \rceil, 1 \le f \le s$;
$M^*(c_fd_f) = \lceil \sqrt{(M(c_f)^2 + M(d_f)^2)/2} \rceil, 1 \le f \le s$;
Thus $M^*(cc_f) \neq M^*(dd_f) \neq M^*(cd) \neq M^*(c_fd_f)$ are distinct for each f, $T^2(K_{1,s})$ is VORSMG. Figure 2 shows VORSMG labeling of $T^2(K_{1,4})$.

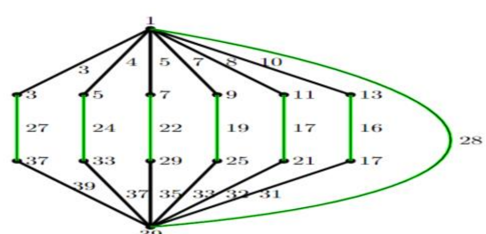

Figure 2:

Theorem 3. The 2-tuple graph of Coconut tree C(s, t) is VORSMG.
Proof.The graph $T^2(C(s, t))$ has "2(s+ t)"vertices and "3(s+ t) – 2"edges.
Let $\{ c_1 , c_2 , . . . c_t \}$ be the vertices of $P_t$of C(s, t)and the pendant vertices attached to the vertex "c_t" are $\{ c'_1 , c'_2 , . . . c'_t\}$.
Let $\{ d_1 , d_2 , . . . d_s \}$be the vertices of$P_s$of $C(s, t)^*$ and the pendant vertices attached to the vertex "d_t" are $\{d'_1 , d'_2 , . . . d'_t\}$. We define the vertex labeling $M:V\rightarrow\{1, 3, 5, \ldots, 2q +1\}$ as follows.

$M(c_1) = 6 (s+ t) + 3$;
$M(c'_f) =2f+1$, $1 \leq f \leq t$;
$M(d_1) = 6 (s+ t) + 5$;
$M(d_1) = 6t+ 1$;
$M(c_f) =M(c_{f-1}) – 4$, $2 \leq f\leq s– 1$;
$M(d_f) =M(d_{f-1}) – 4$, $2 \leq f \leq s$;
$M(c_s) =1$;
$M(d'_f) =M(d'_{f-1}) – 4$, $2 \leq f \leq t$;$M(d'_1) =M(d_s) – 2$;
The corresponding edge labeling obtained as
$M^*(c_fc_{f+1}) =\lceil \sqrt{(M(c_f)^2+ M(c_{f+1})^2)/2} \rceil$;
$M^*(d_fd_{f+1}) =\lceil \sqrt{(M(d_f)^2+ M(d_{f+1})^2)/2} \rceil$;
$M^*(c_sc'_f) =\lceil \sqrt{(M(c_s)^2+ M(c'_f)^2)/2} \rceil$;
$M^*(d_sd'_f) =\lceil \sqrt{(M(d_s)^2+ M(d'_f)^2)/2} \rceil$;
Thus the edges are distinct, $T^2(C(s, t))$ is VORSMG.
Figure 3 shows VORSMG labeling of $T^2(C(6, 4))$.

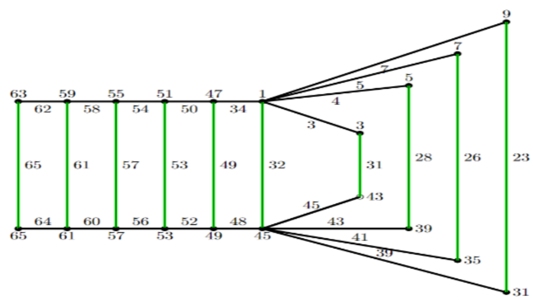

Figure 3:

Theorem 4. The 2-tuple graph of Slanting Ladder (SL_s) is VORSMG.
Proof.The graph $T^2(SL_s)$ has "4s" vertices and "8m –6"edges.
Let $\{ c_1 , c_2 , . . . c_s \}$, $\{ c'_1 , c'_2 , . . . c'_s \}$ are the vertices of path $P_s$and $P'_s$of SL_s.
Let$\{d_1 ,d_2 , . . . d_s \}$, $\{ d'_1 , d'_2 , . . . d'_s \}$ are the vertices of path P_s and $P'_s$of SL^*_s.
We define the vertex labeling $M:V\rightarrow\{1, 3, 5, \ldots, 2q +1\}$as follows:
$M(c_1) = 3$;
$M(c_f) =M(c_{f-1}) +4$, $2 \leq f\leq s$;
$M(d_1) = 1$;
$M(d_f) =M(d_{f-1}) +4$, $2 \leq f\leq s$;
$M(c'_1) =16m – 11$;
$M(c'_f) =M(c'_{f-1}) – 4$, $2 \leq f\leq s$;
$M(d'_1) =16m – 13$;$M(d'_f) =M(d'_{f-1}) – 4$, $2 \leq f\leq s$;
The corresponding edge labeling obtained as
$M^*(c_fc_{f+1}) =\lceil \sqrt{(M(c_f)^2+ M(c_{f+1})^2)/2} \rceil$;
$M^*(d_fd_{f+1}) =\lceil \sqrt{(M(d_f)^2+ M(d_{f+1})^2)/2} \rceil$;

$M^*(c_f c'_{f+1}) = \lceil \sqrt{(M(c_f)^2 + M(c'_{f+1})^2)/2}\, \rceil$;
$M^*(d_{f+1} d'_f) = \lceil \sqrt{(M(d_{f+1})^2 + M(d'_f)^2)/2}\, \rceil$;
$M^*(c'_f d'_f) = \lceil \sqrt{(M(c'_f)^2 + M(d'_f)^2)/2}\, \rceil$;
$M^*(c_f d_f) = \lceil \sqrt{(M(c_f)^2 + M(d_f)^2)/2}\, \rceil$;
Thus, the edges are distinct, $T^2(SL_m)$is VORSMG. A VORSMG of $T^2(SL_s)$is given in Figure 4.

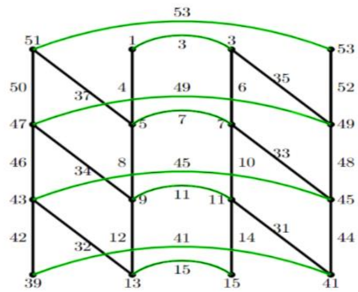

Figure 4:

Theorem 5.The 2-tuple graph of ladder L_s is VORSMG.
Proof.$T^2(L_s)$has"4s"vertices and"8s –4"edges.
Let{ $c_1, c_2, \ldots c_s$ }and{ $c'_1, c'_2, \ldots c'_s$ } be the vertices of pathP_s and P'_s of L_s .
Let {$d_1, d_2, \ldots d_s$ } and{ $d'_1, d'_2, \ldots d'_s$ } be the vertices of path P_s and $P'_s$of L^*_s.
We define the vertex labeling$M:V \to \{1, 3, 5, \ldots, 2q+1\}$ as follows:
$M(c_1) = 1$;
$M(c_f) = M(c_{f-1}) + 4,\ 2 \le f \le s$;
$M(d_1) = 3$;
$M(d_f) = M(d_{f-1}) + 4,\ 2 \le f \le s$;
$M(c'_1) = 16s - 7$;
$M(c'_f) = M(c'_{f-1}) - 4,\ 2 \le f \le s$;
$M(d'_1) = 16s - 9$;
$M(d'_f) = M(d'_{f-1}) - 4,\ 2 \le f \le s$;
The corresponding edge labeling obtained as
$M^*(c_f c_{f+1}) = \lceil \sqrt{(M(c_f)^2 + M(c_{f+1})^2)/2}\, \rceil$;
$M^*(d_f d_{f+1}) = \lceil \sqrt{(M(d_f)^2 + M(d_{f+1})^2)/2}\, \rceil$;
$M^*(c'_f c'_{f+1}) = \lceil \sqrt{(M(c'_f)^2 + M(c'_{f+1})^2)/2}\, \rceil$;
$M^*(d'_f d'_{f+1}) = \lceil \sqrt{(M(d'_f)^2 + M(d'_{f+1})^2)/2}\, \rceil$;
$M^*(c'_f d'_f) = \lceil \sqrt{(M(c'_f)^2 + M(d'_f)^2)/2}\, \rceil$;
$M^*(c_f d_f) = \lceil \sqrt{(M(c_f)^2 + M(d_f)^2)/2}\, \rceil$;
Thus the edges are distinct, $T^2(L_s)$is VORSMG. A VORSMG of $T^2(L_s)$is given in Figure 5.

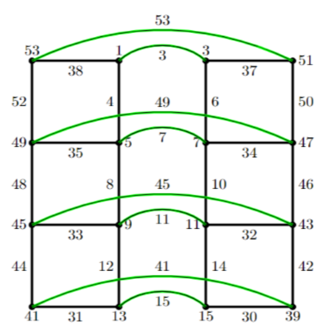

Figure 5:

Theorem 6.The 2-tuple graph of cycle $C_s$is a VORSMG graph.
Proof. The graph $T^2(C_s)$has "2s" vertices and "3s" edges. Let { c_1 , c_2 , . . . c_s} and { d_1 , d_2 , . . . d_s} representthe vertices of the cycle $C_s$and C^*_s.
The graph$T^2(C_s)$ is obtained by joining the vertices of"c_f" to "d_f".
We define the vertex labelingM:V→{1, 3, 5, …, 2q +1}asfollows:
Case i) when s is odd
$M(c_1) = 6s – 3$;
$M(c_f) = M(c_{f–1}) – 12,\ 2 \le f \le (s + 1)/2$;
$M(c_{(s+3)/2}) = M(c_{(s+1)/2}) + 16$; $M(c_f) = M(c_{f–1}) + 12,\ (s+5)/2 \le f \le s$;
$M(d_1) = 6s – 5$.
$M(d_f) = M(d_{f–1}) – 12,\ 2 \le f \le (s - 1)/2$;$M(d_{(s+1)/2}) = M(d_{(s - 1)/2}) – 8$;
$M(d_{(s+3)/2}) = M(d_{(s + 1)/2}) + 12$;
$M(d_f) = M(d_{f–1}) + 12,\ (s +5)/2 \le f \le s$;
Case ii) when m is even
$M(c_1) = 6s – 3$;
$M(c_f) = M(c_{f–1}) – 12,\ 2 \le f \le s/2$;
$M(c_{(s/2)+1}) = M(c_{s/2}) + 4$; $M(c_f) = M(c_{f–1}) + 12,\ (s/2) + 2 \le f \le s$;
$M(d_1) = 6s – 5$;
$M(d_f) = M(d_{f–1}) – 12,\ 2 \le f \le s/2$;
$M(d_f) = M(d_{f–1}) + 12,\ s/2 \le f \le s$;
$M(d_{(s/2)+1}) = M(d_{s/2}) + 4$.
The corresponding edge labeling obtained as
$M^*(c_fc_{f+1}) = \lceil \sqrt{(M(c_f)^2 + M(c_{f+1})^2)/2} \rceil$, $1 \le f \le s – 1$;
$M^*(d_fd_{f+1}) = \lceil \sqrt{(M(d_f)^2 + M(d_{f+1})^2)/2} \rceil$, $1 \le f \le s$;
$M^*(c_fd_f) = \lceil \sqrt{(M(c_f)^2 + M(d_f)^2)/2} \rceil$, $1 \le f \le s$; Thus $M^*(c_fc_{f+1}) \neq M^*(d_fd_{f+1}) \neq M^*(c_fc'_{f+1}) \neq M^*(c_fd_f)$ are distinct for each f, $T^2(C_s)$is VORSMG. A VORSMG of $T^2(C_8)$ is given in Figure 6.

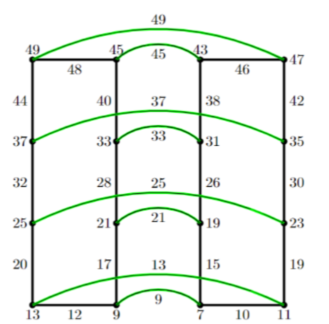

Figure 6:

Observation
If $T^2(G)$is VORSMG then G is VORSMG.

CONCLUSION

We defined new graceful labeling called vertex odd root square mean graceful labeling and applied for some 2-tuple standard graphs. Our future work is to apply this labeling for various families of graphs and in various field.

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