Mostar related indices and its polynomial of Cellulose

P. Kandan[a,b] T. Monica[a], and S. Subramanian[c]
[a]Department of Mathematics, Annamalai University, Annamalai Nagar- 608 002, India.
[b]Department of Mathematics, Government Arts College, Chidambaram-608 102, India.
[c]Hindustan College of Arts and Science, Chennai 608 103, India.
Correspondence should be addressed to: P.Kandan, kandan2k@gmail.com

Abstract

Cellulose is one of the natural bio-polymers which have been extensively employed in various fields due to their valuable and remarkable chemical and physical properties. As cellulose is a key ingredient in many products, its applications have been recognized across many industries like pharmaceutical, bio-fuels, textiles, and etc. Since graphs have such a wide range of applications, the study of graphs using chemistry attracts a lot of researchers worldwide. Studying topological indices of a chemical graph, which describe the structure of molecules, is one such application. In this paper, topological indices based on the bond have been used to compute the cellulose chemical structure using the Mostar index, a weighted version of the Mostar index and its polynomial.

Keywords: Mostar index, Mostar polynomial, Cellulose

1. Introduction and Preliminaries

Most of the compounds considered are based on carbon. We often use the term chemical graph (molecular structure) of chemical compounds represented in a graph where atoms are represented by vertices and bonds are represented by edges. It has received considerable attention both in the context of complex networks and in more classical applications of chemical graph theory. Most of the compounds considered are based on carbon. We often use the term chemical graph (molecular structure) of chemical compounds represented in a graph where atoms are represented by vertices and bonds are represented by edges. It has received considerable attention both in the context of complex networks and in more classical applications of chemical graph theory.

This is useful for modeling any number of systems whose structure and function depends on the connection pattern of components. In response to environmental concerns, modern demands in materials science have stimulated the development of various bio-based materials in which cellulose and its derivatives play a major role. Cellulose's unique and excellent chemical and physical properties have made widely recognized in a wide range of applications.

The main reason for this is that the strong bonds between cellulose molecules make them very rigid. Chemically, they are the most abundant organic compounds on earth. Due to its complex intermolecular and extra molecular interactions, cellulose molecules are insoluble in common polar solvents such as water, alcohols, and amines, so they are held tightly together byhydrogen bonding, forming the desired shape. It becomes difficult to mold. In [7,13] the closed-form formulas for the primary, secondary, modified and extended Zagreb indices, the inverse and general randic indices, the symmetric division deg, the harmonic and inverse sum indices, and the secondary atomic bond indices are followed by motivated the introduction of one-sided polyomino chain and one-sided hexagonal chain problems have been studied and cellulose $(C_6H_{10}O_5)_d$ is one of the most common organic polymers on earth. It is an important structural component of the major cell walls of green plants, algae, and oomycetes. Recently, In [4,6,18,22] authors investigated various cellulose extractor techniques that are built by incorporating various procedures including oxidation, etherification, and esterification to transform the generated cellulose into modified cellulose and noncrystalline cellulose. Its modified versions are non-toxic, biodegradable polymers with high tensile and compressive strengths, and they are widely used in a variety of industries, including the pharmaceutical sector and nanotechnology. Both chemical and physical processes could be used to modify cellulose to make it appropriate for certain uses and add value to it. While chemical modification comprises acid hydrolysis, oxidation, esterification and etherification, amidation, carbamation, and UV or laser therapy, physical transformation involves plasma and corona discharge, UV or laser therapy. Both in the context of complex networks and in more advanced methods of chemical graph theory, it garnered a great deal of interest. Some of the well-known bond-additive distance-based indices, such as the Padmakar-Ivan index (PI index), Szeged index, and Mostar index, use polynomials to measure the peripherality of individual bonds (i.e., edges), then add up their individual contributions to produce a total measure of peripherality for a given graph.

We provided the essential definitions in this section. The graphs that are taken into consideration in this article are finite, connected, and simple graphs G =(V(G),E(G)) that are used to study the chemical graph of cellulose G={ $G_1,G_2,...,G_d$:x_i|i=1,2,...,d-1}.The graph was created from the graphs G_1 ,G_2 ,...,G_d by the linking vertex x_i∈[$G_i G_{i+1}^{d-1}$] for all i=1,2,...,d-1. The notation V(G) stands for vertex and E(G) stands for the edge of the graph G. We determine the symbol $\wedge_u$, the degree of a vertex u∈V(G) is the number of edges associated with u. Topological indices of graphs are denoted by contributions of edges according to the terminology used in graph theory. The Wiener index is one of the oldest and most widely investigated topological indices, introduced by Wiener [35] in 1947 for the study of the boiling points of paraffins. The number of vertices in T which are closer to u than to v is denoted by $\prod_u$(e|T). For trees T the Wiener index is defined as follows

$$W(T) = \sum_{e=uv\in E(T)} \prod_u(\mathrm{e|T})\prod_v(\mathrm{e|T})$$

where $\prod_v$(e|T) is defined analogously. This equivalence for the Wiener index of trees served as inspiration, in [16] Gutman put forward to the Szeged index defined as, see[11,27,29,30].

$$S_Z(G) = \sum_{e=uv\in E(T)} \prod_u(\mathrm{e|G})\prod_v(\mathrm{e|G})$$

$$S_{Z_w}(G) = \sum_{e=uv\in E(T)} ((\wedge_u\,(\mathrm{e|G}) + \wedge_v\,(\mathrm{e|G}))\prod_u(\mathrm{e|G})\prod_v(\mathrm{e|G})$$

The Szeged index discovered uses for modelling the physicochemical characteristics and physiological actions of organic compounds acting as pharmaceuticals or containing pharmacological activity, conjugates of PI related papers[25,24], and its mathematical representations may be found in [7,22]. Based on the significant success of the Szeged and Wiener indices, the Padmakar-Ivan (PI) vertex version proposed by Khalifeh et al., [28] and the weighted Padmakar-Ivan index are defined as follows

$$PI(G) = \sum_{e=uv\in E(T)} (\prod_u(\mathrm{e|G}) + \prod_v(\mathrm{e|G}))$$

$$PI_W(G) = \sum_{e=uv\in E(T)} ((\wedge_u\,(\mathrm{e|G}) + \wedge_v\,(\mathrm{e|G}))(\prod_u(\mathrm{e|G}) + \prod_v(\mathrm{e|G}))$$

The deep concentration of discriminating capability of the Padmakar-Ivan index in QSPR/QSAR research was described in [26,21], while Gutman and Ashrafi's thorough investigation of the index's fundamental mathematical features and applications in chemistry and nanoscience appeared in [17]. A bond additive structural invariant known as the Mostar index was established by Dosslic et al.,[12], to measure how far a graph is from being distance-balanced, the Mostar index is defined as follows;

$$Mo(G) = \sum_{e=uv\in E(T)} |\prod_u(\mathrm{e|G}) - \prod_v(\mathrm{e|G})|$$

$$Mo_A(G) = \sum_{e=uv\in E(T)} \big((\wedge_u\,(\mathrm{e|G}) + \wedge_v\,(\mathrm{e|G})\big)|\prod_u(\mathrm{e|T}) - \prod_v(\mathrm{e|G})|$$

$$Mo_M(T) = \sum_{e=uv\in E(T)} \big((\wedge_u\,(\mathrm{e|G}).\wedge_v\,(\mathrm{e|G})\big)|\prod_u(\mathrm{e|G}) - \prod_v(\mathrm{e|G})|$$

It measures the degree of peripherality of certain edges and the graph overall. Besides, along with the increasing the interest for context of complex networks, chemical graph theory has become a very important topic due to its various promising potential applications in real life (see[8,9,12,23,33]). There is a new method for calculating the weighted version of the Mostar invariants for Phthalocyanines dendrimers [19]. Recently, the author's studied the eigenvalue-based energy of HA-PTX conjugates and molecular structural analysis with the degree-based index. The ISI index and ISI energy of molecular structures of HA-PTX conjugates are developed in [10]. The following literatures contains a number of further findings about various topological indices under various graphical operations [3,20,34]. In [2,15,23,31,32,36], the authors were concerned about PI and Szeged polynomial. In any network, the PI polynomial is based on measuring opposing edge strips, put x=1, PI (G, 1)=1 was developed PI(G)=PI'(G,1)$ and Sz(G)=Sz'(G,1). This polynomial, represented by the symbol PI (G, x), counts the non-equidistant edges in G.

$$PI(G,x) = \sum_{e=uv\in E(G)} x^{(\Pi_u(e|G)+\Pi_v(e|G))}$$

The Szeged polynomial defined as

$$Sz(G,x) = \sum_{e=uv\in E(G)} x^{(\Pi_u(e|G)\Pi_v(e|G))}$$

In Ali et al., [1] introduced the most typical one, and we use it here because the invariant emerges as the evaluation of its derivative at x = 1. The Mostar polynomial Mo (G, x) of a graph G is defined as.

$$Mo(G,x) = \sum_{e=uv\in E(G)} x^{|\Pi_u(e|G)-\Pi_v(e|G)|}$$

The Mostar index can be found by calculating the value of the Mostar polynomial's derivative at x = 1 hence,

Mo (G) = Mo'(G,1).

$$Mo_A(G,x) = \sum_{e=uv\in E(G)} x^{((\wedge_u(e|G)+\wedge_v(e|G))|\Pi_u(e|G)-\Pi_v(e|G)|}$$

$$Mo_M(G,x) = \sum_{e=uv\in E(G)} x^{((\wedge_u(e|G).\wedge_v(e|G))|\Pi_u(e|G)-\Pi_v(e|G)|}$$

where $\wedge_u$ ($\wedge_v$)number of edges incident to u(v).

2. MAIN RESULT

In this part, we examine the Mostar index, its weighted version, and its cellulose polynomials. When (CL_d =CL), for instance CL_d=G, declared Cellulose: the bond-additive indices of Cellulose Figure (a) shows the unit of cellulose molecular structure with linear iteration d. Figure (b) corresponds to the units of chemical graphs of cellulose for d=1 and d=3. Let's begin our discussion with the division of the edge set ECL_d) on the basis of vertices' degree in order to reach the desired outcome.

For the sake of convenience, we'll suppose that Ce= CL_d and CL(CL_i) are the vertex-induced subgraphs of CL, where CL_i is the ith units of CL_d. By analysing and observing the chemical graph structure, we find that the edge of E(CL_d) may be classified into five groups based on the degree as shown in the table below.

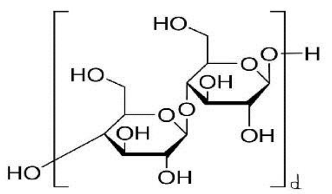
$E_1(CL_d)$={e=uv∈E(CL_d): $\wedge_u (CL_d) = 1, \wedge_v (CL_d) = 2$ } |$E_1(CL_d)$|=2d

$E_2(CL_d)$={e=uv∈E(CL_d): $\wedge_u (CL_d) = 1, \wedge_v (CL_d) = 3$ } |$E_2(CL_d)$|=4d+2

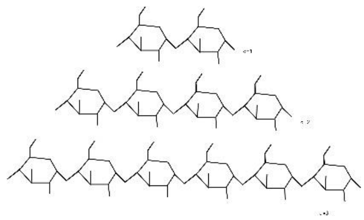
$E_3(CL_d)$={e=uv∈E(CL_d): $\wedge_u (CL_d) = 2, \wedge_v (CL_d) = 3$ } |$E_3(CL_d)$|=10d-3

$E_4(CL_d)$={e=uv∈E(CL_d): $\wedge_u (CL_d) = 3, \wedge_v (CL_d) = 3$ } |$E_4(CL_d)$|=8d

It can be observed that in general |V(CL_d)|=22d+1, |E(CL_d)|=24d.



(A) The molecular structure of cellulose



(B) The chemical graph of cellulose

Moreover, we can now determine the final observation regarding the Mostar index of cellulose graphs.

E	($\wedge_u, \wedge_v$)	edge	Π_u(e|CL_d)	Π_v(e|CL_d)	$\sum_i$ | Π_u (e|CL_d) −Π_v (e|CL_d)|
E_1	(1,2)	2d	1	22d	2d(22d-1)
E_2	(1,3)	4d+1	1	22d	(4d+2) (22d-1)
E_3	(2,3) (3,2)	2d d	2 22i-15	22d-1 22d+16-22i	2d(22d-3)

	(2,3)	d	22i-16	22d+17-22i	$\begin{cases} 11d^2 & if\ d\ is\ even \\ \frac{(d+1)(11d-2)}{2}+\frac{(d-1)(11d+2)}{2} & if\ dis\ odd \end{cases}$
	(3,2)	d	22i-11	22d+12-22i	$11d^2$
	(2,3)	d	22i-10	22d+11-22i	$\begin{cases} 11d^2 & if\ d\ is\ even \\ \frac{(d+1)(11d-10)}{2}+\frac{(d-1)(11d+10)}{2} & if\ dis\ odd \end{cases}$
	(3,2)	d	22d+18-22i	22i-17	$\begin{cases} 11d^2 & if\ d\ is\ even \\ \frac{(d-1)(11d+10)}{2}+\frac{(d+1)(11d-10)}{2} & if\ dis\ odd \end{cases}$
	(2,3)	d	22d+17-22i	22i-16	$\begin{cases} 11d^2 & if\ d\ is\ even \\ \frac{(d+1)(11d+2)}{2}+\frac{(d-1)(11d-2)}{2} & if\ dis\ odd \end{cases}$
	(2,3)	d-1	22(i-1)	22d+1-22(i-1)	$11d^2$ $\begin{cases} \frac{d}{2}(11d-21)+\frac{(d-2)(11d-1)}{2} & if\ d\ is\ even \\ 11(d-1)^2 & if\ dis\ odd \end{cases}$
	(3.2)	d-1	22(i-1)+1	22d-22(i-1)	$\begin{cases} \frac{d-2}{2}(11d-1)+\frac{d(11d-1)}{2} & if\ d\ is\ even \\ 11(d-1)^2 & if\ dis\ odd \end{cases}$
E_4	(3,3)	3d	22d+17-22i	22d-16	$11d^2$
		d	22i-15	22d+16-22i	$\begin{cases} 11d^2 & if\ d\ is\ even \\ \frac{(d+1)(11d-2)}{2}+\frac{(d-1)(11d+2)}{2} & if\ dis\ odd \end{cases}$
		3d	22i-5	22d+6-22i	$11d^2$
		d	22i-4	22d+5-22i	$\begin{cases} 11d^2 & if\ d\ is\ even \\ \frac{(d-1)(11d-2)}{2}+\frac{(d+1)(11d+2)}{2} & if\ dis\ odd \end{cases}$

2.1 Mostar index

We can get result as follows, first we applying edge partition method on distance, and degree based techniques. In ordered to calculate the bond-additive topological indices in the reference [7,8,9,13,14,22]. For information on recent studies that examined several gear and wheel-related graphs [12,25,26]. To determine the Mostar index, its polynomial and its weighted version.

Theorem 2.1. Let the chemical graph of Cellulose of CL_d then

$$\mathrm{Mo}(CL_d)=\begin{cases} 2d(22d-1)+2d(2d-3)+(4d+2)(22d-1)+66d^2+2\left(\frac{d}{2}(11d-21)+\frac{(d-2)}{2}(11d-1)\right) \\ +88d^2\ if\ d\ is\ even \\ 2d(22d-1)+2d(2d-3)+(4d+2)(22d-1)+22d^2+2\big((d+1)(11d-5)+(d-1)(11d+1)\big) \\ +88d^2\ if\ d\ is\ odd \end{cases}$$

Proof. By the definition of Mostar index of cellulose, using the edge partition and applying the Table., we have

$$\mathrm{Mo}(CL_d)=\sum_{uv=e\in E(CL_d)} |\ \Pi_u\ (CL_d)\ -\Pi_v\ (CL_d)|$$

$$=\sum_{uv=e\in E_1\cup E_2\cup E_3\cup E_4} |\ \Pi_u\ (CL_d)\ -\Pi_v\ (CL_d)|$$

Case(i): d is even

We have calculated the above summation separately, for the edge partition

For edge partition E_1 :

$$\text{Mo}(CL_d) = \sum_{uv=e\in E_1(CL_d)} |\ \Pi_u\ (CL_d)\ -\Pi_v\ (CL_d)|$$

$$= \sum_{\substack{i=1 \\ uv=e\in E_1(CL_d)}} |1-22d|$$

=2d(22d-1)

For edge partition E_2 :

$$\text{Mo}(CL_d) = \sum_{uv=e\in E_2(CL_d)} |\ \Pi_u\ (CL_d)\ -\Pi_v\ (CL_d)|$$

$$= \sum_{\substack{i=1 \\ uv=e\in E_2(CL_d)}} |1-22d|$$

=(4d+2)(22d-1)

For the edge partitionE_3:

$$\sum_{\substack{i=1 \\ e=uv\in E_3(e|CL_d)}}^{d} |\ \Pi_u(e|CL_d) - \Pi_v(e|CL_d)| \ = \sum_{\substack{i=1 \\ e=uv\in E_3(e|CL_d)}}^{d} |2-(22d-1)|$$

$$+ \sum_{\substack{i=1 \\ e=uv\in E_3(e|CL_d)}}^{d} |(22i-15)-(22d+16-22i)| + \sum_{\substack{i=1 \\ e=uv\in E_3(e|CL_d)}}^{d} |(22i-16)-(22d+17-22i)|$$

$$+ \sum_{\substack{i=1 \\ e=uv\in E_3(e|CL_d)}}^{d} |(22i-11)-(22d+12-22i)| + \sum_{\substack{i=1 \\ e=uv\in E_3(e|CL_d)}}^{d} |(22i-10)-(22d+11-22i)|$$

$$+ \sum_{\substack{i=1 \\ e=uv\in E_3(e|CL_d)}}^{d} |(22d+18-22i)-(22i-17)| + \sum_{\substack{i=1 \\ e=uv\in E_3(e|CL_d)}}^{d} |(22d+17-22i)-(22i-16)|$$

$$+ \sum_{\substack{i=1 \\ e=uv\in E_3(e|CL_d)}}^{d} |(22i)-(22d+1-22i)| + \sum_{\substack{i=1 \\ e=uv\in E_3(e|CL_d)}}^{d} |(22i+1)-(22d-22i)|$$

= 2d(22d-3)+11d^2+11d^2+11d^2+11d^2+11d^2+11d^2+$\frac{d}{2}(11d-21)+\frac{(d-2)(11d-1)}{2}$

$+\frac{(d-2)}{2}(11d-1)+\frac{d(11d-21)}{2}$

= 22d (22d-3)+66d^2+d(11d-21)+(d-2)(11d-1).

For the edge partitionE_4:

$$\sum_{\substack{i=1\\ e=uv\in E_4(G)}}^{d} |\Pi_u\ (e|CL_d)-\Pi_v\ (e|CL_d)|$$

$$= 3(11d^2) + \frac{(d+1)(11d-2)}{2}+\frac{(d-1)(11d+2)}{2}+3(11d^2) + \frac{(d-1)(11d-2)}{2} + \frac{(d+1)(11d+2)}{2}$$

$$= 33d^2 + 33d^2 + (d+1)(11d) + (d-1)(11d)$$

$$= 66d^2 + 22d^2 = 88d^2$$

Hence summarizing these values obtained here E_1, E_2, E_3 and E_4, we have

Mo(CL_d)=Mo(E_1)+Mo(E_2)+Mo(E_4)+Mo(E_4)

Mo(CL_d)=2d(221d-1)+2d(22d-3)+(4d+2)(22d-1)+6.11d^2+d(11d-21)+(d-2)(11d-1)+88d^2

=2d(221d-1)+2d(22d-3)+(4d+2)(22d-1)+ d(11d-21)+(d-2)(11d-1)+154d^2

Case(ii): d is odd

We have calculated the above summation separately, for the edge partition

For edge partitionE_1:

$$\sum_{\substack{i=1\\ e=uv\in E_1(G)}}^{d} |\Pi_u\ (e|CL_d)-\Pi_v\ (e|CL_d)| = 2d(22d-1)$$

For edge partitionE_2:

$$\sum_{\substack{i=1\\ e=uv\in E_2(G)}}^{d} |\Pi_u\ (e|CL_d)-\Pi_v\ (e|CL_d)| = (4d+2)(22d-1)$$

For edge partitionE_3:

$$\sum_{\substack{i=1\\ e=uv\in E_3(G)}}^{d} |\Pi_u\ (e|CL_d)-\Pi_v\ (e|CL_d)|$$

= 2d(22d-3)+($\frac{(d+1)(11d-2)}{2}+\frac{(d-1)(11d+2)}{2}+11d^2+\frac{(d+1)(11d-10)}{2}+\frac{(d-1)(11d+10)}{2}$)

$+\frac{(d-1)(11d+10)}{2}+\frac{(d+1)(11d-10)}{2})+\frac{(d+1)(11d+2)}{2}+\frac{(d-1)(11d-2)}{2}+11d^2+11(d-1)^2+11(d-1)^2$

=2d(22d-3)+2((d+1)(11d-5)+(d-1)(11d+5))+22d^2+2(11$(d-1)^2$)

For edge partitionE_4:

$$\sum_{\substack{i=1\\e=uv\in E_4(G)}}^{d} |\Pi_u\,(e|CL_d) - \Pi_v\,(e|CL_d)|$$

$$= 3(11d^2) + \frac{(d+1)(11d-2)}{2} + \frac{(d-1)(11d+2)}{2} + 3(11d^2) + \frac{(d-1)(11d-2)}{2} + \frac{(d+1)(11d+2)}{2}$$

$$= 33d^2 + 33d^2 + (d+1)(11d) + (d-1)(11d)$$

$$= 66d^2 + 22d^2 = 88d^2$$

Hence summarized the edge partitions, E_1, E_2, E_3 and E_4 we have

Mo(CL_d)=Mo(E_1)+Mo(E_2)+Mo(E_4)+Mo(E_4)

=2d(22d-1)+(4d+2)(22d-1)+2d(22d-3)+22d^2+2((d+1)(11d-5)+(d-1)(11d+5))+88d^2.

Hence the theorem

Now using the Table.2., we computing following results

2.2 Weighted version of Mostar index

Theorem.2.2. Let the chemical group of cellulose of CL_d then

$$Mo_A(CL_d)=\begin{cases} 6d(22d-1)+4(4d+2)(22d-1)+5\big(2d(22d-3)\big)+66d^2+d(11d-21)+ \\ (d-2)(11d-1)+88d^2 \qquad if\ d\ is\ even \\ 6d(22d-1)+4(4d+2)(22d-1)+5\big(2d(22d-3)\big)+22d^2+2(d+1)(11d-5)+(d-1) \\ (11d-5)+2(11(d-1)^2)+6(88d^2) \qquad if\ d\ is\ odd \end{cases}$$

Proof: by definition of additively weighted Mostar index and using edge partitio0n of Cellulose and from the Table. 1., we have

$$Mo_A(CL_d) = \sum_{\substack{i=1\\e=uv\in E_i(G)}}^{4} (\wedge_u\ (e|CL_d) + \wedge_v\ (e|CL_d)|)|\Pi_u\,(e|CL_d) - \Pi_v\,(e|CL_d)|$$

$$= \sum_{e=uv\in E_1(G)} (1+2)|\Pi_u\,(e|CL_d) - \Pi_v\,(e|CL_d)| + \sum_{e=uv\in E_2(G)} (1+3)|\Pi_u\,(e|CL_d) - \Pi_v\,(e|CL_d)|$$

$$+ \sum_{e=uv\in E_3(G)} (2+3)|\Pi_u\,(e|CL_d) - \Pi_v\,(e|CL_d)| + \sum_{e=uv\in E_4(G)} (3+3)|\Pi_u\,(e|CL_d) - \Pi_v\,(e|CL_d)|$$

Case(i): d is even

We have calculated the above summation separately, for the edge partition.

For edge partition E_1:

$$\sum_{e=uv\in E_1(G)} (1+2)|\Pi_u\,(e|CL_d) - \Pi_v\,(e|CL_d)| = 6d(22d+1)$$

For edge partition E_2

$$\sum_{e=uv\in E_2(G)} (1+3)|\Pi_u\,(e|CL_d) - \Pi_v\,(e|CL_d)| = 4(4d+1)(22d-1)$$

For edge partitionE_3:

$$\sum_{e=uv\in E_3(G)} (2+3)|\Pi_u\ (e|CL_d) - \Pi_v\ (e|CL_d)|$$
$$= \quad 5\big(2d(22d-3)\big) + 6d^2 + d(11d-21) + (d-2)(11d-1)$$

For edge partitionE_4:

$$\sum_{e=uv\in E_4(G)} (3+3)|\Pi_u\ (e|CL_d) - \Pi_v\ (e|CL_d)| = 6(88d^2)$$

Hence by summation separately, for the edge partitionE_1, E_2, E_3, and E_4

$Mo_A\ (CL_d)$ =$Mo_A\ (E_1)$+$Mo_A\ (E_2)$+$Mo_A\ (E_3)$+$Mo_A\ (E_4)$

$Mo_A\ (CL_d)$=6d(22d-1)+4(4d+2)(22d-1)+5(2d(22d-3))+66$d^2 + d(11d-21) + d-2)(11d-1) + 6(88d^2)$

Case(ii): d is odd

We have calculated the above summation separately, for the edge partition.

For edge partition E_1:

$$\sum_{e=uv\in E_1(G)} (1+2)|\Pi_u\ (e|CL_d) - \Pi_v\ (e|CL_d)| = 6d(22d+1)$$

For edge partition E_2

$$\sum_{e=uv\in E_2(G)} (1+3)|\Pi_u\ (e|CL_d) - \Pi_v\ (e|CL_d)| = \quad 4(4d+1)(22d-1)$$

For edge partitionE_3:

$$\sum_{e=uv\in E_3(G)} (2+3)|\Pi_u\ (e|CL_d) - \Pi_v\ (e|CL_d)|$$
$$= 5(2d(22d-3) + 2\big((d+1)(11d-5) + (d-1)(11d+5)\big) + 22d^2 + 2(11(d-1)^2$$

For edge partitionE_4:

$$\sum_{e=uv\in E_4(G)} (3+3)|\Pi_u\ (e|CL_d) - \Pi_v\ (e|CL_d)| = 6(88d^2)$$

Hence by summation separately, for the edge partitionE_1, E_2, E_3, and E_4

$Mo_A\ (CL_d)$=$Mo_A\ (E_1)$+$Mo_A\ (E_2)$+$Mo_A\ (E_3)$+$Mo_A\ (E_4)$

=$6d(22d+1) +\ 4(4d+1)(22d-1)$ +$5(2d(22d-3) + 2\big((d+1)(11d-5) + (d-1)(11d+5)\big) + 22d^2 + 2(11(d-1)^2) + 6(88d^2)$

Hence the theorem.

Theorem2.2. Let the chemical group of cellulose of CL_d then

$$Mo_M(CL_d)=\begin{cases} 4d(22d-1)+3(4d+2)(22d-1)+6\big(2d(22d-3)\big)+66d^2+d(11d-21)+ \\ (d-2)(11d-1))+9(88d^2) \quad if\ d\ is\ even \\ 4d(22d-1)+3(4d+2)(22d-1)+6(2d(22d-3)+22d^2+2(d+1)(11d-5)+(d-1) \\ (11d-5)+2(11(d-1)^2))+6(88d^2) \quad if\ d\ is\ odd \end{cases}$$

Proof: By definition of Multiplicatively weighted Mostar index and using edge partitio0n of Cellulose and from the Table. 1, we have

$$Mo_M(CL_d) = \sum_{\substack{i=1 \\ e=uv\in E_i(G)}}^{4} (\wedge_u (e|CL_d).\wedge_v\ (e|CL_d)|)|\Pi_u (e|CL_d) - \Pi_v (e|CL_d)|$$

$$= \sum_{e=uv\in E_1(G)} (1.2)|\Pi_u (e|CL_d) - \Pi_v (e|CL_d)| + \sum_{e=uv\in E_2(G)} (1.3)|\Pi_u (e|CL_d) - \Pi_v (e|CL_d)|$$

$$+ \sum_{e=uv\in E_3(G)} (2.3)|\Pi_u (e|CL_d) - \Pi_v (e|CL_d)| + \sum_{e=uv\in E_4(G)} (3.3)|\Pi_u (e|CL_d) - \Pi_v (e|CL_d)|$$

Case(i): d is even

We have calculated the above summation separately, for the edge partition.

For edge partition E_1:

$$\sum_{e=uv\in E_1(G)} (1.2)|\Pi_u (e|CL_d) - \Pi_v (e|CL_d)| = 6d(22d+1)$$

For edge partition E_2:

$$\sum_{e=uv\in E_2(G)} (1.3)|\Pi_u (e|CL_d) - \Pi_v (e|CL_d)| = 3(4d+2)(22d-1)$$

For edge partitionE_3:

$$\sum_{e=uv\in E_3(G)} (2.3)|\Pi_u (e|CL_d) - \Pi_v (e|CL_d)|$$

$$= 6\big(2d(22d-3)\big)+6d^2+d(11d-21)+(d-2)(11d-1))$$

For edge partitionE_4:

$$\sum_{e=uv\in E_4(G)} (3.3)|\Pi_u (e|CL_d) - \Pi_v (e|CL_d)| = 9(88d^2)$$

Hence by summation separately, for the edge partitionE_1, E_2, E_3, and E_4

Mo_M (CL_d) =Mo_M (E_1)+Mo_M (E_2)+Mo_M (E_3)+Mo_M (E_4)

Mo_M (CL_d)= 4d(22d-1)+3(4d+2)(22d-1)+6(2d(22d-3))+66$d^2+d(11d-21)+d-2)(11d-1))+$ $9(88d^2)$

Case(ii): d is odd

We have calculated the above summation separately, for the edge partition.

For edge partition E_1:

$$\sum_{e=uv\in E_1(G)} (1.2)|\Pi_u\,(e|CL_d) - \Pi_v\,(e|CL_d)| = 6d(22d+1)$$

For edge partition E_2

$$\sum_{e=uv\in E_2(G)} (1+3)|\Pi_u\,(e|CL_d) - \Pi_v\,(e|CL_d)| = \quad 4(4d+1)(22d-1)$$

For edge partitionE_3:

$$\sum_{e=uv\in E_3(G)} (2.3)|\Pi_u\,(e|CL_d) - \Pi_v\,(e|CL_d)|$$
$$= 6(2d(22d-3) + 2\big((d+1)(11d-5) + (d-1)(11d+5)\big) + 22d^2 + 2(11(d-1)^2))$$

For edge partitionE_4:

$$\sum_{e=uv\in E_4(G)} (3.3)|\Pi_u\,(e|CL_d) - \Pi_v\,(e|CL_d)| = 9(88d^2)$$

Hence by summation separately, for the edge partitionE_1, E_2, E_3, and E_4

$Mo_M\,(CL_d)$ =$Mo_M\,(E_1)$+$Mo_M\,(E_2)$+$Mo_M\,(E_3)$+$Mo_M\,(E_4)$

$= \; 4d(22d+1) + \; 3(4d+2)(22d-1)+6(2d(22d-3)$

$+2\big((d+1)(11d-5) + (d-1)(11d+5)\big) + 22d^2 + 2(11(d-1)^2) + 9(88d^2)$

Hence the theorem.

2.3. Mostar Polynomial

Theorem. 2.4. Let the chemical graph of Cellulose of CL_d then

$Mo(Cl_d, x) \; = 2\mathrm{d}x^{22d-1} + \; (4\mathrm{d}+2)x^{22d-1} \; + (2\mathrm{d})x^{22d-3}$

$$+2(\sum_{i=1}^{\frac{d}{2}} x^{22d+31-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+31))} + \; 5 \; \sum_{i=1}^{\frac{d}{2}} x^{22d+33-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+33)}$$

$$+ \; \sum_{i=1}^{\frac{d}{2}} x^{22d+23-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+23)} + \sum_{i=1}^{\frac{d}{2}} x^{22d+21-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+21)}$$

$$+\sum_{i=1}^{\frac{d}{2}} x^{22d+35-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+35)} + \quad + 2(\sum_{i=1}^{\frac{d}{2}} x^{22d+1-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+1)})$$

$$+ 3\sum_{i=1}^{\frac{d}{2}} x^{22d+11-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+11)} + \sum_{i=1}^{\frac{d}{2}} x^{22d+9-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+9)} \ if\ d\ is\ even$$

$$=2dx^{22d-1}+ (4d+2)x^{22d-1} + (2d)x^{22d-3}$$

$$+2\sum_{i=1}^{\frac{d+1}{2}} x^{22d+31-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+31)} + 5 \sum_{i=1}^{\frac{d+1}{2}} x^{22d+33-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+33)}$$

$$+ \sum_{i=1}^{\frac{d+1}{2}} x^{22d+23-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+23)} + \sum_{i=1}^{\frac{d+1}{2}} x^{22d+21-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+21)}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{22d+35-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+35)} + 2(\sum_{i=1}^{\frac{d+1}{2}} x^{22d+1-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+1)})$$

$$+ 3\sum_{i=1}^{\frac{d+1}{2}} x^{22d+11-44i} \sum_{\frac{i=d+3}{2}}^{d} x^{44i-(22d+11)} + \sum_{i=1}^{\frac{d+1}{2}} x^{22d+9-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+9)} \ if\ d\ is\ odd$$

Proof: By the definition of Mostar polynomial and using edge partition of cellulose, using edge partition and from the Table 1., we have

$$Mo(CL_d, x) = \sum_{e=uv\in E(G)} x^{|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

$$= \sum_{e=uv\in E_1(G)} x^{|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} + \sum_{e=uv\in E_2(G)} x^{|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

$$+ \sum_{e=uv\in E_3(G)} x^{|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} + \sum_{e=uv\in E_4(G)} x^{|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

We have calculated the above summation separately, for the edge partition.

For edge partition E_1:

$$\sum_{e=uv\in E_1(G)} x^{|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} = \sum_{i=1}^{d} x^{|1-22d|} = 2d\, x^{22d-1}$$

For edge partition E_2:

$$\sum_{e=uv\in E_1(G)} x^{|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} = \sum_{i=1}^{d} x^{|1-22d|} = (4d+2)\, x^{22d-1}$$

For edge partition E_3:

$$\sum_{e=uv\in E_1(G)} x^{|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} = 2dx^{22d-3} + \sum_{i=1}^{d} x^{|(22i-15)-(22d+16-24)|}$$

$$+ \sum_{i=1}^{d} x^{|(22i-16)-(22d+17-22i|} + \sum_{i=1}^{d} x^{|(22i-11)-(22d+12-22i)|} + \sum_{i=1}^{d} x^{|(22i-10)-(22d+11-22i)|}$$

$$+ \sum_{i=1}^{d} x^{|(22d+18-22i)-(22i-17)|} + \sum_{i=1}^{d} x^{|(22d+17-22i)-(22i-16)|}$$

$$+ \sum_{i=1}^{d} x^{|22i-(22d+1-22i)|} + \sum_{i=1}^{d} x^{|(22i+1)-(22d-22i)|}$$

Case(I)) d is even : $= 2dx^{22d-3} + \sum_{i=1}^{\frac{d}{2}} x^{22d+31-44i} \quad \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+31)}$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{22d+33-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+33)} + \sum_{i=1}^{\frac{d}{2}} x^{22d+23-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+23)}$$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{22d+21-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+21)} + \sum_{i=1}^{\frac{d}{2}} x^{22d+35-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+35)}$$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{22d+33-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+33)} + \sum_{i=1}^{\frac{d}{2}} x^{22d+1-44i} \sum_{i=\frac{d+2}{2}}^{d-1} x^{44i-(22d+1)}$$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{22d+1-44i} \sum_{i=\frac{d+2}{2}}^{d-1} x^{44i-(22d+1)}$$

$$= 2dx^{22d-3} + \sum_{i=1}^{\frac{d}{2}} x^{22d+31-44i} \quad \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+31)}$$

$$+ 2\left(\sum_{i=1}^{\frac{d}{2}} x^{22d+33-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+33)}\right) + \sum_{i=1}^{\frac{d}{2}} x^{22d+23-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+23)}$$

$$+\sum_{i=1}^{\frac{d}{2}} x^{22d+21-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+21)} + \sum_{i=1}^{\frac{d}{2}} x^{22d+35-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+35)}$$

$$+ 2(\sum_{i=1}^{\frac{d}{2}} x^{22d+1-44i} \sum_{i=\frac{d+2}{2}}^{d-1} x^{44i-(22d+1)})$$

Case(ii) d is odd= $2dx^{22d-3} + \sum_{i=1}^{\frac{d+1}{2}} x^{22d+31-44i} \sum_{\frac{d+3}{2}}^{d} x^{44i-(22d+31)}$

$$+ \sum_{i=1}^{\frac{d+1}{2}} x^{22d+33-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+33)} + \sum_{i=1}^{\frac{d+1}{2}} x^{22d+23-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+23)}$$

$$+\sum_{i=1}^{\frac{d-1}{2}} x^{22d+21-44i} \sum_{i=\frac{d+1}{2}}^{d} x^{44i-(22d+21)} + \sum_{i=1}^{\frac{d+1}{2}} x^{22d+35-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+35)}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{22d+33-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+33)} + \sum_{i=1}^{\frac{d-1}{2}} x^{22d+1-44i} \sum_{i=\frac{d+1}{2}}^{d} x^{44i-(22d+1)}$$

$$+\sum_{i=1}^{\frac{d-1}{2}} x^{22d+1-44i} \sum_{i=\frac{d+1}{2}}^{d} x^{44i-(22d+1)}$$

$$= 2dx^{22d-3} + \sum_{i=1}^{\frac{d+1}{2}} x^{22d+31-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+31)}$$

$$+ 2(\sum_{i=1}^{\frac{d+1}{2}} x^{22d+33-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+33)}) + \sum_{i=1}^{\frac{d+1}{2}} x^{22d+23-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+23)}$$

$$+\sum_{i=1}^{\frac{d-1}{2}} x^{22d+21-44i} \sum_{i=\frac{d+1}{2}}^{d} x^{44i-(22d+21)} + \sum_{i=1}^{\frac{d+1}{2}} x^{22d+35-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+35)}$$

$$+ 2(\sum_{i=1}^{\frac{d-1}{2}} x^{22d+1-44i} \sum_{i=\frac{d+1}{2}}^{d-1} x^{44i-(22d+1)})$$

For edge partition E_4

$$\sum_{e=uv\in E_4(G)} x^{|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

$$= 3\sum_{i=1}^{d} x^{|(22d+17-22i)-(22i-16)|} + \sum_{i=1}^{d} x^{|(22i-15)-(22d+16-22i)|}$$

$$+ 3\sum_{i=1}^{d} x^{|(22i-5)-(22d+6-22i)|} + \sum_{i=1}^{d} x^{|(22i-4)-(22d+5-22i)|}$$

Case(i): d is even

$$= 3\sum_{i=1}^{\frac{d}{2}} x^{22d+33-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+33)} + \sum_{i=1}^{\frac{d}{2}} x^{22d+31-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+31)}$$

$$+3\sum_{i=1}^{\frac{d}{2}} x^{22d+11-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+11)} + \sum_{i=1}^{\frac{d}{2}} x^{22d+9-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+9)}$$

Case(i): d is odd

$$= 3\sum_{i=1}^{\frac{d+1}{2}} x^{22d+33-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+33)} + \sum_{i=1}^{\frac{d+1}{2}} x^{22d+31-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+31)}$$

$$+3\sum_{i=1}^{\frac{d+1}{2}} x^{22d+11-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+11)} + \sum_{i=1}^{\frac{d+1}{2}} x^{22d+9-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+9)}$$

Hence summarized the edge partition are E_1, E_2, E_3, and E_4.

Mo (CL_d, x) =Mo (E_1)+Mo (E_2)+Mo (E_3)+Mo (E_4)

Case(a): d is even

$$\text{Mo}(Cl_d, x) = 2\text{d}x^{22d-1} + (4\text{d}+2)x^{22d-1}$$

$$+(2\text{d})x^{22d-3} + 2\left(\sum_{i=1}^{\frac{d}{2}} x^{22d+31-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+31)}\right) + 5\sum_{i=1}^{\frac{d}{2}} x^{22d+33-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+33)}$$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{22d+23-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+23)} + \sum_{i=1}^{\frac{d}{2}} x^{22d+21-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+21)} +$$

$$\sum_{i=1}^{\frac{d}{2}} x^{22d+35-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+35)} + 2(\sum_{i=1}^{\frac{d}{2}} x^{22d+1-44i} \sum_{i=\frac{d+2}{2}}^{d-1} x^{44i-(22d+1)})$$

$$+ 3\sum_{i=1}^{\frac{d}{2}} x^{22d+11-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+11)} + \sum_{i=1}^{\frac{d}{2}} x^{22d+9-44i} \sum_{i=\frac{d+2}{2}}^{d} x^{44i-(22d+9)}$$

Case(b): if d is odd

$$Mo(CL_d, x) = 2dx^{22d-1} + (4d+2)x^{22d-1} + (2d)x^{22d-3}$$

$$+2\sum_{i=1}^{\frac{d+1}{2}} x^{22d+31-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+31)} + 5\sum_{i=1}^{\frac{d+1}{2}} x^{22d+33-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+33)}$$

$$+ \sum_{i=1}^{\frac{d+1}{2}} x^{22d+23-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+23)} + \sum_{i=1}^{\frac{d+1}{2}} x^{22d+21-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+21)}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{22d+35-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+35)} + 2(\sum_{i=1}^{\frac{d+1}{2}} x^{22d+1-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+1)})$$

$$+ 3\sum_{i=1}^{\frac{d+1}{2}} x^{22d+11-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+11)} + \sum_{i=1}^{\frac{d+1}{2}} x^{22d+9-44i} \sum_{i=\frac{d+3}{2}}^{d} x^{44i-(22d+9)}$$

Weighted version of Mostar polynomial:

Theorem. 2.5. Let the chemical graph of Cellulose of CL_d then

$$Mo_A(Cl_d, x) = 2dx^{3(22d-1)} + (4d+2)x^{4(22d-1)}$$

$$+(2d)x^{5(22d-3)} + \sum_{i=1}^{\frac{d}{2}} x^{5(22d+31-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+31)}$$

$$+ 2\sum_{i=1}^{\frac{d}{2}} x^{5(22d+33-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+33))} + \sum_{i=1}^{\frac{d}{2}} x^{5(22d+23-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+23))}$$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{5(22d+21-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+21))} + \sum_{i=1}^{\frac{d}{2}} x^{5(22d+35-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+35))}$$

$$+\ 2\left(\sum_{i=1}^{\frac{d}{2}} x^{5(22d+1-44i)} \sum_{i=\frac{d+2}{2}}^{d-1} x^{5(44i-(22d+1))}\right) +\ 3\sum_{i=1}^{\frac{d}{2}} x^{6(22d+11-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+11))}$$

$$+3\sum_{i=1}^{\frac{d}{2}} x^{6(22d+33-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+33))} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+31-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+31))}$$

$$+\sum_{i=1}^{\frac{d}{2}} x^{6(22d+9-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+9))}\ if\ d\ is\ even$$

$$=2dx^{3(22d-1)} + (4d+2)x^{4(22d-1)} + (2d)x^{5(22d-3)}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+31-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+31))} +\ 2\sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+33-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+33))}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+23-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+23))} + \sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+21-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+21))}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+35-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+35))} +\ 2\left(\sum_{i=1}^{\frac{d-1}{2}} x^{5(22d+1-44i)} \sum_{i=\frac{d+3}{2}}^{d-1} x^{5(44i-(22d+1))}\right)$$

$$+\ 3\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+11-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+11))} +\ 3\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+33-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+33))}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+31-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+31))} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+9-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+9))}\ if\ d\ is\ odd$$

Proof: By the definition of additively weighted Mostar polynomial and using edge partition of cellulose from the Table 1., we have

$$Mo_A(CL_d, x) = \sum_{e=uv\in E(G)} x^{(\Lambda_u(e|CL_d)+\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

$$= \sum_{e=uv\in E_1(G)} x^{(\Lambda_u(e|CL_d)+\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} + \sum_{e=uv\in E_2(G)} x^{(\Lambda_u(e|CL_d)+\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

$$+\sum_{e=uv\in E_3(G)} x^{(\Lambda_u(e|CL_d)+\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} + \sum_{e=uv\in E_4(G)} x^{(\Lambda_u(e|CL_d)+\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

We have calculated the above summation separately, for the edge partition.

For edge partition E_1:

$$\sum_{e=uv\in E_1(G)} x^{(\Lambda_u(e|CL_d)+\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} = \sum_{i=1}^{d} x^{(1+2)|1-22d|} \quad = 2d\, x^{3(22d-1)}$$

For edge partition E_2:

$$\sum_{e=uv\in E_2(G)} x^{(\Lambda_u(e|CL_d)+\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} = \sum_{i=1}^{d} x^{(1+3)|1-22d|} \quad = (4d+2)\, x^{4(22d-1)}$$

For edge partition E_3:

$$\sum_{e=uv\in E_3(G)} x^{(\Lambda_u(e|CL_d)+\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

$$= \quad 2dx^{(2+3)22d-3} + \sum_{i=1}^{d} x^{(2+3)|(22i-15)-(22d+16-24)|}$$

$$+ \sum_{i=1}^{d} x^{(2+3)|(22i-16)-(22d+17-22i|} + \sum_{i=1}^{d} x^{(2+3)|(22i-11)-(22d+12-22i)|}$$

$$+ \sum_{i=1}^{d} x^{(2+3)|(22i-10)-(22d+11-22i)|}$$

$$+ \sum_{i=1}^{d} x^{(2+3)|(22d+18-22i)-(22i-17)|} + \sum_{i=1}^{d} x^{(2+3)|(22d+17-22i)-(22i-16)|}$$

$$+ \sum_{i=1}^{d} x^{(2+3)|22i-(22d+1-22i)|} + \sum_{i=1}^{d} x^{(2+3)|(22i+1)-(22d-22i)|}$$

Case(i): d is even

$$= \quad 2dx^{5(22d-3)} + \sum_{i=1}^{\frac{d}{2}} x^{5(22d+31-44i)} \quad \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+31))}$$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{5(22d+33-44i)} \quad \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+33))} + \sum_{i=1}^{\frac{d}{2}} x^{5(22d+23-44i)} \quad \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+23))}$$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{5(22d+21-44i)} \quad \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+21))} + \sum_{i=1}^{\frac{d}{2}} x^{5(22d+35-44i)} \quad \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+35))}$$

$$+\sum_{i=1}^{\frac{d}{2}} x^{5(22d+33-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+33))} + \sum_{i=1}^{\frac{d}{2}} x^{5(22d+1-44i)} \sum_{i=\frac{d+2}{2}}^{d-1} x^{5(44i-(22d+1))}$$

$$+\sum_{i=1}^{\frac{d}{2}} x^{5(22d+1-44i)} \sum_{i=\frac{d+2}{2}}^{d-1} x^{5(44i-(22d+1))}$$

$$= \quad 2dx^{5(22d-3)} + \sum_{i=1}^{\frac{d}{2}} x^{5(22d+31-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+31))}$$

$$+ \ 2(\sum_{i=1}^{\frac{d}{2}} x^{5(22d+33-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+33))}) + \sum_{i=1}^{\frac{d}{2}} x^{5(22d+23-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+23))}$$

$$+\sum_{i=1}^{\frac{d}{2}} x^{5(22d+21-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+21))} + \sum_{i=1}^{\frac{d}{2}} x^{5(22d+35-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{5(44i-(22d+35))}$$

$$+ 2(\sum_{i=1}^{\frac{d}{2}} x^{5(22d+1-44i)} \sum_{i=\frac{d+2}{2}}^{d-1} x^{5(44i-(22d+1))})$$

Case(ii) d is odd

$$= \quad 2dx^{5(22d-3)} + \sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+31-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+31))}$$

$$+ \ \sum_{i}^{\frac{d+1}{2}} x^{5(22d+33-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+33))} + \sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+23-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+23))}$$

$$+\sum_{i=1}^{\frac{d-1}{2}} x^{5(22d+21-44i)} \sum_{i=\frac{d+1}{2}}^{d} x^{5(44i-(22d+21))} + \sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+35-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+35))}$$

$$+\sum_{i}^{\frac{d+1}{2}} x^{5(22d+33-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+33))} + \sum_{i}^{\frac{d-1}{2}} x^{5(22d+1-44i)} \sum_{\frac{d+1}{2}}^{d} x^{5(44i-(22d+1))}$$

$$+\sum_{i=1}^{\frac{d-1}{2}} x^{5(22d+1-44i)} \sum_{i=\frac{d+1}{2}}^{d-1} x^{5(44i-(22d+1))}$$

$$= \quad 2dx^{5(22d-3)} + \sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+31-44i)} \sum_{\frac{d+3}{2}}^{d} x^{5(44i-(22d+31))}$$

$$+ \quad 2(\sum_{i}^{\frac{d+1}{2}} x^{5(22d+33-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+33))}) + \sum_{i}^{\frac{d+1}{2}} x^{5(22d+23-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+23))}$$

$$+\sum_{i}^{\frac{d-1}{2}} x^{5(22d+21-44i)} \sum_{i=\frac{d+1}{2}}^{d} x^{5(44i-(22d+21))} + \sum_{i}^{\frac{d+1}{2}} x^{(22d+35-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{5(44i-(22d+35))}$$

$$+ 2(\sum_{i}^{\frac{d-1}{2}} x^{5(22d+1-44i)} \sum_{\frac{d+1}{2}}^{d-1} x^{5(44i-(22d+1))})$$

For edge partition E_4

$$\sum_{e=uv\in E_4(G)} x^{(\Lambda_u(e|CL_d)+\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

$$= \quad 3 \sum_{i}^{d} x^{(3+3)|(22d+17-22i)-(22i-16)|} + \sum_{i}^{d} x^{(3+3)|(22i-15)-(22d+16-22i)|}$$

$$+ \quad 3 \sum_{i}^{d} x^{(3+3)|(22i-5)-(22d+6-22i)|} + \sum_{i}^{d} x^{(3+3)|(22i-4)-(22d+5-22i)|}$$

Case(i): d is even

$$= 3\sum_{i=1}^{\frac{d}{2}} x^{6(22d+33-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+33))} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+31-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+31))}$$

$$+3\sum_{i=1}^{\frac{d}{2}} x^{6(22d+11-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+11))} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+9-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+9))}$$

Case(ii): d is odd

$$= 3\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+33-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+33))} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+31-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+31))}$$

$$+3\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+11-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+11))} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+9-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+9))}$$

Hence summarized the edge partition are E_1, E_2, E_3, and E_4.

$Mo_A\ (CL_d, x) = Mo_A\ (E_1) + Mo_A\ (E_2) + Mo_A\ (E_3) + Mo_A\ (E_4)$

Case(a): d is even

$$Mo_A(Cl_d, x) \ = 2dx^{3(22d-1)} + \ (4d+2)x^{4(22d-1)} + (2d)x^{5(22d-3)}$$

$$+\sum_{i=1}^{\frac{d}{2}} x^{5(22d+31-44i)} \sum_{\frac{d+2}{2}}^{d} x^{5(44i-(22d+31))} + 2\sum_{i=1}^{\frac{d}{2}} x^{5(22d+33-44i)} \sum_{\frac{d+2}{2}}^{d} x^{5(44i-(22d+33))}$$

$$+ \ \sum_{i=1}^{\frac{d}{2}} x^{5(22d+23-44i)} \sum_{\frac{d+2}{2}}^{d} x^{5(44i-(22d+23))} + \ \sum_{i=1}^{\frac{d}{2}} x^{5(22d+21-44i)} \sum_{\frac{d+2}{2}}^{d} x^{5(44i-(22d+21))}$$

$$+\sum_{i=1}^{\frac{d}{2}} x^{5(22d+35-44i)} \sum_{\frac{d+2}{2}}^{d} x^{5(44i-(22d+35))} + \ 2(\sum_{i=1}^{\frac{d}{2}} x^{5(22d+1-44i)} \sum_{\frac{d+2}{2}}^{d} x^{5\left(44i-(22d+1)\right)})$$

$$3\sum_{i=1}^{\frac{d}{2}} x^{6(22d+33-44i)} \sum_{\frac{d+2}{2}}^{d} x^{6\left(44i-(22d+33)\right)} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+31-44i)} \sum_{\frac{d+2}{2}}^{d} x^{6\left(44i-(22d+31)\right)}$$

$$+ 3\ \sum_{i=1}^{\frac{d}{2}} x^{6(22d+11-44i)} \sum_{\frac{d+2}{2}}^{d} x^{6(44i-(22d+11))} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+9-44i)} \sum_{\frac{d+2}{2}}^{d} x^{6(44i-(22d+9))}$$

Case(b): if d is odd

$Mo_A(CL_d, x) = 2dx^{3(22d-1)} +$

$(4d+2)x^{4(22d-1)} + (2d)x^{5(22d-3)} + 2\sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+31-44i)} \sum_{\frac{d+3}{2}}^{d} x^{5(44i-(22d+31))}$

$$+ 2\sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+33-44i)} \sum_{\frac{d+3}{2}}^{d} x^{5(44i-(22d+33))} + \ \sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+23-44i)} \sum_{\frac{d+3}{2}}^{d} x^{5(44i-(22d+23))}$$

$$+ \ \sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+21-44i)} \sum_{\frac{d+3}{2}}^{d} x^{5(44i-(22d+21))} \ + \sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+35-44i)} \sum_{\frac{d+3}{2}}^{d} x^{5(44i-(22d+35))}$$

$$+\ 2\left(\sum_{i=1}^{\frac{d+1}{2}} x^{5(22d+1-44i)} \sum_{\frac{d+3}{2}}^{d} x^{5(44i-(22d+1))}\right) +\ 3\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+33-44i)} \sum_{\frac{d+3}{2}}^{d} x^{6(44i-(22d+33))}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+31-44i)} \sum_{\frac{d+3}{2}}^{d} x^{6(44i-(22d+31))}$$

$$+\ 3\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+11-44i)} \sum_{\frac{d+3}{2}}^{d} x^{6(44i-(22d+11))} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+9-44i)} \sum_{\frac{d+3}{2}}^{d} x^{6(44i-(22d+9))}$$

Theorem. 2.6. Let the chemical graph of Cellulose of CL_d then

$$\mathrm{Mo}_M(Cl_d, x) \ = 2\mathrm{d}x^{2(22d-1)} +\ (4\mathrm{d}+2)x^{4(22d-1)} + (2\mathrm{d})x^{6(22d-3)}$$

$$+\sum_{i=1}^{\frac{d}{2}} x^{6(22d+31-44i)} \sum_{\frac{d+2}{2}}^{d} x^{6(44i-(22d+31)} +\ 2\sum_{i=1}^{\frac{d}{2}} x^{6(22d+33-44i)} \sum_{\frac{d+2}{2}}^{d} x^{6(44i-(22d+33))}$$

$$+\ \sum_{i=1}^{\frac{d}{2}} x^{6(22d+23-44i)} \sum_{\frac{d+2}{2}}^{d} x^{6(44i-(22d+23))} +\ \sum_{i=1}^{\frac{d}{2}} x^{6(22d+21-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+21))}$$

$$+\sum_{i=1}^{\frac{d}{2}} x^{6(22d+35-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6\left(44i-(22d+35)\right)} +\ 2\left(\sum_{i=1}^{\frac{d}{2}} x^{6(22d+1-44i)} \sum_{i=\frac{d+2}{2}}^{d-1} x^{6\left(44i-(22d+1)\right)}\right)$$

$$+\ 3\sum_{i=1}^{\frac{d}{2}} x^{9(22d+33-44i)} \sum_{\frac{d+2}{2}}^{d} x^{9\left(44i-(22d+33)\right)} + \sum_{i=1}^{\frac{d}{2}} x^{9(22d+31-44i)} \sum_{\frac{d+2}{2}}^{d} x^{9(44i-(22d+31))}$$

$$+\ 3\sum_{i=1}^{\frac{d}{2}} x^{9(22d+11-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{9(44i-(22d+11))} +\ \sum_{i=1}^{\frac{d}{2}} x^{9(22d+9-44i)} \sum_{\frac{d+2}{2}}^{d} x^{9(44i-(22d+9))}\ \ if\ d\ is\ even$$

$$=2\mathrm{d}x^{2(22d-1)} + (4\mathrm{d}+2)x^{3(22d-1)} + (2\mathrm{d})x^{6(22d-3)}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+31-44i)} \sum_{\frac{d+3}{2}}^{d} x^{6(44i-(22d+31))} + 2 \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+33-44i)} \sum_{\frac{d+3}{2}}^{d} x^{6(44i-(22d+33))}$$

$$+ \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+23-44i)} \sum_{\frac{d+3}{2}}^{d} x^{6(44i-(22d+23)0)} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+21-44i)} \sum_{\frac{d+3}{2}}^{d} x^{6(44i-(22d+21))}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+35-44i)} \sum_{\frac{d+3}{2}}^{d} x^{6(44i-(22d+35))} + 2(\sum_{i=1}^{\frac{d-1}{2}} x^{6(22d+1-44i)} \sum_{\frac{d+3}{2}}^{d} x^{6(44i-(22d+1))})$$

$$+ 3 \sum_{i=1}^{\frac{d+1}{2}} x^{9(22d+11-44i)} \sum_{\frac{d+3}{2}}^{d} x^{9(44i-(22d+11))} + 3 \sum_{i=1}^{\frac{d+1}{2}} x^{9(22d+33-44i)} \sum_{\frac{d+3}{2}}^{d} x^{9(44i-(22d+33))}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{9(22d+31-44i)} \sum_{\frac{d+3}{2}}^{d} x^{9(44i-(22d+31))} + \sum_{i=1}^{\frac{d+1}{2}} x^{9(22d+9-44i)} \sum_{\frac{d+3}{2}}^{d} x^{9(44i-(22d+9))} \quad if\ d\ is\ odd$$

Proof: By the definition of Multiplicatively weighted Mostar polynomial and using edge partition of cellulose, from the Table 1., we have

$$Mo_M(CL_d, x) = \sum_{e=uv\in E(G)} x^{(\Lambda_u(e|CL_d).\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

$$= \sum_{e=uv\in E_1(G)} x^{(\Lambda_u(e|CL_d).\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} + \sum_{e=uv\in E_2(G)} x^{(\Lambda_u(e|CL_d).\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

$$+ \sum_{e=uv\in E_3(G)} x^{(\Lambda_u(e|CL_d).\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} + \sum_{e=uv\in E_4(G)} x^{(\Lambda_u(e|CL_d).\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

We have calculated the above summation separately, for the edge partition.

For edge partition E_1:

$$\sum_{e=uv\in E_1(G)} x^{(\Lambda_u(e|CL_d).\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} = \sum_{i=1}^{d} x^{(1.2)|1-22d|} = 2d\, x^{2(22d-1)}$$

For edge partition E_2:

$$\sum_{e=uv\in E_2(G)} x^{\Lambda_u(e|CL_d).\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|} = \sum_{i=1}^{d} x^{(1.3)|1-22d|} = (4d+2)\, x^{3(22d-1)}$$

For edge partition E_3:

$$\sum_{e=uv\in E_3(G)} x^{(\Lambda_u(e|CL_d).\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

$$= \quad 2dx^{(2.3)22d-3} + \sum_{i=1}^{d} x^{(2.3)|(22i-15)-(22d+16-24)|}$$

$$+ \sum_{i=1}^{d} x^{(2.3)|(22i-16)-(22d+17-22i|} + \sum_{i=1}^{d} x^{(2.3)|(22i-11)-(22d+12-22i)|}$$

$$+ \sum_{i=1}^{d} x^{(2.3)|(22i-10)-(22d+11-22i)|}$$

$$+ \sum_{i=1}^{d} x^{(2.3)|(22d+18-22i)-(22i-17)|} + \sum_{i=1}^{d} x^{(2.3)|(22d+17-22i)-(22i-16)|}$$

$$+ \sum_{i=1}^{d} x^{(2.3)|22i-(22d+1-22i)|} + \sum_{i=1}^{d} x^{(2.3)|(22i+1)-(22d-22i)|}$$

Case(i): d is even

$$= \quad 2dx^{6(22d-3)} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+31-44i)} \quad \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+31))}$$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{6(22d+33-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+33))} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+23-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+23))}$$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{6(22d+21-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+21))} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+35-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+35))}$$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{6(22d+33-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+33))} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+1-44i)} \sum_{i=\frac{d+2}{2}}^{d-1} x^{6(44i-(22d+1))}$$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{6(22d+1-44i)} \sum_{i=\frac{d+2}{2}}^{d-1} x^{6(44i-(22d+1))}$$

$$= \quad 2dx^{6(22d-3)} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+31-44i)} \quad \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+31))}$$

$$+ \quad 2\left(\sum_{i=1}^{\frac{d}{2}} x^{6(22d+33-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+33))}\right) + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+23-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+23))}$$

$$+ \sum_{i=1}^{\frac{d}{2}} x^{6(22d+21-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+21))} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+35-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+35))}$$

$$+ 2\left(\sum_{i=1}^{\frac{d}{2}} x^{6(22d+1-44i)} \sum_{i=\frac{d+2}{2}}^{d-1} x^{6(44i-(22d+1))}\right)$$

Case(ii) d is odd

$$= \quad 2dx^{6(22d-3)} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+31-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+31))}$$

$$+ \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+33-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+33))} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+23-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+23))}$$

$$+ \sum_{i=1}^{\frac{d-1}{2}} x^{6(22d+21-44i)} \sum_{i=\frac{d+1}{2}}^{d} x^{6(44i-(22d+21))} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+35-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+35))}$$

$$+ \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+33-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6\left(44i-(22d+33)\right)} + \sum_{i=1}^{\frac{d-1}{2}} x^{6(22d+1-44i)} \sum_{i=\frac{d+1}{2}}^{d} x^{6\left(44i-(22d+1)\right)}$$

$$+ \sum_{i=1}^{\frac{d-1}{2}} x^{6(22d+1-44i)} \sum_{i=\frac{d+1}{2}}^{d} x^{6(44i-(22d+1))}$$

$$= \quad 2dx^{6(22d-3)} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+31-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+31))}$$

$$+ \quad 2\left(\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+33-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+33))}\right) + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+23-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+23))}$$

$$+ \sum_{i=1}^{\frac{d-1}{2}} x^{6(22d+21-44i)} \sum_{i=\frac{d+1}{2}}^{d} x^{6\left(44i-(22d+21)\right)} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+35-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6\left(44i-(22d+35)\right)}$$

$$+2(\sum_{i=1}^{\frac{d-1}{2}} x^{6(22d+1-44i)} \sum_{i=\frac{d+1}{2}}^{d} x^{6(44i-(22d+1))})$$

For edge partition E_4

$$\sum_{e=uv\in E_4(G)} x^{(\Lambda_u(e|CL_d).\Lambda_v(e|CL_d))|\Pi_u(e|CL_d)-\Pi_v(e|CL_d)|}$$

$$= 3 \sum_{i=1}^{d} x^{(3.3)|(22d+17-22i)-(22i-16)|} + \sum_{i=1}^{d} x^{(3.3)|(22i-15)-(22d+16-22i)|}$$

$$+ 3 \sum_{i=1}^{d} x^{(3.3)|(22i-5)-(22d+6-22i)|} + \sum_{i=1}^{d} x^{(3.3)|(22i-4)-(22d+5-22i)|}$$

Case(i): d is even

$$= 3\sum_{i=1}^{\frac{d}{2}} x^{9(22d+33-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{9(44i-(22d+33))} + \sum_{i=1}^{\frac{d}{2}} x^{9(22d+31-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{9(44i-(22d+31))}$$

$$+3\sum_{i=1}^{\frac{d}{2}} x^{9(22d+11-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{9(44i-(22d+11))} + \sum_{i=1}^{\frac{d}{2}} x^{9(22d+9-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{9(44i-(22d+9))}$$

Case(i): d is odd

$$= 3\sum_{i=1}^{\frac{d+1}{2}} x^{9(22d+33-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{9(44i-(22d+33))} + \sum_{i=1}^{\frac{d+1}{2}} x^{9(22d+31-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{9(44i-(22d+31))}$$

$$+3\sum_{i=1}^{\frac{d+1}{2}} x^{9(22d+11-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{9(44i-(22d+11))} + \sum_{i=1}^{\frac{d+1}{2}} x^{9(22d+9-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{9(44i-(22d+9))}$$

Hence summarized the edge partition are E_1, E_2, E_3, and E_4.

$\mathrm{M}o_M\ (CL_d, x)$ =$\mathrm{M}o_M\ (E_1)$+$\mathrm{M}o_M\ (E_2)$+$\mathrm{M}o_M\ (E_3)$+$\mathrm{M}o_M\ (E_4)$

Case(a): d is even

$$\mathrm{M}o_M(Cl_d, x) = 2\mathrm{d}x^{2(22d-1)} + (4\mathrm{d}+2)x^{3(22d-1)}$$

$$+(2\mathrm{d})x^{6(22d-3)} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+31-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+31))}$$

$$+2 \sum_{i=1}^{\frac{d}{2}} x^{6(22d+33-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+33))} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+23-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+23))}$$

$$+\sum_{i=1}^{\frac{d}{2}} x^{6(22d+21-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+21))} + \sum_{i=1}^{\frac{d}{2}} x^{6(22d+35-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+35))}$$

$$+ 2(\sum_{i=1}^{\frac{d}{2}} x^{6(22d+1-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{6(44i-(22d+1))}) + 3\sum_{i=1}^{\frac{d}{2}} x^{9(22d+33-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{9(44i-(22d+33))}$$

$$+\sum_{i=1}^{\frac{d}{2}} x^{9(22d+31-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{9(44i-(22d+31))}$$

$$+3\sum_{i=1}^{\frac{d}{2}} x^{9(22d+11-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{9(44i-(22d+11))} + \sum_{i=1}^{\frac{d}{2}} x^{9(22d+9-44i)} \sum_{i=\frac{d+2}{2}}^{d} x^{9(44i-(22d+9))}$$

Case(b): if d is odd

$$\mathrm{Mo}_M(CL_d, x)=2\mathrm{d}x^{2(22d-1)}$$

$$+(4\mathrm{d}+2)x^{3(22d-1)} + (2\mathrm{d})x^{6(22d-3)} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+31-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+31))}$$

$$+2\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+33-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+33))} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+23-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+23))}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+21-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+21))} + \sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+35-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+35))}$$

$$+ 2(\sum_{i=1}^{\frac{d+1}{2}} x^{6(22d+1-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{6(44i-(22d+1))}) + 3\sum_{i=1}^{\frac{d+1}{2}} x^{9(22d+33-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{9(44i-(22d+33))}$$

$$+\sum_{i=1}^{\frac{d+1}{2}} x^{9(22d+31-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{9(44i-(22d+31))}$$

$$+3\sum_{i=1}^{\frac{d+1}{2}} x^{9(22d+11-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{9(44i-(22d+11))} + \sum_{i=1}^{\frac{d+1}{2}} x^{9(22d+9-44i)} \sum_{i=\frac{d+3}{2}}^{d} x^{9(44i-(22d+9))}$$

Conclusion

In this study, we primarily calculate the bond-additive based indices such as Mostar index, weighted version of Mostar index and their polynomials of cellulose graphs using chemical graph analysis are distance calculation. Our theoretical formulations demonstrate the great potential for practical implementation in pharmacy and chemical engineering. In future this work may extend to other indices.

References:

[1] A. Ali, and T. Dosli., Mostar index: Results and perspectives, Applied Mathematics and Computation, 404, (2021), 126245.

[2] M. Arockiaraj, S. Mushtaq, S. Klavzar, J. Celin Fiona, K. Balasubramanian, Szeged-like topological indices and the efficacy of the cut method: The case of melen structures, Discrete Math. Lett., 9, (2022), 49-56.

[3] P.Ali, S. Ajaz , K.O.A. Kirmani, F. Rugaie, Azam Kwong, Degree-based topological indices and polynomials of hyaluronic acid-curcumin conjugates, Saudi Pharmaceutical Journal, 28, (2020), 1093-1100.

[4] M. Asif, and M. Hussain, Topological characterization of cellulose network, Comp. J. Combin. Math., 2, (2020), 21-30.

[5] A.R. Ashrafi, M. Ghorbani, and M. Jalali , The PI and Edge Szeged Polynomials of an Infinite Family of Fullerenes Fullerenes, Nanotubes and Carbon Nanostructures, 18(2), (2010), 1536-4046.

[6] R. Cruz-Medina , D. A. Ayala-Hernández .et., Curing of Cellulose Hydrogels by UV Radiation for Mechanical Reinforcement, MDIP,13, (2021),23-42.

[7] M. Ahmad, Saeed , M. Javaid and E. Bonyah, Molecular Descriptor Analysis of Certain Isomeric Natural Polymers, Hindawi Journal of Chemistry,26, (2021), Article ID 9283246.

[8] Ö Colakoğlu Havare , Mostar Index (Mo) and Edge (Mo) Index of some Cycle Related Graphs Romanian journal of Mathematics and computer Science, 10, (2020), 53-66.

[9] Ö Colakogğlu Havare, Mostar Index of bridge graph, TWMS J.App.and Eng. Math., 11, (2021), 587-697.

[10] Ö Çolakoglu, The inverse sum indeg index (ISI) and (ISI) energy of Hyaluronic Acid-Paclitaxel molecules used in anticancer drugs, Open J. Discret. Appl. Math., 4(3), (2021), 72-81.

[11] A. Dobrynin, The Szeged and Wiener indices of line graphs, MATCH Commun. Math. Com-put. Chem., 79, (2018), 743-756.

[12] T. Dosilic, I. Martinjak,S. Skrekovski, S. Tipuric,S. Spuzevic, and I. Zubac, Mostar index, journal of Mathematical Chemistry, 56, (2018), 2995-3013.

[13] W. Gao and Wang .W, Second atom-bond connectivity index of special chemical molecular structures, Journal of Chemistry, 2014, (2014), Article ID 906254.

[14] A. Ghalavand , A,R. Ashrafi and M. Nehaad , On Mostar and Edge Mostar indices of graph, Topological Indices and Application of Graph Theory, 2021, (2021), Article ID 6651220.

[15] M. Ghorbani and M. Jalali, The Vertex PI, Szeged and Omega Polynomials of Carbon Nanocones CNC4[n], MATCH Commun. Math. Comput. Chem., 62, (2009), 353-362.

[16] I. Gutman and A. Dobrynin, The Szeged index - a success story, Graph Theory Notes New York, 34, (1998), 37-44.

[17] I. Gutman, and A.R. Ashrafi , On the PI Index of Phenylenes and their Hexagonal Squeezes, MATCH Commun. Math. Comput. Chem., 60, (2008), 135-142.

[18] P.K. Gupta., S.S. Raghunath ,D.V. Prasanna ,P. Venkat,V. Shree, C. Chithananthan, S. Choudhary, K. Surender and K. Geetha, An Update on Overview of Cellulose, Its Structure and Applications, web science, intech open, (2019), 1-23.

[19] K. Imrana ,S. Akhterb,F. Yasmeenc, and K. Alic, The Weighted Mostar Invariants of Phthalocyanines, Triazine- Based and

Nanostar Dendrimers,Polycyclic Aromatic Compounds, 42, (2022), 1-18.

[20] A. Jahanbani, Z. Shao and S.M. Sheikholeslami, Calculating degree based multiplicative topological indices of Hyaluronic Acid-Paclitaxel conjugates, molecular structure in cancer treatment, 39(14), (2021), 5304-5313.

[21] S.A.K. Kirmani., P. Ali and P. Azam, Topological indices and QSPR/QSAR analysis of some antiviral-drugs being investigated for the treatment of COVID-19 patients, Int J Quantum Chem., 121, (2021), e26594.

[22] A.J. Khalaf ., et al., Degree-based topological indices and polynomial of cellulose, journal of prime in mathematics, 17(1), (2021), 70-78.

[23] Kandan .P and Subramanian .S, On Mostar Index of Graph, Advance in Mathematics:Scientific Journal, 10, (2021), 2115-2126.

[24] P. Kandan, S. Subramanian and P. Rajesh Calculating PI related indices and their polynomial of Hyaluronic Acid and Conjugates , Journal of the Indonesian Mathematical Society, 28(2), 194-214.,2022

[25] P. Kandan ,S. Subramanian and P. Rajesh, Weighted Mostar index of certain graphs, Advance in Mathematical: Scientific Journal, 10(9), (2021),3093-3111.

[26] P. Kandan and S. Subramanian, Mostar index of Conical and generalized gear graph, Commun. Combin., Crypotogr. and Computer Sci, 2, (2022), 1-9.

[27] P.V. Khadikar, S. Karmarkar, V.K. Agrawal, J. Singh, A. Shrivastava,I. Lukovi, and M.V. Diudea, Szeged index Applications for drug modeling, Lett. Drug Design Disc., 2, (2005), 606-624.

[28] P.V. Khadikar and S. Karmarkar, A novel PI index and its applications to QSPR/QSAR studies, J. Chem. Inf. Comput. Sci., 41, (2001), 934-949.

[29] M.H. Khalifeh , H. Yousefi, A.R. Ashrafi, Vertex and edge PI-indices of Cartesian Product graphs, Discrete Appl. Math. 156, (2008), 1780-1789.

[30] M. Knor, R. Skrekovski, A.T. Tepeh Mathematical Aspects of Wiener Index, Ars. math. contemp. 11, (2016), 327-352.

[31] M. Nadeem, A. Yousaf, H. Alolaiyan and A. Razaq, Certain polynomials and related topological indices for the series of benzenoid graphs, scientific reports, 9(1), Article number: 9129, (2019), 1-9.

[32] M. Mirzargar, PI, Szeged and Edge Szeged Polynomials of a Dendrimer Nanostar, MATCH Commun. Math. Comput. Chem., 62, (2009), 363-370.

[33] N. Tranik, Computing the Mostar index in networks with application to molecular graphs, Iranian J. math. chem. 12, (2021), 1-18.

[34] J. Wang, Y.Wang, and L. Zheng, Computational on the topological indices of hyaluronic acid, Journal of Applied Analysis and Computation, 3(10), (2020), 1193-1198.

[35] H. Wiener, Structural determination of paraffin boiling points, J. Am. Chem. Soc., 69, (1947), 17-20.

[36] P. Kandan, S. Subramanian and P. Rajesh, Some Bond-Additive indices and its Polynomial of Cellulose , Journal of the Indonesian Mathematical Society, (Accepted)