τ^*-semi-generalized -closed sets in Topological spaces

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Abstract

We establish a new collection of sets termed τ^* semi-generalized closed and open sets in (M, τ^*) topological spaces. In this research and examine some of its characteristics.

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1 Introduction

Levine[4][5] explored the most fundamental features and introduced the notions of generalized closed sets in (M, τ) in topological spaces. Dunham[3] proposed the cl^* and topology τ^* and investigated their characteristics. Generalized semi-closed sets in topological spaces were introduced by and studied by Arya[1]. Semi-generalized closed sets were first introduced to topology by B.K. Lahiri and P. Bhattacharya[2]. In the weaker topological space (M, τ^*), Pushpalatha et al.[6] presented τ^* generalized closed sets. In $(M, \tau*)$ topological space Saranya et al.[7] presented τ^*-generalized-semi-closed sets. The purpose of this study is to define the notions of τ^*-semi-generalized-closed sets in (M, τ^*) topological spaces.

2 Preliminaries

M and N are topological spaces on which no speration axioms are assumed unless otherwise explicity stated. For a subset E of a topological space

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M, $int(E)$, $int^*(E)$, $cl(E)$, $cl^*(E)$, $scl^*(E)$ and A^c denote the interior, interior*, closure, closure*, semi closure* and complement of E respectively.

Definition 2.1. *E of a topological space (M, τ) is called a generalized closed[3] if $cl(E) \subseteq K$ whenever $E \subseteq K$ and K is open in M.*

Note For a subset E of a topological space (M, τ), the generalized closure of E denoted by $cl^*(E)$[2] is defined as the intersection of all g-closed sets containing E.

Definition 2.2. *Let (M, τ) be a topological space. Then the collection $\{K : cl^*(K^c) = (K^c)\}$ is a topology on M and is denoted as τ^*[4] that is $\tau^* = \{K : cl^*(K^c) = (K^c)\}$*

Remark For a subset E of M

$cl^*(E) = \cap\{O : O \supset E, O^c \in \tau^*\}$

$int^*(E) = \cup\{K : K \subset E, K \in \tau^*\}$

Definition 2.3. *A subset E of a topological space M is said to be τ^*-semi-closed set (briefly τ^*-s-closed) if $int^*(cl^*(E)) \subseteq E$. The complement of a τ^*-s-closed set is called the τ^*-semi-open set (briefly τ^*-s-open).*

For a subset E of a topological space (M, τ^*), the generalized semi closure of E denoted as $scl^*(E)$ is defined as the intersection of all semi closed sets in τ^* containing E

Definition 2.4. *A subset E of a topological space M is said to be τ^*-generalized-semi-closed set[7] (briefly τ^*-gs-closed) if $scl^*(E) \subseteq K$ whenever $E \subseteq K$ and K is τ^*-open.*

3 τ^*-Semi-Generalized-Closed Sets in Topological Spaces

In this section, we introduce the concept of τ^*-semi-generalized-closed sets in topological spaces.

Definition 3.1. *A subset E of a topological space M is said to be τ^* -semi-generalized-closed set (briefly τ^*-sg-closed) if $scl^*(E) \subseteq K$ whenever $E \subseteq K$ and K is τ^*-s- open . The complement of a τ^*-sg- closed set is called the τ^*-semi-generalized-open set (briefly τ^*-sg-open).*

Theorem 3.1. *Every closed set in M is τ^*-sg-closed*

Proof *Let E be a closed set. Let $E \subseteq K$ where K is τ^*-s-open. Since E is closed, $cl(E) = E \subseteq K$. But $scl^*(E) \subseteq cl(E)$. Hence we have $scl^*(E) \subseteq K$. Therefore E is τ^*-sg- closed.*

Theorem 3.2. *Every semi closed set in M is τ^*-sg-closed*
Proof *Let E be a semi closed set. Let $E \subseteq K$ where K is τ^*-s-open. Since E is semi closed. But $scl^*(E) \subseteq scl(E) \subseteq K$. Hence we have $scl^*(E) \subseteq K$. Therefore E is τ^*-sg-closed.*

Theorem 3.3. *Every τ^*-closed set in M is τ^*-sg-closed*
Proof *Let E be a τ^* closed set. Let $E \subseteq K$ where K is τ^*-s-open. Since E is τ^* closed. $scl^*(E) \subseteq cl^*(E)$. Therefore $scl^*(E) \subseteq K$. Hence E is τ^*-sg- closed.*

The converse is not true.

Example 3.1. *Consider the topological spaces $M = \{1, 2, 3\}$ with topology $\tau = \{M, \emptyset, \{1\}, \{1, 2\}\}$. Then the set $\{1\}$ is τ^*-sg-closed but not closed, semi-closed and τ^*-closed.*

Theorem 3.4. *Every τ^*-sg-closed set in M is τ^*-gs-closed*
The proof is straight forward from the definition. The converse is not true.

Example 3.2. *Consider the topological spaces $M = \{1, 2, 3\}$ with topology $\tau = \{M, \emptyset, \{1\}, \{1, 2\}\}$. Then the set $\{1, 2\}$ is τ^*-gs closed but not τ^*-sg-closed.*

Theorem 3.5. *If a subset E of X is τ^*-sg-closed and $E \subseteq F \subseteq scl^*(E)$ then F is τ^*-sg-closed.*
proof *Let E be a τ^*-sg- closed set and $E \subseteq F \subseteq scl^*(E)$. Let P be a τ^*-s-open set of M such that $F \subseteq$. Since E is τ^*-sg- closed, we have $scl^*(E) \subseteq P$. Now $scl^*(E) \subseteq scl^*(F) \subseteq scl^*[scl^*(E)] = scl^*(E) \subseteq P$. That is $scl^*(F) \subseteq P$ whenever P is τ^*-s-open. Therefore F is τ^*-sg-closed.*

Theorem 3.6. *Let E be an τ^*-sg-closed set. Then $scl^*(E) - E$ contains no non empty τ^*-s-closed set in M.*
Proof *Given E is a τ^*-sg- closed set. Let U be a non empty τ^*-s-closed subset of $scl^*(E) - E$. Now $U \subseteq scl^*(E) - E$. Then $U \subseteq scl^*(E) \cap E^c$, since $scl^*(E) - E = scl^*(E) \cap E^c$. Therefore $U \subseteq scl^*(E)$ and $U \subseteq E^c$. Therefore $E \subseteq U^c$. Since U^c is a τ^*-s-open set and E is τ^*-sg-closed, $scl^*(E) \subseteq U^c$. That is $U \subseteq [scl^*(E)]^c$. Hence $U \subseteq scl^*(E) \cap [scl^*(E)]^c = \emptyset$. That is $U = \emptyset$, a contradiction. Thus $scl^*(E) - E$ contains no non-empty τ^*-s-closed set in M.*

Corollary 3.1. *Let E be a τ^*-sg-closed set. Then E is τ^*-s-closed if and only if $scl^*(E) - E$ is τ^*-s-closed.*
proof *Given E is a τ^*-sg-closed set. If E is τ^*-s-closed, then we have $scl^*(E) - E = \emptyset$ which is τ^*-s-closed set. Conversely , let $scl^*(E) - E$ be a τ^*-s-closed. Then, by Theorem 3.6, $scl^*(E) - E$ does not contain any non-empty τ^*-s-closed subset and therefore $scl^*(E) - E = \emptyset$. This implies that $E = scl^*(E)$ and so E is τ^*-s closed set.*

Theorem 3.7. *The union of two τ^*-sg-closed sets is again a τ^*-sg-closed.*

proof *Assume that E and F are τ^*-sg-closed sets in M. Let K be an τ^*-s-open set in M such that $E \cup F \subseteq K$. Then $E \subseteq K$ and $F \subseteq K$. Since E and F are τ^*-sg-closed, $scl^*(E) \subseteq K$ and $scl^*(F) \subseteq K$. Hence $scl^*(E \cup F) = scl^*(E) \cup scl^*(F) \subseteq K$. Therefore $E \cup F$ is τ^*-sg-closed.*

Theorem 3.8. *For each $x \in M$ either the set $M - \{x\}$is τ^*-sg-closed in M or τ^*-s-open .*

Proof *Suppose $M - \{x\}$ is not τ^*-s-open . Then M is the only τ^*-s-open set containing $M - \{x\}$. This implies $scl^*(M - \{x\}) \subseteq X$. Hence $M - \{x\}$ is a τ^*-sg-closed in M.*

Remark We deduce the following consequences from the aforementioned observations.

closed $\rightarrow \tau^*$*closed* $\rightarrow \tau^*$-sg- closed $\rightarrow \tau^*$-gs- closed

The reverse implication is not true.

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