TWO DIMESIONAL STEADY FLOW OF STABLY STRATIFIED INCOMPESSIBLE INVISCID TOWARDS A SINK AND BIOMEDICAL APPLICATIONS

Dr. Rakesh Dube[1], Dr. Leena Bhardwaj[2]*

[1]Professor, SRM University Delhi-NCR, Sonepat, Haryana, India, Email: deansh@srmuniversity.ac.in; duberakesh@gmail.com

[2]*Assistant Professor, Department of Forensic Science, Amity University, Manesar, Haryana

***Corresponding Author: -** Dr. Leena Bhardwaj
*Assistant Professor, Department of Forensic Science, Amity University, Manesar, Haryana, Email: lbhardwaj@ggn.amity.edu

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Abstract:
Yih (1958), Deller (1959) , Kao (1964-65) and Dube (2002) considered the problem of two dimensional steady flow in a semi-infinite channel formed by $0 \le y \le a$, $-\infty \le x \le 0$ (the x-axis being horizontal and y-axis being vertical) of steady stratified incompressible inviscid fluid towards line sink across the bottom at $x = 0, y = 0$. Yin tackled the problem by assuming that the sink does not affect the density and velocity for upstream. He reduced the real flow to the pseudo-flow and solved the non-dimensionalzed equation.

for the pseudo-stream function. A solution was obtained for ψ valid for the Froude number $F > \frac{1}{\pi}$. If $F < \frac{1}{\pi}$, Yih showed that upstream waves will occur, which may violate the upstream conditions. He, however, argued that when $F < \frac{1}{k}$, an exact solution can still be obtained if waves are allowed upstream and if $\frac{d\rho}{d\psi}$ is still constant. Very large value of F will imply small stratification and so at $F = \infty$, the streamline pattern has practically no difference from that of potential flow of homogeneous fluid. As F decreases, an eddy develops near the upper corner which gradually elongates as $F \to \frac{1}{\pi}$. Yih also predicted that for $F < \frac{1}{\pi}$, flow separation may occur where an almost stagnant zone of fluid lies in the upper region while the fluid in the lower region flows towards the sink.

Debler in his experiment showed that $F = \frac{1}{\pi}$ is indeed a critical Froude number, above the Yih's solution is valid and below which the flow separates into a flowing zone towards the sink and a stagnant region about it.
Kao suggested that the stagnation of fluid above the flowing zone may be avoided by introducing a fictitious sink distribution at $x = 0$ from $y = a\,(a > 0)$ to $y = d$. He showed that the streamline at $y = a$ will divide the flow field into two: the flow due to sink at the origin and the flow due to the fictitious sink distribution. He also introduced a free streamline (a line of velocity discontinuity) which separates the flowing zone from the completely stagnant regime so as to obtain the mathematical solution of the problem posed by the sink flow with a stagnant zone.
Pao & Kao (1974) discussed the unsteady flow in a semi-infinite channel towards a line sink placed at the lower corner. They also studied the steady state flow pattern for $F = 0.32$ and found that the flow in the upper corner is slowed down substantially even to become almost stagnant but no corner eddy appears. This is in contradiction to Yih's result for the same value of F
Due to the symmetry of the flow about $x = 0$; Yih's studied the problem only for the semi-infinite channel flow towards the line sink placed at the origin. The fluid is bound by a wall at $x = 0$, and so he took the part of the $y - axis$ in the channel to be a streamline. The flow is also experimentally verified by the Debler for a vertical wall at $x = 0$. But if $x = 0$ is not a wall and if we consider the flow in the whole channel from

$x = \pm\infty$ towards the sink at the origin, it is doubtful whether the part of the $y-axis$ mentioned above will be a streamline. The reason may be that there is nothing y to support the weight of the fluid column (narrow) just above the sink, and hence the fluid coming from $x = \pm\infty$ towards the sink may not be able to come in contact with the y -axis except converting at the sink (i.e. at the origin). This will cause the existence of a vacuum like core (not occupied by fluid) around the y -axis above the origin, and this needs a deeper examination, which we consider only a little bit in the present place of work. Yih never mentioned about the behavior of the flow when such cases happen and it is also doubtful whether his solution will remain valid even if such cases arise. It is under the physical idea of free fall (due to gravity) mentioned above and also on Pao & Kao's contradiction to Yih's result on the formation of eddy at the upper corner that we propose here to restudy Yih's problem with a slight modification by taking the channel as infinite.

Nomenclature:

F	**:**	Froude number
F_0	:	Ordinary Froude number
$\bar{q}$	:	Velocity vector
$\bar{q}'$	:	Dimension of velocity of pseudo velocity
$\bar{F}$	:	External force other than gravitational force
$\bar{g}$	:	Acceleration due to gravity
p	:	Pressure
ρ	:	Density
ρ_0	:	Reference density
U	:	Horizontal velocity
U_0	:	Representative velocity
d	:	Reference length
g	:	Acceleration due to gravitation
ψ	:	Stream function
ψ'	:	Pseudo stream function
$\bar{k}$	:	Unit vector perpendicular to the plane of the motion
δ	:	Variation of height of the streamline.

Formulation of the problem: Equation and boundary conditions

With the x -axis horizontal and y -axis vertical, we consider the steady flow of a stably stratified fluid under the influence of gravity in an infinite channel formed by the horizontal planes $y = 0$ and $y = d$, where d is depth of the channel. The flow is considered to be two-dimensional and also is towards a sink placed at the origin $(x = 0, y = 0)$ lying at the bottom of the channel as shown in the figure-1 :

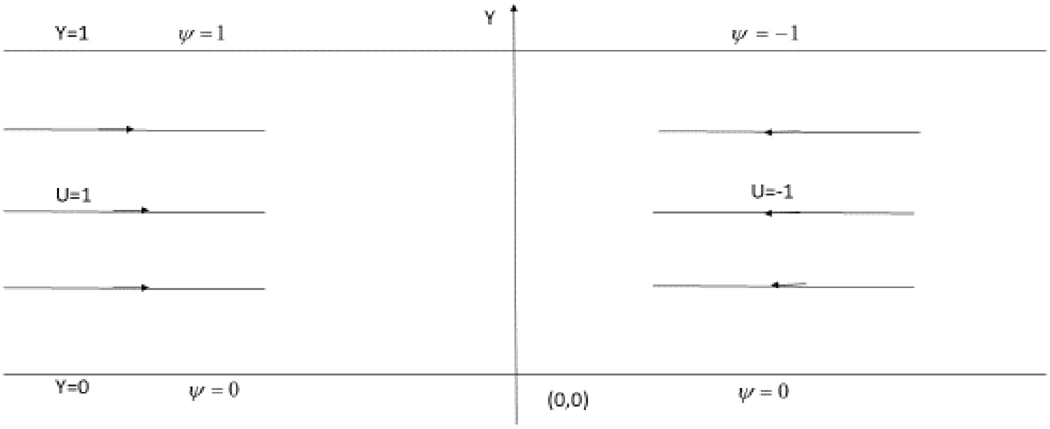

Fig-1 : Non-dimentionalized Pseudo-flow Model

The problem of the stratified flow will be studied by considering the pseudo-flow obtained through the transformation viz

$$\bar{q} = \left(\frac{\rho}{\rho_0}\right)^{-1/2} \bar{q}' \qquad (1)$$

here $\bar{q}$ is the true velocity (i.e. the velocity in the actual flow) and $\bar{q}'$, the pseudo-velocity (i.e. the velocity in the flow associated with the actual flow) ; ρ is the density and ρ_0 some reference density which in our problem, is taken to be the density at the lower boundary (i.e. at $y = 0$).

Since $\bar{q}'$ satisfies $\nabla.\bar{q}' = 0$, a stream function ψ' can be defined such that $\bar{q}' = -\bar{k} \times \nabla \psi'$; this stream function will be called "pseudo-stream function." The pseudo-flow will be described by the pseudo-stream function ψ' . The equation satisfied by ψ' is given by

$$\nabla^2 \psi' + \frac{1}{\rho_0}\frac{d\rho}{d\psi'} gy = \frac{1}{\rho_0}\frac{dH}{d\psi'} \qquad (2)$$

where $$H = p + \frac{1}{2}\rho_0 q'^2 + \rho g y \ . \qquad (3)$$

The equation (2) is the basic equation for the problem.

We make the following assumptions:

(i) that at $x = \pm\infty$, the pseudo-flow has the constant velocity $(\mp U_0)$ parallel to the x -axis, and

(ii) that at infinity, the density ρ decreases linearly with the height so that at $x = \pm\infty$, and

$$\rho = \rho_0\left(1 - \alpha \frac{Y}{d}\right) \qquad (4)$$

where $\alpha = \left(1 - \frac{\rho_1}{\rho_0}\right)$, ρ_1 being the density at the upper boundary (i.e. at $y = d$)

These assumptions imply parallel actual flow with non-zero vorticity at infinity; also they will make liner the equation (2) for the pseudo-stream function ψ' .

Taking d as the reference length and U_0 as the reference velocity , the non-demintionalized liner equation describing the pseudo-low in the left half of the channel is obtained as

$$\left(\nabla^2 + \beta^2\right)\psi = \beta^2 y \ , (x < 0) \qquad (5)$$

where $$\beta = F^{-1} = \left(\frac{U_0^2}{\alpha g d}\right)^{-1/2}.$$

The equation (5) will not represent the equation of the flow in the right half of the channel. Symmetry consideration of the flow however show that $\psi(-x, y) = -\psi(x, y)$ for $x \neq 0$. Thus the equation for the flow in the right half of the channel is given by

$$\left(\nabla^2 + \beta^2\right)\psi = -\beta^2 y \ ,(x > 0). \qquad (6)$$

The stream function ψ is anti-symmetric about the y-axis, but the vertical component (v) of the velocity is symmetrical about the said axis.

Since $v = -\frac{\partial \psi}{\partial x}$, the equation for the vertical component of the velocity is

$$\left(\nabla^2 + \beta^2\right)v = 0. \qquad (7)$$

This equation is true on both halves of the channel, which means unlike ψ, v satisfied a single equation for both halves of the channel.

It is, therefore, found more convenient and simpler to deal with the equation for v viz. equation (7) than the equations (5) and (6) for ψ. Considerations of the boundary considerations of v and ψ also supports this. Once v is determined from the equation (7), then we can determine ψ and the other flow variables. The task is thus reduced to the determination of v from the equation (7) , subject to the appropriate boundary conditions. The boundary conditions of v are set as follows:

$$\left.\begin{aligned} v = 0 \quad at \ x = \pm\infty \\ v = 0 \quad at \ \ y = 1 \\ v = 0 \quad at \ y = 0, x \neq 0 \end{aligned}\right\} \qquad (8)$$

As regards the boundary condition at $x = 0$, it is observed that the uppermost streamlines do not reach the y-axis except meeting it at the origin and as such , no specific value can be attached to v for $x = 0, y \neq 0$. But near the sink at the origin, v is of the form

$$v \sim \frac{y}{r^2}, (r^2 = x^2 + y^2).$$

So we may write it as $$v = -\lambda \frac{y}{r^2} \qquad (9)$$

where $-\lambda$ is the non-dimensional sink strength.

In view of the above, we can also write near the sink ($x = 0, y = 0$)

$$\psi = -\lambda \tan^{-1}\left(\frac{y}{x}\right) = -\lambda\theta \ ,(0 \leq \theta \leq \frac{\pi}{2}). \qquad (10)$$

By our assumption at $x = \infty$, the non-dimensional pseudo-velocity is constant being unity and parallel to the x-axis. So that $x = \infty, \psi = -y$. Therefore the lowest streamline $\psi = 0$ (corresponding to $y = 0$) goes to the origin along $\theta = 0$ while the uppermost streamline $\psi = -1$(corresponding to $y = 1$) goes to the origin along $\theta = \frac{\pi}{2}$. Hence , it follows from the equation (10) that $\lambda = \frac{\pi}{2}$. Hence , it follows from equation (10) that $\lambda = \frac{2}{\pi}$.

With this value of λ, the equation (9) becomes

$$v = -\frac{2}{\pi}\frac{y}{r^2} \ \ (\text{near } x = 0, y = 0). \qquad (11)$$

This is also true for the left half of the channel.

The equations (8) and (10) can together be replaced by $v = A\delta(x) \ at \ y = 0, \forall x$, where A is a real constant to be determined and $\delta(x)$ is the Dirac delta function.

We now put together all the boundary conditions of v as

$$\left.\begin{aligned} v=0 \quad at\ x=\pm\infty \\ v=0 \quad at\ \ y=1 \\ v=A\delta(x)\ \ at\ \ y=0,\forall x \end{aligned}\right\} \qquad (12)$$

Solution of $v\ \ for\ \ \beta<\pi$ **:**

In tracking the semi-infinite problem, Yih found that if $F\left(=\beta^{-1}\right)>\dfrac{1}{n}$, an analytical solution can be obtained , but if $F<\dfrac{1}{\pi}$, the upstream waves may occur and hence will violate the assumptions taken for the problem. Here in our problem, we may expect the same situation. So , we shall first try to investigate in detail the case for $F>\dfrac{1}{\pi}$, i.e. for $\beta<\pi$.

Though the flow in the present problem is symmetrical about $x=0$, Yih's solution for ψ can not be used at once; for the boundary conditions in Yih's problem and the present problem are not same in nature. The present problem is studied with the possible physical idea that the fluid does not come in contract with the y-axis except converging at the origin, and this idea leaves, unlike Yih's problem, ψ and hence v unspecified at $x=0$.

The direct attempt to solve the equation (7) by the method of separation of variables fails , for the constants appearing in the solution cannot fully be determined with the boundary conditions available, i.e. with the boundary conditions (12) only . Here we try to solve the equation (7) with the help of integral transform. This method can , in fact, remove the un-specification of the boundary condition on the y-axis.

To get the solution of the equation (7), we use the two-way Fourier transform. The two-way Fourier transform of $v(x,y)$ with respect to x is given by

$$\bar{v}(t,y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} v(x,y)\exp(-itx)dx\ . \qquad (13)$$

With this Fourier transform of $v(x,y)$, one finds from the equation (7)

$$\frac{1}{\sqrt{2\pi}}\left[\left(\frac{\partial v}{\partial x}+ity\right)\exp(-itx)\right]_{x=-\infty}^{\infty}-t^2\bar{v}+\frac{d^2\bar{v}}{dy^2}+\beta^2\bar{v}=0\ . \qquad (14)$$

At $x=\pm\infty, v=0$ (first condition of the boundary condition of (12); and also at $x=\pm\infty, \dfrac{\partial v}{\partial x}=0$ as v should vanish smoothly. However, if the imaginary part of t say η , be opposite in sign to that of x , then it is doubly sure that the integrated part of the above equation will be zero. Here we shall choose η as above. In fact, this restriction of η is necessary for the convergence of the integral in equation (13).

With the choice of η as stated above, the integrated part of equation (14) contributes nothing and accordingly that equation reduces to

$$\frac{d^2\bar{v}}{dy^2}-(t^2-\beta^2)\bar{v}=0. \qquad (15)$$

To solve this equation, we note the conditions to be satisfied by $\bar{v}$; they are

$$\text{at } y=1,\bar{v}=0 \text{ and at } y=0,\bar{v}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} A\delta(x)\exp(-itx)dx=\frac{A}{\sqrt{2\pi}} \qquad (16)$$

The solution of the equation (15) satisfying the above conditions given in the equation (15) is

$$\bar{v}(t,y)=\frac{A}{\sqrt{2\pi}}\frac{\sinh\left[\left(t^2-\beta^2\right)^{1/2}(1-y)\right]}{\sinh\left(t^2-\beta^2\right)^{1/2}}\ . \qquad (17)$$

To find $v(x,y)$, we are to take the inversion of $\bar{v}(t,y)$ with respect to t . The inverse Fourier transform of $\bar{v}(t,y)$ is

$$v(x,y)=\frac{A}{2\pi}\int_{-\infty+iy}^{\infty+iy}\frac{\sinh\left[(t^2-\beta^2)^{1/2}(1-y)\right]}{\sinh(t^2-\beta^2)^{1/2}}\exp(itx)dt. \qquad (18)$$

Here the integral is taken along the line $t=i\gamma$ from $-\infty\ to\ +\infty$. The convergence of the integral requires that both x and imaginary t should be of the same sign. We impose the restriction of $\gamma(=\operatorname{Im}t)$ such that no pole of integrand lies within the infinite strip between the real axis and the line $t=i\gamma$. The reason for this restriction on the imaginary part of t will be seen soon.

We now examine the general poles of the function

$$\frac{\sinh\left[(t^2-\beta^2)^{1/2}(1-y)\right]}{\sinh(t^2-\beta^2)^{1/2}}\exp(itx).$$

The points $t=\pm\beta$ are obviously not singularities. The singularities will be gven by the zeros of $\sinh\left(t^2-\beta^2\right)^{1/2}$ other than $t=\pm\beta$.

If we put $t=\xi+i\eta$, then we can write $\left(t^2-\beta^2\right)^{1/2}=C+iD$, where

$$C^2=\frac{1}{2}\left[(\xi^2-\eta^2-\beta^2)+\left\{(\xi^2-\eta^2-\beta^2)^2+4\xi^2\eta^2\right\}^{1/2}\right],$$

$$D^2=\frac{1}{2}\left[-(\xi^2-\eta^2-\beta^2)+\left\{(\xi^2-\eta^2-\beta^2)^2+4\xi^2\eta^2\right\}^{1/2}\right]$$

and therefore $\sinh(t^2-\beta^2)^{1/2}=\sinh(C+iD)=\sinh C\cos D+i\cosh C\sin D$.

So $\sinh(t^2-\beta^2)^{1/2}$ will be zero when $C=0$ and $D=0\ or\ \pm n\pi$, $(n=1,2,3,...)$.

Both $C=0\ and\ D=0$ gives $t=\pm\beta$ (a real quantity)

If $C=0\ and\ D=\pm n\pi$, then one finds $D^2=n^2\pi^2=\eta^2+\beta^2$ $(as\,C=0\ \ only\ \ when\,\xi=0)$,

whence $\eta=\pm(n^2\pi^2-\beta^2)^{1/2}$, $n=1,2,3,...$

By our assumption, $\beta<\pi$, and so $n^2\pi^2-\beta^2>0$ $(n=1,2,3,...)$ making η always real.

The above analysis shows that the poles of the function

$$\frac{\sinh\left[(t^2-\beta^2)^{1/2}(t-y)\right]}{\sinh(t^2-\beta^2)^{1/2}}\exp(itx)$$

are given by (excluding the non-singular poles $t=\pm\beta$).

$t=\pm i(n^2\pi^2-\beta^2)^{1/2}$, $n=1,2,3,...$ (purely imaginary) .

The lowest pole on the positive side of the imaginary axis is at $t=i(\pi^2-\beta^2)^{1/2}$.

So , if we choose γ (occurring in the limit of the integral given in the equation (18)) in such a way that $0<\gamma<(\pi^2-\beta^2)^{1/2}$, then , no pole of the integrand lies within the strip of the complex t-plane bounded by the line from $-\infty+iy\ to\ \infty+iy$ is the same as the integral taken along the line just above the real axis from $-\infty\ to\ +\infty$ (by Cauchy's theorem , the integrals $\int_{-R}^{-R+i\gamma}$ and $\int_{R}^{R+i\gamma}$ vanish as $R\to\infty$, each being of the order $o(e^{-R})$).

Hence with the restriction , the equation (18) becomes

$$v=\frac{A}{2\pi}\int_{-\infty+iO}^{\infty+iO}\frac{\sinh\left[(t^2-\beta^2)^{1/2}(1-y)\right]}{\sinh(t^2-\beta^2)^{1/2}}\exp(itx)dt. \qquad (19)$$

In the integral on the right hand side, t is purely real and the integral is taken along the line just above the real axis. This will ensure the value of v for $x>0$, i.e. the value of v on the right half of the channel.

For the evaluation of the integral, we first note that the integral $\int_{-\infty+iO}^{\infty+iO}$ is the same as the integral $\int_{-\infty}^{\infty}$.

So, to evaluate the integral, we consider a contour C consisting of the real axis from $t=-R_N \ \ to \ \ t=R_N$ and a semi-circle of radius R_N on the upper half of the real axis. Here we shall choose R_N to be large enough so that all the N poles viz $-i(n^2\pi^2-\beta^2)^{1/2}$,$(n=1,2,3,...,N)$ lie (without any exception) within C .
Then by the Cauchy's Residue theorem,

$$\int_C \frac{\sinh(t^2-\beta^2)^{1/2}(1-y)}{\sinh(t^2-\beta^2)}\exp(itx)dt=\int_{-R_N}^{R_N}+\int_{ltl=R_N}=2i\pi\sum \text{Re}\,sidues\,at\,poles\,with\,C$$

$$=2i\pi\sum_{n=1}^{N}\left[\frac{\left\{\sinh(t^2-\beta^2)(1-y)\right\}(t^2-\beta^2)^{1/2}}{\cosh(t^2-\beta^2)^{1/2}}\exp(itx)\right]_{t=(n^2\pi^2-\beta^2)^{1/2}}$$

$$=2\pi^2\sum_{n=1}^{N}\frac{n\sin n\pi y}{(n^2\pi^2-\beta^2)^{1/2}}\exp\left[-x(n^2\pi^2-\beta^2)^{1/2}\right] \qquad (20)$$

On the semi-circle, $t=R_Ne^{i\varphi}$, where φ is the argument of t, we find

$$\int_{ltl=R_N}\frac{\sinh\left[(t^2-\beta^2)^{1/2}(1-y)\right]}{\sinh(t^2-\beta^2)^{1/2}}\exp(itx)dt$$

$$=i\int_0^{\pi}\frac{\sinh\left[(R_N^2e^{2i\varphi}-\beta^2)^{1/2}(1-y)\right]}{\sinh\left[(R_N^2e^{2i\varphi}-\beta^2)^{1/2}\right]}\exp[-xR_N(\sin\varphi-i\cos\varphi]R_Ne^{i\varphi}d\varphi$$

The amplitude of the integrand is found to be order $o(R_Ne^{-R_N})$ and hence as $R_N\to\infty$ the above integral tends to zero.
Therefore, making $N\to\infty$, implies $R_N\to\infty$, we get from the equation (20)

$$\int_{-\infty}^{\infty}\frac{\sinh\left[\left(t^2-\beta^2\right)^{1/2}(1-y)\right]}{\sinh\left[\left(t^2-\beta^2\right)^{1/2}\right]}\exp(itx)dt$$

$$=2\pi^2\sum_{n=1}^{\infty}\frac{n\sin n\pi y}{(n^2\pi^2-\beta^2)^{1/2}}\exp\left[-x(n^2\pi^2-\beta^2)^{1/2}\right]$$

Therefore , denoting by v_+ the vertical component of the pseudo-velocity for $x>0$ (i.e. for the right half of the channel), we get from the equation (19)

$$v_+=\pi A\sum_{n=1}^{\infty}\frac{n\sin n\pi y}{(n^2\pi^2-\beta^2)^{1/2}}\exp\left[-x(n^2\pi^2-\beta^2)^{1/2}\right],(x>0) \qquad (21)$$

Since $v(x,y)$ is symmetrical about $x=0$, the vertical velocity for the left half of the channel $(x<0)$ is obtained simply by changing $x\ to\ -x$ in the equation (21) . Denoting this velocity by v_- , we have

$$v_-=\pi A\sum_{n=1}^{\infty}\frac{n\sin n\pi y}{(n^2\pi^2-\beta^2)^{1/2}}\exp\left[x(n^2\pi^2-\beta^2)^{1/2}\right],(x<0) \qquad (22)$$

The expressions (21) and (22) for $v_+ \ and \ v_-$ can now be put in a single expression as

$$v=A\sum_{n=1}^{\infty}\frac{n\sin n\pi y}{(n^2\pi^2-\beta^2)^{1/2}}\exp\left[-lxl(n^2\pi^2-\beta^2)^{1/2}\right] . \qquad (23)$$

Next, we proceed to evaluate A, which has so far been remaining undermined. To do this, we first consider the value of v near the origin given by the equation (23) and then compare it with the actual value near the origin ,i.e.

$$v=-\frac{2}{\pi}\frac{y}{r^2} \qquad \text{(equation (11))}.$$

Because of the summation nature of the expression for v , it is not so easy to obtain at once from the equation (23) the form to which the expression will reduce when $x\ and\ y$ are both small. Here, we try to obtain it by referring to the equation satisfied by v near the origin.

The general equation for v is the equation (7), viz., $\left(\nabla^2+\beta^2\right)v=0$.

In this equation, we see that $\frac{\nabla^2 v}{\beta^2 v} \sim \frac{1}{r^2\beta^2} >> 1$, near the origin.

(as near the origin, the distance from the origin $r<<1$ and β remains finite).

Hence, near the origin, the equation of v approximates to $\nabla^2 v=0$.

(This shows though indirectly that near the origin the flow is irrotational implies that the flow near the origin is sink flow)

It is expression for v (equation (23)) is reduced to such a form near the origin as satisfying the above equation viz. $\nabla^2 v=0$, then since in the reduced form the factor $\sin n\pi y$ is still to remain unchanged (as suggested by the boundary conditions at $y=0\ and\ y=1$), the exponential part involving x in the reduced form cannot contain β , and consequently there is no reason for inclusion of β in the denominator also.

Therefore, near the origin, the expression for v given by the equation (24) will reduce to (dropping β)

$$v=A\sum_{n=1}^{\infty}\sin n\pi y\ \exp(-n\pi lxl)=A\frac{\exp(-\pi lxl)\sin\pi y}{1+\exp(-2\pi lxl)-2\exp(-\pi lxl)\cos\pi y} \quad (24)$$

Hence when $x\ and\ y$ are both small , the expression (24) reduces to $v=A\frac{y}{r^2}$.

Comparing this with the actual value of v near the origin i.e. with the equation (1) we find $A=-2$.

With this value of A , we have from the equation (23)

$$v=-2\pi\sum_{n=1}^{\infty}\frac{n\sin\pi ny}{(n^2\pi^2-\beta^2)^{1/2}}\exp\left[-lxl(n^2\pi^2-\beta^2)^{1/2}\right]. \quad (25)$$

The solution given by the above equation is valid for the entire region of flow. The solution shows that v can never be positive for all x and $0\le y\le 1$.

Conclusion:

We have considered the problem of steady two-dimensional flow of a stably stratified, incompressible inviscid fluid towards a sink situated at the middle point (Origin) of the lower horizontal boundary of an infinite channel. The problem is investigated by the physical flow to a pseudo-flow as Yih did in the semi-infinite channel problem. The solution of the pseudo-flow is obtained first by the linearizing its equation and then by using the Fourier transforms.

The pseudo-flow is characterized by one parameter β , which is the inverse of the internal Froude number F . When $\beta=0$, the stratified fluid flow reduces to the homogeneous irrotational fluid flow(unaffected directly by gravity).

Our solution for β for one-half of the channel agrees well with that of Yig. But when $\beta\neq 0$, our solution differs greatly from Yih. It is found that as soon as β becomes non-zero, a central core in the form of α cone not occupied by the fluid starts developing around the vertical line through the sink (origin). The top of the core becomes wider and wider as β goes on increasing towards π . The reason for the development of core may be attributed to gravity effect. The abovementioned concepts may have its widespread biomedical applications, like diagnostic and therapeutic intervention of cardiac diseases, and/or therapeutic potency of drugs, may effect half life of drugs.

References:

1. Craper, G.D., A three dimensional solution for waves in the Lee of mountains. J Fluid Mechanics. 1959.
2. Debler, W.R., Stratified flow into a line sink. J. Eng. Mech. Div., Proc. ASCE, 1959.
3. Dube, Rakesh, On the flow of a viscous incompressible fluid through a rectilinear pipe, Proceedings of National Academy of Sciences, India , Sec-A, 70(A) ,IV, 2000.

4. Dube, Rakesh, Transient force convection energy equation for periodic variation of inlet temperature in channel of uniform square cross section, Springer India, Proceedings of National Academy of Sciences, India, 72(A),II, 2002.
5. Lamb, H., Hydrodynamics, Cambridge University Press, Lond, 1932.
6. Long, R.R., Some aspects of the flow of stratified fluids, I.A. Theoretical Investigation, Tellus , 1953.
7. Milne, J.W. and Thomson, C.B.E, Theoretical Dynamics, MacMillan & Co., New York, 1960
8. Oberhettinger, F, Fourier expansions: A collection of Formulas, Academic Press, New York, 1973.
9. Yih, C.S., On the flow of a stratified fluid, Proc. 3rd U.S. Congr. Appl. Mech., 1958.
10. Yih, C.S. , Effect of density variation on fluid flows, J.Fluid Mech, 1960.
11. Yih, C.S., Dynamics of Non-homogeneous fluids, MacMillan & Co., New York, 1965
12. Yih , C. S., Stratified Flows, Annual Review of Fluid Mechanics, Vol-1, 1969.