INHOMOGENEOUS BIANCHI TYPE VI$_0$ COSMOLOGICAL MODEL FOR PERFECT FLUID DISTRIBUTION IN GENERAL RELATIVITY

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Abstract
A string cosmological model based on Bianchi VI$_0$ inhomogeneous bifurcation theory has been explored in general relativity, with a perfect fluid distribution model. To accomplish the task of obtaining the deterministic solution to Einstein's field equations, we suppose that the inhomogeneous generalization of Bianchi type VI$_0$ with density and pressure may be used, and the model obtained will be the expanding, shearing, and spinning of the universe. In addition, talk about the model's physical and geometrical properties so that you can better understand how it works.

Keywords: Bianchi Type VI$_0$, space-time, cosmic-sting, and general relativity.

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1. Introduction

Our study is an investigation of the inhomogeneous string dust cosmological model proposed by Bianchi VI_0 in general relativity. In the framework of chaotic cosmology, there is a belief among scholars (Misner, 1968; MacCallium, 1971; Collins & Hawking, 1973) that considerable amounts of inhomogeneity and anisotropy were present in the early cosmos. This belief is based on the chaotic cosmology theory. On the other hand, because of the process of cosmic expansion, there is the chance that the universe will evolve towards homogeneity and isotropy at some point in the future. The pursuit of knowledge regarding the processes that led to the homogeneity and isotropization of the cosmos is one of the most significant challenges facing theoretical cosmology today. Studying perturbed FRW models (Lifshitz, 1946; Fall, 1979) is one method for tackling this problem. Constructing exact inhomogeneous extensions of specific Bianchi-types cosmological models (Belinskii and Khalatnikov, 1969; Belinskii et al., 1972; Barrow & Tipper, 1979) is another method. Both of these methods have been developed in recent decades. Matzner and Centrella (1979) conducted research that investigated the propagation of inhomogeneity non the form of pulses. Wainwright et al. (1979) developed accurate solutions that generalise Bianchi types-III, V, and VII models for vacuum and a stiff ideal fluid. In addition, Carmeli et al. (1981) have presented novel inhomogeneous generalisations of the Bianchi types-III, V, and VU models when a massless scalar field is present. These models were developed in the presence of the scalar field.

This research study focuses on a generalised form of the Bianchi type VI0 cosmological model, which was created by using accurate inhomogeneous solutions derived from Einstein's field equations. Specifically, the work examines the implications of this version of the model.

The model that was created as a consequence may be described as being non-rotating, expanding, and shearing. In order to give deeper insight into the model's specific characteristics, the physical qualities of the model as well as its geometric attributes are fully examined.

2. Field Equations

We consider "Bianchi type VI_0 " space-time metric

$$ds^2 = a^2(x,t)\{dx^2 - dt^2\} + e^{2x}b^2(t)dy^2 + e^{-2x}c^2(t)dz^2 \quad [2.1]$$

The Einstein field equation for perfect fluid distribution are

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j = -8\pi\left[(\rho+p)v_iv_j - pg_{ij}\right] \quad [2.2]$$

And $$T_i^j = -8\pi\left[(\rho+p)v_iv_j - pg_{ij}\right]$$

Where (ρ, p, v_i are density, pressure, and Unit flow vector respectively.) Assuming the comoving coordinates are $v_1 = v_2 = v_3 = 0$, and $v_4 = a$

Equation (2.2) and (2.3) for the space-time (2.1), lead to

$$\frac{1}{a^2}\left[\frac{b_{44}}{b} + \frac{c_{44}}{c} - \frac{a_4}{a}\left(\frac{b_4}{b} + \frac{c_4}{c}\right) + \frac{b_4c_4}{bc} + 1\right] = -8\pi p \quad [2.4]$$

$$\frac{1}{a^2}\left[\left(\frac{a_4}{a}\right)_4 - \left(\frac{a_4}{a}\right)_1 + \frac{b_{44}}{b} - 1\right] = -8\pi p \quad [2.5]$$

$$\frac{1}{a^2}\left[\left(\frac{a_4}{a}\right)_4 - \left(\frac{a_4}{a}\right)_1 + \frac{c_{44}}{c} - 1\right] = -8\pi p \quad [2.6]$$

$$\frac{1}{a^2}\left[1-\frac{a_4}{a}\left(\frac{b_4}{b}+\frac{c_4}{c}\right)-\frac{b_4c_4}{bc}\right]=-8\pi\rho \qquad [2.7]$$

$$\left(\frac{b_4}{b}-\frac{c_4}{c}\right)-\frac{a_1}{a}\left(\frac{b_4}{b}+\frac{c_4}{c}\right)=0 \qquad [2.8]$$

The suffices 4 and 1 after (a, b and c) denote partial differentiation w.r.t. t and x respectively.

3. Solution of Field Equations

From equation (2.5) and (2.6) we have

$$\frac{b_{44}}{b}=\frac{c_{44}}{c} \qquad [3.1]$$

Equation (2.8) leads to

$$\frac{a_1}{a}=\frac{\left(\frac{b_4}{b}-\frac{c_4}{c}\right)}{\left(\frac{b_4}{b}+\frac{c_4}{c}\right)}=f(t) \qquad [3.2]$$

where f(t) is some function of time alone. By equation (3.2) leads to

$$cb_4-bc_4=q \qquad [3.4]$$

From the equation (3.1)

$cb_4-bc_4=q$ [3.4]

Where q is a constant

Equation (3.2) and (3.4) give

$$f(t)=\frac{q}{(bc)_4} \qquad [3.5]$$

From equations (2.4),(2.6),(2.7), (3.3) and (3.4) , we have

$$x\left[f_{44}+f_4\left(\frac{b_4}{b}+\frac{c_4}{c}\right)\right]-\left[\frac{b_{44}}{b}+\frac{b_4c_4}{bc}+2\right]=0 \qquad [3.6]$$

From which we conclude that

$$f_{44}+f_4\left(\frac{b_4}{b}+\frac{c_4}{c}\right)=0 \qquad [3.7]$$

Equation (3.7) on integration yield

$$F_4=\frac{\alpha}{bc} \text{ where } \alpha \text{ is constant} \qquad [3.8]$$

From the equation (3.5) and (3.8) we get

$$F(t)=\beta(bc)^{\frac{\alpha}{q}} \text{ where } \beta \text{ is constant} \qquad [3.9]$$

From the equation (3.5) and (3.9) we get

$$bc=(LT)^{\frac{q}{\alpha+q}} \text{ where } L=\left[\frac{\alpha+q}{\beta}\right]^{\frac{q}{\alpha+q}} \qquad [3.10]$$

Equation (3.4) and (3.10) give

$$\frac{b}{c}=\alpha_0\exp\left[\frac{q(\alpha+q)}{\alpha L}.T^{\frac{\alpha}{\alpha+q}}\right] \qquad [3.11]$$

where α0 is a constant

From the equation (3.3), (3.9), and (3.10), we have

$$a(x,t) = \exp\left[x\beta\left(LT\right)^{\frac{\alpha}{\alpha+q}}\right]$$

$$a(x,t) = \exp\left[x\beta\left(LT\right)^{n}\right] \qquad [3.12]$$

From the equation (3.10) and (3.11)we have

$$b(t) = \left(LT\right)^{\frac{q}{2(\alpha+q)}} (\alpha_0)^{\frac{1}{2}} \exp\left[\frac{q(\alpha+q)}{2\alpha L} T^{\frac{\alpha}{\alpha+q}}\right]$$

$$b(t) = \left(LT\right)^{\frac{m}{2}} (\alpha_0)^{\frac{1}{2}} \exp\left[\gamma T^{n}\right] \qquad [3.13]$$

And

$$c(t) = \left(LT\right)^{\frac{q}{2(\alpha+q)}} (\alpha_0)^{-\frac{1}{2}} \exp\left[\frac{-q(\alpha+q)}{2\alpha L} T^{\frac{\alpha}{\alpha+q}}\right]$$

$$c(t) = \left(LT\right)^{\frac{m}{2}} (\alpha_0)^{-\frac{1}{2}} \exp\left[-\gamma T^{n}\right] \qquad [3.14]$$

Where $\frac{\alpha}{\alpha+q} = n$, $\frac{q}{\alpha+q} = m$ and $\frac{q}{2nL} = \gamma$

By transformation of coordinate & constants, then the line element reduces to the form

$$\begin{aligned} ds^2 &= \exp\left[x\beta\left(LT\right)^{n}\right]\left(dx^2 - dt^2\right) \\ &+ (LT)^{m} \alpha_0 \exp\left[\gamma . T^{n}\right] e^{2x} dy^2 \\ &+ \left(LT\right)^{m} (\alpha_0)^{-1} \exp\left[\gamma . T^{n}\right] . e^{-2x} dz^2 \end{aligned} \qquad [3.15]$$

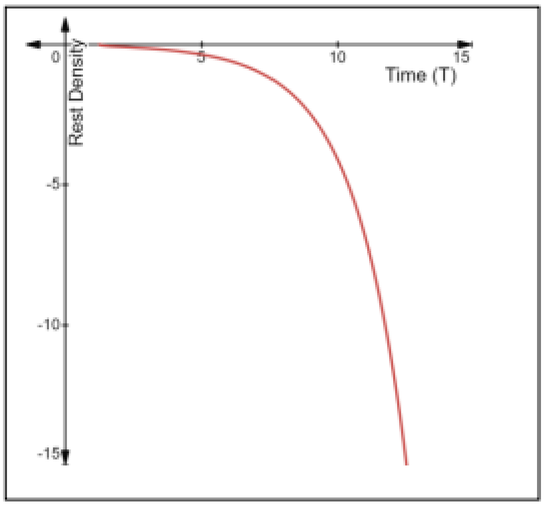
This might be thought of as an inhomogeneous generalisation of the ideal fluid distribution model proposed for Bianchi type VI0.

4. Geometrical and Physical Properties

The model's dimensional and topological characteristics are described as follows: Both the tension density (λ) and the rest density (ρ) of the string are

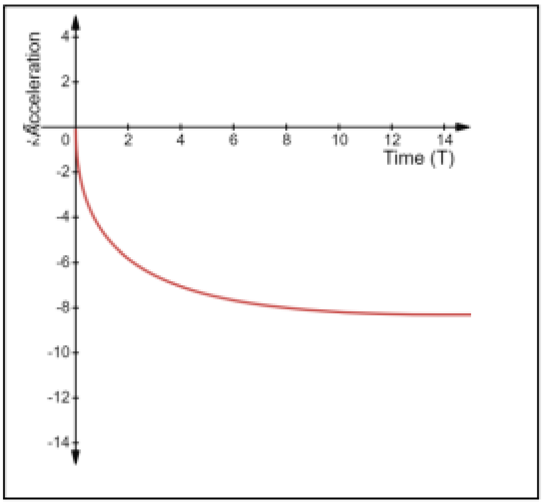
$$\lambda = \frac{-1}{8\pi \exp\left[2x\beta(LT)^{n}\right]}\left[\begin{aligned} & x\beta n(n-1)L^{2n+1}T^{n-1}\beta nL^{n}T^{n-1} \\ & + \frac{m(m-2)}{4T^2} + \gamma m T^{m-1}(m-1+mT^{m-1} \\ & + \gamma T^{2m+1} - 1 \end{aligned}\right] \qquad [4.1]$$

$$\rho = \frac{-1}{8\pi \exp\left[2x\beta(LT)^{n}\right]}\left[\begin{aligned} & 1 - \frac{x\beta n m L^{n-1}T^{n-2}}{2} \\ & - \frac{1}{T^2}\left(\frac{m^2}{4} - \gamma^2 m^2 T^{2m}\right) \end{aligned}\right] \qquad [4.2]$$



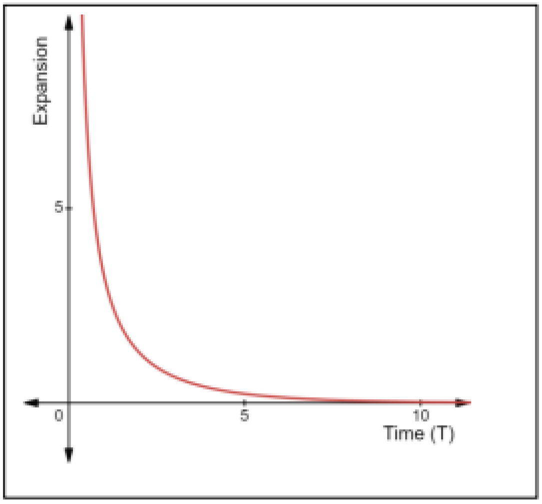
Graph 4.1 : Plot of Time vs. Rest Density Component of the acceleration and its magnitude

$$\dot{v}_i = \frac{-m}{T^{n-1}\exp(x\beta(LT)^n)} \quad [4.3]$$



Graph 4.2 : Plot of Time vs. Acceleration Magnitude of ω=0, than expansion (θ) is given by

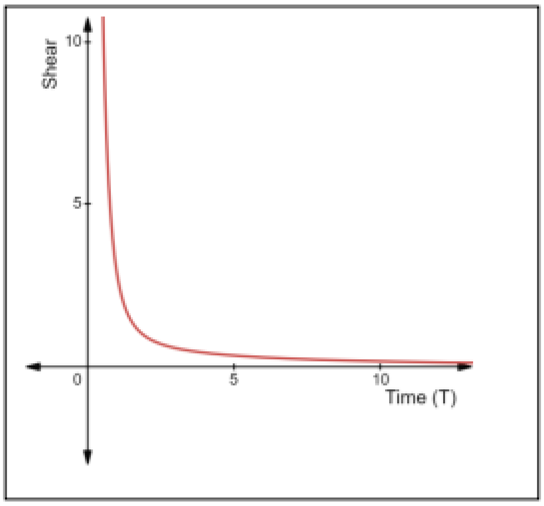
$$\theta = \frac{1}{\exp[x\beta(LT)^n]}\left[x\beta n(LT)^{n-1} + \frac{m}{T}\right] \quad [3.4]$$



Graph 4.3 : Plot of Time vs. Expansion Shear (σ) is given by

$$\sigma = \frac{1}{\exp[x\beta(LT)^{\frac{n}{2}}]}\left[\begin{array}{l}\left(x\beta nL^{n}T^{n-1}\right)^{2}+\left(\frac{\sqrt{5}m}{2T^{2}}\right)^{2}\\ +3x\beta mnL^{n}T^{n-2}-\gamma^{2}m^{2}T^{2(m-1)}\end{array}\right]^{1/2}$$

[3.5]



Graph 4.4 : Plot of Time vs. Shear

5. Conclusion

The requirement that the model (2.23) must fulfil is that of having a low amount of energy ($- < < \rho\ \rho$ p). If the criterion is met ($\rho > p$), then the condition is met ($(n-1) > 0$). The presence of a singularity in the model when T = 0 is evidence of an initial singularity. According to the findings of the research, incoherent matter is relatively little in comparison to stiff fluid both close to the initial singularity and asymptotically.

When we look at (3.4), it is easy to see that we need to have in order for expansion (θ) to be positive and continue to rise even when T = 0. In the region denoted by, the essential condition ($\rho > 0$) has been shown to be met

$$\frac{1}{2}x\beta nmL^{n}T^{n-2}+\frac{m^{2}}{4T^{2}}-\gamma^{2}m^{2}T^{2(m-1)}>1$$

The region close to the singularity is still limited. The singularity denoted by the coordinate T=0 is absent from this area. At time equal to zero, the model begins with a big-bang at T = 0, and it continues to expand until it reaches a point where it is valid asymptotically ($T \to =$). It is possible that the dispersion of the material is best understood as a two-component fluid consisting of pressure-less dust and stiff fluid travelling at equal velocity. Ratio of p/ρ generally places equal weight on both p one $T \to 0$ and $T \to \infty$ the other. When this condition holds, the flow vector cannot be negative when$mm \neq 0$. The acceleration approaches zero as it approaches the initial singularity, although it is not quite zero at that point.

6. References

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