A STUDY OF MCDM MODEL UNDER A QUAD FUZZY SET ENVIRONMENT

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Abstract

In today's world, decision-makers face a complex and uncertain problem due to various attributes, such as the indefinite mind of an individual and uncertain information. A fuzzy set developed a suitable method to deal with inaccurate perspectives. The paper aims to develop a novel fuzzy extended set, the Quad fuzzy set used as a powerful tool compared to the Pythagorean fuzzy set (PFS) and Fermatean fuzzy set (FFS). We introduce the arithmetic and geometric operators for the Quad Fuzzy Set. Further, a case study problem is demonstrated using Analytic Hierarchy Process (AHP), a Decision-Making Method. Moreover, the study compares the best crop variety of sugarcane grown in Tamil Nadu. Finally, a clarified numerical model of selecting a best choice in crop variety of sugarcane is demonstrated with its practicability and effectiveness.

Keywords: Criteria, linguistic variable, Fuzzy set, Arithmetic Operations,sugarcane cultivation, attributes.

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1. Introduction

Multi-criteria decision-making methods are providing to be a potential tool for investigating and obtaining the solution for complex real time problems due to their fundamental capability to evaluate preferences with respect to different criteria for selection of the best preferences. MCDM (Multi-criteria decision-making) problems have many features such as presence of various disproportionate criteria, unsimiliar measure of variables among the unreasonable criteria, existence of realistic ideas of several alternatives. These decision-making problems describing multidimensional situations are solved by various MCDM methods. In MCDM, alternatives are evaluated and ranked primarily for decision-making purposes. This is bring outthe different mathematical techniquesto the considered methods.

India is one of the leading countries in the world in cultivation and Inputs into sugarcane production. In Tamil Nadu, Sugarcane is a conventional agricultural crop grown all over the state. The tropical climate of Southern state plays an important role in cultivation of sugarcane. The months of December to May is the growing season in the state. In certain districts like Trichy, Tanjore, Perambalur, Karur, Erode and Coimbatore the cultivation of sugarcane isalmost throughout the year. The growth of the cane consists a series of criteria which makes the decision maker (DM) to analyse the comparative study of the different sugarcane varieties.

Tamil Nadu is the third region to grow and harvest sugarcane with respect to the area and production as per the Recent survey which is estimated about 3,52,000 hectares area of cultivation of cane.Lokhande[11] wrote a Study of Indian Sugar Industry in Post Reforms Era.Bajpai[5] et.al., described Dynamics of sugar productivity and factors affecting the production.Edalatpanah S.A. et.al [6]explained Data Envelopment Analysis and Efficiency of firms : A Goal Programming Approaching Method. Malarkodi[13]et.al.,surveyed Sustaining sugarcane production in Western parts of Tamil Nadu. New varieties of hybrid breed and plantations are introduced to improvise the horizon of the sugar production. Tamilselvi[19] clearly studied about the Sugarcane: Tamil Nadu leads the way towards major yields.Agyan Panda et.al.[1] defined and showed a concept to Improve Crop Production Through WSN: An Approach of Smart Agriculture .

Fuzzy Sets is a valid form to express the uncertain information. Set theory assumes that elements in a set either belong to the set or do not belong to it as a result of a bivalent condition. Contrary to fuzzy set theory, fuzzy set theory allows for gradual assessment of the membership of elements in a set; this is explained by the use of a membership function valued within the real unit interval [0,1]. In the early 1970s, Zadeh introduced fuzzy set theory [24], which can be described as important medium to understand uncertain information. Orthopair Fuzzy sets depends on the membership along with grades are combined values in the interval ranging between 0 to 1.Yager[21] developed a collections of uncertain pair called as q-rung orthopair.This defined collections is structured in terms of q^{th} power for degree of membership and q^{th}power for the degree of non membership whichnot greater than 1.The general class is noted to increase the acceptability of the membership support.This helps many researchers to explore the q-rung orthopair fuzzy sets (q-ROFS).Mainly, Orthopair fuzzy set is the Atanasov’s Classic Intuitionistic Fuzzy sets(IFSs)[23].In 2013, Yager[22] discussed the concept of Pythagorean Fuzzy Set (PFS) in their study ,which paved a needful technique to deal with vague information. The Pythagorean Fuzzy set points out to be investigated within a short duration. Garg[7] introduced Pythagorean Fuzzy information aggregation with Einstein Operations. In uncertainity concepts ,a new type of fuzzy set named the Fermatean Fuzzy set (FFS)was developed by Senapati & Yager[17].The Fermatean Membership Grade(FMG) Functions to be higher than the grades of Pythagorean Membership was explained in 2019 by Senapati and Yager[16].FFS is considered as the type of q-ROFS describing q to be equal to 3 with their functional numbers are denoted as Fermataen Fuzzy numbers (FFN).An example for the reason to develop the FFS.A situation based to give his priority for an alternative y_j for the Criterion C_i might allow the performance of the degree where the criterion C_i satisfies as 0.9 and consequently where the criterion C_i dissatisfied by the alternative y_j as 0.6.We observe 0.9+0.6>1 but as a result $0.9^3+0.6^3<1$.

Senapati and Yager [16] showed the numerical examples working over the fermaten fuzzy numbers with its application.Novel cubic fermatean fuzzy soft idea structures was studied by Manemaran and Nagaraj [12]. In 2020, the classic q-rung picture fuzzy Yager operator with aggregation and its types gave a clear concept to assessment of the data by the decision makers. This particular types of q-rung picture Yager Aggregation was defined by PeideLiu[15] et.,al. A method for decision making study based on Fermatean fuzzy sets and WASPSAS for Green construction supplier evaluation was considerable laborated by Mehdi Keshavarz Ghorabaee[14] et.,al.Arunodaya Raj Mishra[1] et.al. studied the Fermatean fuzzy CRITIC‑EDAS approach.Zhao [25] et.al. explained the Generalized Aggregation Operators for Intuitionistic Fuzzy sets.Garg et.al[7] discussed on Decision-making analysis based on Fermatean fuzzy Yager aggregation operators. Wei Yang et.al[20] introduced Triangular Single Valued Neutrosophic Data Envelopment Analysis: Application to Hospital Performance Measurement.The concept for power Aggregration Operations and VIKOR methods in complex of q-ROFS was introduced by Gong[9]et.al. The spherical fuzzy set (SFS) is elucidated by the researchers Gundogdu and Kahraman [10] which alters the notion more effective to PFS.

The notion here to employ the evaluation procedure is simple method to rate the weights of the respective criteria. MCDM (modeling, decision making, and management) approaches are more popular and widely accepted for problem solving in complex situations. The AHP algorithm was developed by Professor Thomas L. Saaty as the best way to make decisions when faced with a variety of options.Analytic Hierarchy Process is the best tool to simply the complex unstructured problems, methodologies and dynamics make variables in a hierarchy of ranking. We haveused a MCDM, a useful tool based on AHP. ShafiSalimi,Edalatpanah [16] discussed the concept of Supplier Selection Using Fuzzy AHP Method and D-Numbers.These context shows that Analytic Hierarchy Process is very capable to be adaptable under many problems and environments.

The defined Quad Fuzzy set is explained with Weighted Arithmetic Mean Operator to produce the results for uncertainty.

In this work, we introduce a new kind of fuzzy set named Quad Fuzzy set where the value of q=4 in the collection of the q-Rung Orthopair Fuzzy Set(q-ROFS). The Quad Fuzzy consists to values of Membership Degree(M-D) and Non Membership Degree(NM-D).The aim is to specify the point that Quad Fuzzy Number is capable to support the greater levels of vague concepts.

In section-3, the paper focuses on the fundamental ideas of QFS. In section-4, the designed methods of AHP is explained in QFN. In section-5 presents the conclusions and results with the reference following.

2. Preliminaries:

2.1.Q-Rung Orthopair Fuzzy Set[7]

A q-ROFS A on nonempty set W is defined as

$$A = \{a, \mu_A(a), \upsilon_A(a)\}$$

Where $\mu_A(a)$: W→[0,1] ; $\upsilon_A(a)$: W→[0,1] shows the MD and NMD of an element $a\epsilon$W respectively $W_F(a)=\sqrt[q]{1-\mu_A^q+\upsilon_A^q}$ is value of indeterminacy.

2.2. Intuitionistic Fuzzy Set[3]

An Intuitionistic fuzzy set A on a universe X is an object of the form

$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$

where $\mu_A(x) \in [0,1]$ is called the degree of membership of x in A, and $\nu_A(x) \in [0,1]$ is known to be the degree of non-membership of x in A.

2.3.Pythagorean Fuzzy Set[18]

A Pythagorean fuzzy set P in a finite number in the universe of discourse Y is given as P $= \{< x, \mu_P(a), \nu_P(a) > a \in X\}$

Where $\mu_P(x), \nu_P(x)$:$X \to [0,1]$ be the membership value and non membership value of the element $a \in X$ to the set PFS, respectively, with the condition that

$$0 \le {\mu_P}^2(a) + {\nu_P}^2(a) \le 1.$$

The indeterminacy degree between the membership function is given by

$\pi_P(x) = \sqrt{1 - {\mu_P}^2(a) - {\nu_P}^2(a)}$ where $(\mu_P(a), \nu_P(a))$ called a Pythagorean fuzzy numbers denoted by $PFS = (\mu_P, \nu_P)$.

3. A Novel Set

3.1. QUAD FUZZY SETS

Let Q be a universe of discourse. A Quad fuzzy set F in X is an object in the form $F = \{\langle x, \alpha_F(x), \beta_F(x) : x \in X \rangle\}$ where $\alpha_F(x) : X \to [0,1]$ and $\beta_F(x) : X \to [0,1]$, including the condition, $0 \le (\alpha_F(x))^4 + (\beta_F(x))^4 \le 1$

For every x is in X. The number $\alpha_F(x)$, $\beta_F(x)$ denote, respectively, the value of membership and the degree of non-membership of the element x in the set F.

For any QFS and x ϵ X, $\pi_F(x) = \sqrt[4]{1 - (\alpha_F(x))^4 - (\alpha_F(x))^4}$ is identified as the degree of indeterminacy of x to F.

For convenience ($\alpha_F(x)$, $\beta_F(x)$) a Quad fuzzy number (QFN) denoted by F= (α_F , β_F).

4. Arithmetic Operations for Quad Fuzzy set

Let $Q_1=(\alpha_1, \beta_1)$ and $Q_2=(\alpha_2, \beta_2)$ be any two QFNs then the following operations are valid.

$Q_1 + Q_2 = ({\alpha_1}^4 + {\alpha_2}^4, {\beta_1}^4 + {\beta_2}^4)$

$Q_1 - Q_2 = ({\alpha_1}^4 - {\alpha_2}^4, {\beta_1}^4 - {\beta_2}^4)$

$Q_1 . Q_2 = ((\alpha_1\alpha_2)^4, (\beta_1\beta_2)^4)$

$\frac{Q_1}{Q_2} = (\frac{{\alpha_1}^4}{{\beta_2}^4}, \frac{{\alpha_2}^4}{{\beta_1}^4})$

Example 4.1:

An illustrative Example is given for some operators of QFS, Consider $Q_1=(0.6,0.3)$ and $Q_2=(0.4,0.7)$ be any two QFNs

$Q_1 + Q_2 = (0.623,0.210)$

$Q_1 + Q_2 = (0.240,0.700)$

$Q_1 . Q_2 = (0.193,0.998)$

$\frac{Q_1}{Q_2} = (0.995,0.270)$

4.1.Quad Fuzzy Weight Arithmetic Mean Operators:

Let $F_i=(\mu_a, \nu_a)$ (a=12,3,…..s) be a range of numbers q-RFNs. The QFWAM operator is a defined function $Q^s \rightarrow Q$ such that QFWAMψ ($F_1, F_2,\ldots F_s$)$=\sum_{a=1}^{s}(\psi_a F_a)$,

where $\psi = (\psi_1, \psi_2, \ldots . \psi_s)^T$ is the weight in the form vector of F_i with $\psi_i > 0$ where $\sum_{a=1}^{s} \psi_a = 1$

4.2.Definition:

Let $F_a=<\mu_a, \nu_a>$ be a number of q-RFNs, then aggregated value is given by the QFWAM operator is a q-RFN and

QFWAM ψ ($F_1, F_2,\ldots F_s$)$=\sum_{a=1}^{s}(\psi_i F_i)$

$$\sqrt[q]{\min\left(1, \sum_{i=1}^{\phi} \psi_i {\mu_i}^{q\theta}\right)^{1/\theta}}, \sqrt[q]{1 - min\left(1, \sum_{i=1}^{\phi} \left(\psi_i (1 - {\upsilon_i}^q)\right)^{\theta}\right)^{\frac{1}{\theta}}} \quad (\mathbf{1})$$

4.3.Quad Fuzzy Weight Geometric Mean Operators:

Let $F_i=(\mu_i, \nu_i)$ (i=12,3,…..s) be a range number of q-RFNs. QFWGM operator is a defined function $Q^s \rightarrow Q$ such that

QFWGMψ ($F_1, F_2,\ldots F_s$)$=\otimes_{i=1}^{s}(\psi_i F_i)$,

where $\psi = (\psi_1, \psi_2, \ldots . \psi_s)^T$ is the weight vectors of F_i with $\psi_i > 0; \sum_{i=1}^{s} \psi_i = 1$

4.4.Definition:

Let $F_a=<\mu_a, \nu_a>$ be a number of q-RFNs, then aggregated value shown by the QFWGM is a q-RFN and

QFWGM ψ ($F_1, F_2,\ldots F_s$)$=\otimes_{i=1}^{s}(\psi_i F_i)$,

$$\sqrt[q]{1 - \min\left(1, \sum_{i=1}^{\phi} \psi_i (1 - \mu_i)^{q\theta}\right)^{1/\theta}}, \sqrt[q]{min\left(1, \sum_{i=1}^{\phi} \left(\psi_i ({\upsilon_i}^q)\right)^{\theta}\right)^{\frac{1}{\theta}}} \quad (\mathbf{2})$$

4.5.Score Function For Quad Fuzzy Set:

The distance measures of the Quad Fuzzy set g(A) and g(B) is defined with the membership corresponding to non-membership grades i.e. μ(A) and ν(A). The score function should lies from 0 to 1. We note the score function to be

$$S(A) = \frac{g(A, \upsilon(A))}{g(A, \upsilon(A)) + g(A, \mu(A))} = \frac{1 - \upsilon(A)}{2 - \mu(A) - \upsilon(A)} \quad (\mathbf{3})$$

5. The Designed Algorithm

In the given section,we focus to develop the comparison study between the new fuzzy set named Quad Fuzzy set with its Weighted Arithmetic Mean Operator and Weighted Geometric Mean operator .Wealso evaluate and select the best sugarcane crop in an uncertain environment.The definition of Arithmetic Mean Operator ,Geometric Mean Operator of QFS is discussed in the previous section.

The depiction of the framework is needed for using QRFWMA and QRFWMG . The *figure-1* studies the several steps in the framework.

Assume C_i to be the number of Criteria and A_i denote the alternatives.To understand the procedure easily the proposed method is given below.

Step-1:Group all the crop varieties of sugarcane.The decision makers should define the problem as more knowledge is required to classify the varieties.Next, we evaluate the crop varieties that can be grown in Tamil Nadu.

Step-2:Define the alternatives with respect to favorable and unlikely favorable crop varieties are used in the evaluation process.

Step-3:A set of evaluation using the criteria along the given set of alternatives .The evaluation is defined according to the data taken from the previous studies about Crop varieties . The *table-I* demonstrates the process.

Step-4:The unique technique called AHP is used to determine the Weighted criteria.

Step-5: The sum of weighted criteria assigned by the decision makers is evaluated.By the method of normalization, The points are summarized and the final weighted criteria are formed.

Step-6: We define the Linguistic variable based on the alternatives and corresponding with QFS is shown in *table III.*

Step-7: The given Linguistic Variables are given as Favorable crop variety and unlikely favourable crop variety with QFN is formed by decision maker in this step.

Step-8: In this step, the evaluation is used to classify the criteria and alternatives .In step-5,the defined Linguistic variable based on QFS is used .

Step-9: The following alternatives are ranked with the score functions that are defined in terms of membership and its non-membership degree.

Step-10: Similarly,we construct for Q-RFWG in terms of QFSby using above steps.

The structural outline of the proposed method is presented below

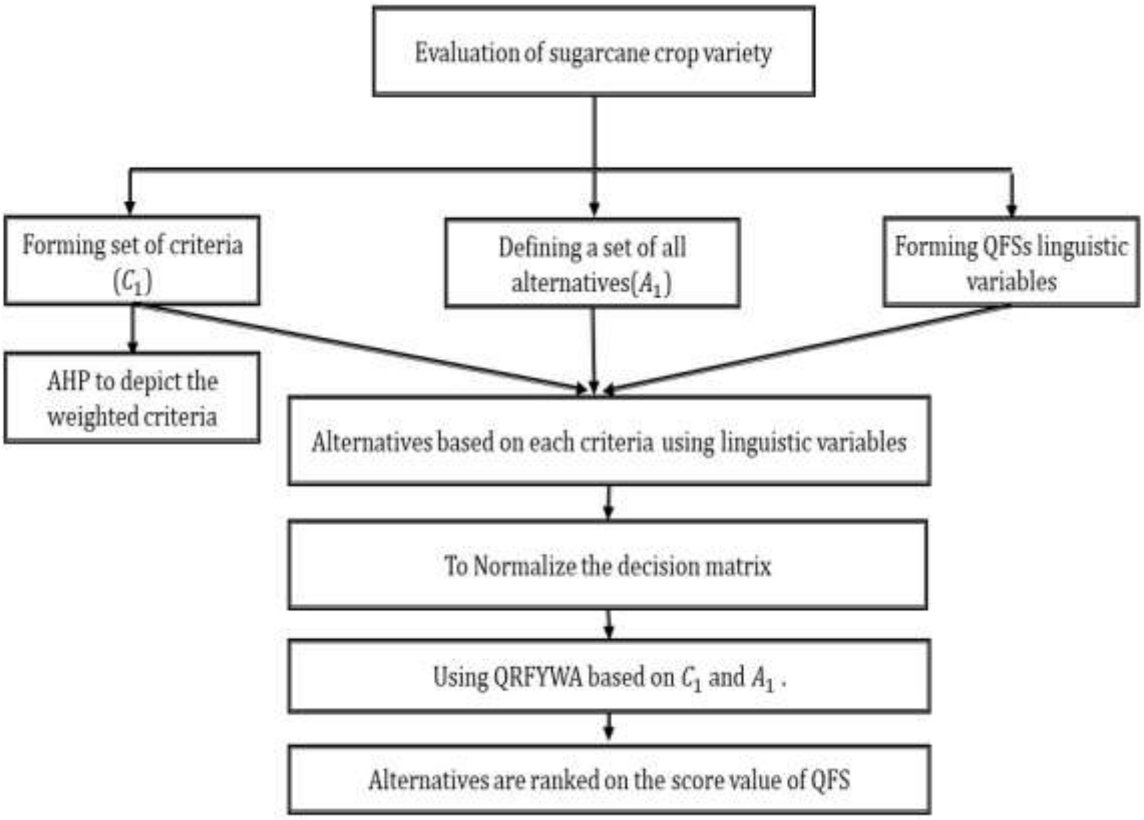

Figure 1. The outline of the proposed method.

5.1 A NUMERICAL ANALYSIS OF THE MCDM PROBLEM WITH QFNS

In this numerical analysis five different crop varieties of sugarcane such as Co 09004, Co C671, Co 94008, Co 11015, Co 86032 which are the attributes. The criteria for the canes are yield of sugarcane, commercially recoverable sugar content CCS and % of sucrose in juice. Assessment values are given in form of linguistic variables denoting favourable varieties and unlikely favourable varieties. In this Fuzzy domain we can assume m alternatives and n criterion.

Criteria / **Alternatives**	***Co 09004 Amritha***	***Coc 671 Karan-1***	***Co 94008 Shyama***	***Co 11015 Atulya***	***Co 86032 Nayana***
Cane Yield t/ha (C_1)	109.85	93.22	93.62	135.70	114.40
CCS t/ha(C_2)	14.56	12.35	11.31	13.12	15.42
Sucrose % in Juice (C_3)	18.94	18.90	17.59	21.46	19.19
Duration in days (C_4)	300	330	360	362	339
Plant Height in cm(C_5)	225.6	107	196	124	179.2

TABLE I.GROUPING OF CROP VARIETY

Cane Trash compost Kg/ha(C_6)	310.2	211	450	88	117

5.2 Analytic Hierarchy Process

5.2.1 Steps for Analytic Hierarchy Process

The initial step in obtaining a consistency value is to determine Consistency Index (CM) using equation

Consistency Measure (CM) = $\frac{\lambda max - m}{m-1}$ **(4)**

Calculation of consistency Measure (CM) is given to be consistency rate whether that will influence the results.

To know if Consistency Measure (CM) is by magnitude certain good enough or not, please note the ratio is considered feasible and standard, if **CR ≤ 0.1**
Determine Consistency Ratio (CR) using Ratio Index equation used

Consistency Ratio CR= Consistency Measure CM/Random Index RI (5)

The acquired Consistency Ratio value is 0.1, indicating that it was possible to use the weight of the created criterion. Priority of the computation results is assigned using the AHP approach according to the criteria shown in Table X.

Criteria / Alternatives	***Co 09004 Amritha***	***Coc 671 Karan-1***	***Co 94008 Shyama***	***Co 11015 Atulya***	***Co 86032 Nayana***
Cane Yield t/ha (C_1)	109.85	93.22	93.62	135.70	114.40
CCS t/ha (C_2)	14.56	12.35	11.31	13.12	15.42
Sucrose % in Juice (C_3)	18.94	18.90	17.59	21.46	19.19
Duration in months (C_4)	10	11	12	10	11
Plant Height in cm(C_5)	225.6	107	196	124	179.2
Cane Trash compost Kg/ha(C_6)	310.2	211	450	88	117
Sum	430.6	277.2	405.5	318.9	358.7

TABLE II. AVERAGE OF THE CROP VARIETY

In the below table,we define the Linguistic variable in terms of four conditions to yield the sugarcane crop

Linguistic Variable	Grade	Range
Favourable variety	F	(0.9, 0.6)
Unlikely Favourable variety	UF	(0.8, 0.3)
Stable variety	S	(0.2,0.6)
Unstable variety	US	(0.1,0.3)

TABLE III.THE USE OF LINGUISTIC VARIABLE

With the use of linguistic variable ,the following table is evaluated .

Decision Maker	A_1	A_2	A_3	A_4	A_5
C_1	F	US	UF	S	F
C_2	F	US	UF	F	S
C_3	F	F	UF	F	F
C_4	US	S	F	US	US
C_5	UF	F	F	UF	F
C_6	F	UF	S	F	S

TABLE IV DECISION MAKER EVALUATION

We proceed to obtain the values of the Quad fuzzy number into a single value by using (1).In terms of QFWAM .To compute ,we assume $\theta = 2$ and q =4.

Criteria/ Alternative	***Co 09004 Amritha***	***Coc 671 Karan-1***	***Co 94008 Shyama***	***Co11015 Atulya***	***Co 86032 Nayana***
Cane Yield t/ha (C_1)	0.2551	0.3362	0.2308	0.4255	0.3189
CCS t/ha (C_2)	0.0338	0.0445	0.0278	0.0097	0.0429
Sucrose % in Juice (C_3)	0.0439	0.0681	0.0433	0.0672	0.0497
Duration in months (C_4)	0.0232	0.0396	0.0295	0.0313	0.0306
Plant Height in cm(C_5)	0.5239	0.3860	0.4833	0.3888	0.4646
Cane Trash compost Kg/ha(C_6)	0.7203	0.7611	1.097	0.2759	0.326

TABLE V. NORMALISED PAIR WISE COMPARISON MATRIX

TABLE VI. CRITERIA WEIGHTS OF THE CROPS

Criteria/ Alternative	***Co09004 Amritha***	***Coc 671 Karan-1***	***Co94008 Shyama***	***Co 11015 Atulya***	***Co 86032 Nayana***	***Criteria Weights***
Cane Yield t/ha (C_1)	0.2551	0.3362	0.2308	0.4255	0.3189	0.3133
CCS t/ha (C_2)	0.0338	0.0445	0.0278	0.0097	0.0429	0.00319
Sucrose % in Juice (C_3)	0.0439	0.0681	0.0433	0.0672	0.0497	0.0544
Duration in months (C_4)	0.0232	0.0396	0.0295	0.0313	0.0306	0.0308
Plant Height in cm(C_5)	0.5239	0.3860	0.4833	0.3888	0.4646	0.44916
Cane Trash compost Kg/ha(C_6)	0.7203	0.7611	1.097	0.2759	0.326	0.2360

The consistency table is evaluated in the view to obtain the caluation for criteria weights

TABLE VII. CONSISTENCY MATRIX

Criteria Weights	0.3133	0.00319	0.0544	0.0308	0.44916	
Criteria	***Co09004 Amritha***	***Coc671 Karan-1***	***Co94008 Shyama***	***Co11015 Atulya***	***Co86032 Nayana***	***Criteria Weights***
Cane Yield t/ha (C_1)	34.41	0.2973	5.0929	4.1811	51.3839	95.36
CCS t/ha (C_2)	4.561	0.03939	0.6152	0.4040	6.9260	12.58
Sucrose % in Juice (C_3)	5.933	0.0602	0.9568	0.6609	0.6193	8.23
Duration in months (C_4)	3.133	0.0350	0.6528	0.308	4.9407	9.0695
Plant Height in cm(C_5)	70.68	0.3413	10.66	3.8192	80.48	165.97
Cane Trash compost Kg/ha(C_6)	97.18	0.6730	24.48	2.7104	52.55	177.54

TABLE VIII.CALCULATION OF $\boldsymbol{\lambda}$

Criteria/ Alternative	***Co09004 Amritha***	***Coc671 Karan-1***	***Co94008 Shyama***	***Co11015 Atulya***	***Co86032 Nayana***	***Weighted sum value***	***Criteria Weights***	$\boldsymbol{\lambda}$
Cane Yield t/ha (C_1)	34.41	0.2973	5.0929	4.1811	51.3839	95.36	0.3133	6.52
CCS t/ha (C_2)	4.561	0.03939	0.6152	0.4040	6.9260	12.58	0.00319	5.58
Sucrose % in Juice (C_3)	5.933	0.0602	0.9568	0.6609	0.6193	8.23	0.0544	7.12
Duration in months (C_4)	3.133	0.0350	0.6528	0.308	4.9407	9.0695	0.0308	6.89
Plant Height in cm(C_5)	70.68	0.3413	10.66	3.8192	80.48	165.97	0.44916	5.89
Cane Trash compost Kg/ha(C_6)	97.18	0.6730	24.48	2.7104	52.55	177.54	0.6360	7.23

λmax =6.53

Consistency Measure (CM) =0.106

N	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	0.41	1.45	1.49

TABLE IX.RANDOM INDEX

Consistency Measure CM=0.106

Consistency Ratio CR= 0.085<0.10

The Process of decision making using AHP based on Criteria weights can be used for further calculation.

Criteria	***Criteria Weights***
Cane Yield t/ha (C_1)	0.3133
CCS t/ha (C_2)	0.0031
Sucrose % in Juice (C_3)	0.0544
Duration in months (C_4)	0.0308
Plant Height in cm(C_5)	0.4491
Cane Trash compost Kg/ha(C_6)	0.2360

TABLE X. SUMMARISED CRITERIA WEIGHT

Using (1) and (2) we determine the values of the pairwise comparison in the terms of the crop variety with its criterion to cultivate the cane crop.

The mathematical induction is takenwhere the best alternative is calculated in $\mathcal{L}_i$ in the following table .

TABLE XI.THE PERFORMANCE VALUES OF THE ALTERNATIVE OF QFWAM FOR

Pairwise Comparison	**Alternatives**	**QFWAM**	**Pairwise Comparison**	**Alternatives**	**QFWAM**
(C_1,A_1)	$\mathcal{L}_1$	(0.8458,0.1178)	(C_2,A_3)	$\mathcal{L}_{16}$	(0.8458,0.1178)
(C_1,A_2)	$\mathcal{L}_2$	(0.823, 0.921)	(C_2,A_4)	$\mathcal{L}_{17}$	(0.823, 0.921)
(C_1,A_3)	$\mathcal{L}_3$	(0.321, 0.864)	(C_2,A_5)	$\mathcal{L}_{18}$	(0.321, 0.864)
(C_1,A_4)	$\mathcal{L}_4$	(0.252, 0.879)	(C_3,A_1)	$\mathcal{L}_{19}$	(0.252, 0.879)
(C_1,A_5)	$\mathcal{L}_5$	(0.912, 0.9823)	(C_3,A_2)	$\mathcal{L}_{20}$	(0.912, 0.9823)
(C_2,A_1)	$\mathcal{L}_6$	(0.620, 0.080)	(C_3,A_3)	$\mathcal{L}_{21}$	(0.620, 0.080)
(C_2A_2)	$\mathcal{L}_7$	(0.124, 0.025)	(C_3,A_4)	$\mathcal{L}_{22}$	(0.124, 0.025)
(C_3,A_5)	$\mathcal{L}_8$	(0.315, 0.538)	(C_5,A_3)	$\mathcal{L}_{23}$	(0.315, 0.538)
(C_4,A_1)	$\mathcal{L}_9$	(0.224, 0.300)	(C_5,A_4)	$\mathcal{L}_{24}$	(0.224, 0.300)
(C_4,A_2)	$\mathcal{L}_{10}$	(0.534, 0.814)	(C_5,A_5)	$\mathcal{L}_{25}$	(0.534, 0.814)
(C_4,A_3)	$\mathcal{L}_{11}$	(0.285, 0.317)	(C_6,A_1)	$\mathcal{L}_{26}$	(0.285, 0.317)
(C_4,A_4)	$\mathcal{L}_{12}$	(0.200, 03.799)	(C_6,A_2)	$\mathcal{L}_{27}$	(0.200, 03.799)
(C_4,A_5)	$\mathcal{L}_{13}$	(0.060, 0.125)	(C_6,A_3)	$\mathcal{L}_{28}$	(0.060, 0.125)
(C_5,A_1)	$\mathcal{L}_{14}$	(0.120, 0.312)	(C_6,A_4)	$\mathcal{L}_{29}$	(0.120, 0.312)
(C_5,A_2)	$\mathcal{L}_{15}$	(0.100, 0.194)	(C_6,A_5)	$\mathcal{L}_{30}$	(0.100, 0.194)

Q=4

TABLE XII.THE SCORE OF S ($\mathcal{L}_i$) OF ALL QFWA

Pairwise Comparison	**Alternatives**	**QFWGM**	**Pairwise Comparison**	**Alternatives**	**QFWGM**
(C_1,A_1)	$\mathcal{L}_1$	(0.351,0.102)	(C_2,A_3)	$\mathcal{L}_{16}$	(0.234,0.212)
(C_1,A_2)	$\mathcal{L}_2$	(0.432,1)	(C_2,A_4)	$\mathcal{L}_{17}$	(0.123,0.756)
(C_1,A_3)	$\mathcal{L}_3$	(0.023,1)	(C_2,A_5)	$\mathcal{L}_{18}$	(0.569,0.25)
(C_1,A_4)	$\mathcal{L}_4$	(1,1)	(C_3,A_1)	$\mathcal{L}_{19}$	(0.456,0.954)
(C_1,A_5)	$\mathcal{L}_5$	(0.3510,1)	(C_3,A_2)	$\mathcal{L}_{20}$	(0.752,0.555)
(C_2,A_1)	$\mathcal{L}_6$	(0.008,0.09)	(C_3,A_3)	$\mathcal{L}_{21}$	(0.369,0.169)
(C_2A_2)	$\mathcal{L}_7$	(0.021,0.162)	(C_3,A_4)	$\mathcal{L}_{22}$	(0.458,0.368)
(C_3,A_5)	$\mathcal{L}_8$	(0.004,0.362)	(C_5,A_3)	$\mathcal{L}_{23}$	(0.528,0.614)
(C_4,A_1)	$\mathcal{L}_9$	(0.023,1)	(C_5,A_4)	$\mathcal{L}_{24}$	(0.125,1)
(C_4,A_2)	$\mathcal{L}_{10}$	(0.045,0.362)	(C_5,A_5)	$\mathcal{L}_{25}$	(0.416,0.347)
(C_4,A_3)	$\mathcal{L}_{11}$	(0.036,0.051)	(C_6,A_1)	$\mathcal{L}_{26}$	(0.895,0.625)

(C_4, A_4)	$\mathcal{L}_{12}$	(0.25,0.1156)	(C_6, A_2)	$\mathcal{L}_{27}$	(1,0.451)
(C_4, A_5)	$\mathcal{L}_{13}$	(0.235,0.042)	(C_6, A_3)	$\mathcal{L}_{28}$	(0.965,0.385)
(C_5, A_1)	$\mathcal{L}_{14}$	(0.054,0.365)	(C_6, A_4)	$\mathcal{L}_{29}$	(0.698,0.148)
(C_5, A_2)	$\mathcal{L}_{15}$	(0.235,0.456)	(C_6, A_5)	$\mathcal{L}_{30}$	(0.567,0.789)

TABLE XIII.PERFORMANCE VALUES OF THE ALTERNATIVE OF QFWGM FOR Q=4.

Alternatives	**S ($\mathcal{L}_i$) of QFWGM**	**Alternatives**	**S ($\mathcal{L}_i$) of QFWGM**
$\mathcal{L}_1$	**0.789**	$\mathcal{L}_{16}$	0.455
$\mathcal{L}_2$	0.654	$\mathcal{L}_{17}$	0.410
$\mathcal{L}_3$	0.024	$\mathcal{L}_{18}$	0.691
$\mathcal{L}_4$	0.101	$\mathcal{L}_{19}$	0.189
$\mathcal{L}_5$	0.236	$\mathcal{L}_{20}$	0.547
$\mathcal{L}_6$	0.765	$\mathcal{L}_{21}$	0.705
$\mathcal{L}_7$	0.526	$\mathcal{L}_{22}$	0.369
$\mathcal{L}_8$	0.504	$\mathcal{L}_{23}$	0.499
$\mathcal{L}_9$	0.474	$\mathcal{L}_{24}$	0.468
$\mathcal{L}_{10}$	0.135	$\mathcal{L}_{25}$	0.478
$\mathcal{L}_{11}$	0.010	$\mathcal{L}_{26}$	0.584
$\mathcal{L}_{12}$	0.235	$\mathcal{L}_{27}$	0.258
$\mathcal{L}_{13}$	0.005	$\mathcal{L}_{28}$	0.433
$\mathcal{L}_{14}$	0.027	$\mathcal{L}_{29}$	0.722
$\mathcal{L}_{15}$	0.697	$\mathcal{L}_{30}$	0.462

TABLE XIV.THE SCORE OF S ($\mathcal{L}_i$) OF ALL QFWGM

Alternatives	**QFWAM**	**QFWGM**	**Ranking Order**
$\mathcal{L}_1$	**0.840**	**0.789**	$A(\mathcal{L}_1) > G(\mathcal{L}_1)$

Alternatives	**S($\mathcal{L}_i$) of QFWAM**	**Alternatives**	**S ($\mathcal{L}_i$) of QFWAM**
$\mathcal{L}_1$	**0.840**	$\mathcal{L}_{16}$	0.782
$\mathcal{L}_2$	0.808	$\mathcal{L}_{17}$	0.521
$\mathcal{L}_3$	0.833	$\mathcal{L}_{18}$	0.738
$\mathcal{L}_4$	0.139	$\mathcal{L}_{19}$	0.237
$\mathcal{L}_5$	0.267	$\mathcal{L}_{20}$	0.599
$\mathcal{L}_6$	0.807	$\mathcal{L}_{21}$	0.768
$\mathcal{L}_7$	0.527	$\mathcal{L}_{22}$	0.545
$\mathcal{L}_8$	0.602	$\mathcal{L}_{23}$	0.559
$\mathcal{L}_9$	0.574	$\mathcal{L}_{24}$	0.566
$\mathcal{L}_{10}$	0.285	$\mathcal{L}_{25}$	0.811
$\mathcal{L}_{11}$	0.488	$\mathcal{L}_{26}$	0.743
$\mathcal{L}_{12}$	0.300	$\mathcal{L}_{27}$	0.423
$\mathcal{L}_{13}$	0.486	$\mathcal{L}_{28}$	0.655
$\mathcal{L}_{14}$	0.458	$\mathcal{L}_{29}$	0.672

$\mathcal{L}_{15}$	0.772	$\mathcal{L}_{30}$	0.525

$\mathcal{L}_{2}$	0.808	0.654	$A(\mathcal{L}_2) > G(\mathcal{L}_2)$
$\mathcal{L}_{3}$	0.833	0.024	$A(\mathcal{L}_3) > G(\mathcal{L}_3)$
$\mathcal{L}_{4}$	0.139	0.101	$A(\mathcal{L}_4) > G(\mathcal{L}_4)$
$\mathcal{L}_{5}$	0.267	0.236	$A(\mathcal{L}_5) > G(\mathcal{L}_5)$
$\mathcal{L}_{6}$	0.807	0.765	$A(\mathcal{L}_6) > G(\mathcal{L}_6)$
$\mathcal{L}_{7}$	0.527	0.526	$A(\mathcal{L}_7) > G(\mathcal{L}_7)$
$\mathcal{L}_{8}$	0.602	0.504	$A(\mathcal{L}_8) > G(\mathcal{L}_8)$
$\mathcal{L}_{9}$	0.574	0.474	$A(\mathcal{L}_9) > G(\mathcal{L}_9)$
$\mathcal{L}_{10}$	0.285	0.135	$A(\mathcal{L}_{10}) > G(\mathcal{L}_{10})$
$\mathcal{L}_{11}$	0.488	0.010	$A(\mathcal{L}_{11}) > G(\mathcal{L}_{11})$
$\mathcal{L}_{12}$	0.300	0.235	$A(\mathcal{L}_{12}) > G(\mathcal{L}_{12})$
$\mathcal{L}_{13}$	0.486	0.005	$A(\mathcal{L}_{13}) > G(\mathcal{L}_{13})$
$\mathcal{L}_{14}$	0.458	0.027	$A(\mathcal{L}_{14}) > G(\mathcal{L}_{14})$
$\mathcal{L}_{15}$	0.772	0.697	$A(\mathcal{L}_{15}) > G(\mathcal{L}_{15})$
$\mathcal{L}_{16}$	0.782	0.455	$A(\mathcal{L}_{16}) > G(\mathcal{L}_1)$
$\mathcal{L}_{17}$	0.521	0.410	$A(\mathcal{L}_2) > G(\mathcal{L}_2)$
$\mathcal{L}_{18}$	0.738	0.691	$A(\mathcal{L}_3) > G(\mathcal{L}_3)$
$\mathcal{L}_{19}$	0.237	0.189	$A(\mathcal{L}_4) > G(\mathcal{L}_4)$
$\mathcal{L}_{20}$	0.599	0.547	$A(\mathcal{L}_5) > G(\mathcal{L}_5)$
$\mathcal{L}_{21}$	0.768	0.705	$A(\mathcal{L}_6) > G(\mathcal{L}_6)$
$\mathcal{L}_{22}$	0.545	0.369	$A(\mathcal{L}_7) > G(\mathcal{L}_7)$
$\mathcal{L}_{23}$	0.559	0.499	$A(\mathcal{L}_8) > G(\mathcal{L}_8)$
$\mathcal{L}_{24}$	0.566	0.468	$A(\mathcal{L}_9) > G(\mathcal{L}_9)$
$\mathcal{L}_{25}$	0.811	0.478	$A(\mathcal{L}_{10}) > G(\mathcal{L}_{10})$
$\mathcal{L}_{26}$	0.743	0.584	$A(\mathcal{L}_{11}) > G(\mathcal{L}_{11})$
$\mathcal{L}_{27}$	0.423	0.258	$A(\mathcal{L}_{12}) > G(\mathcal{L}_{12})$
$\mathcal{L}_{28}$	0.655	0.433	$A(\mathcal{L}_{13}) > G(\mathcal{L}_{13})$
$\mathcal{L}_{29}$	0.672	0.72	$A(\mathcal{L}_{14}) > G(\mathcal{L}_{14})$
$\mathcal{L}_{30}$	0.525	0.462	$A(\mathcal{L}_{15}) > G(\mathcal{L}_{15})$

TABLE XV.RANKING ORDER FOR ARITHMETIC AND GEOMETRIC OPERATORS FOR QFS

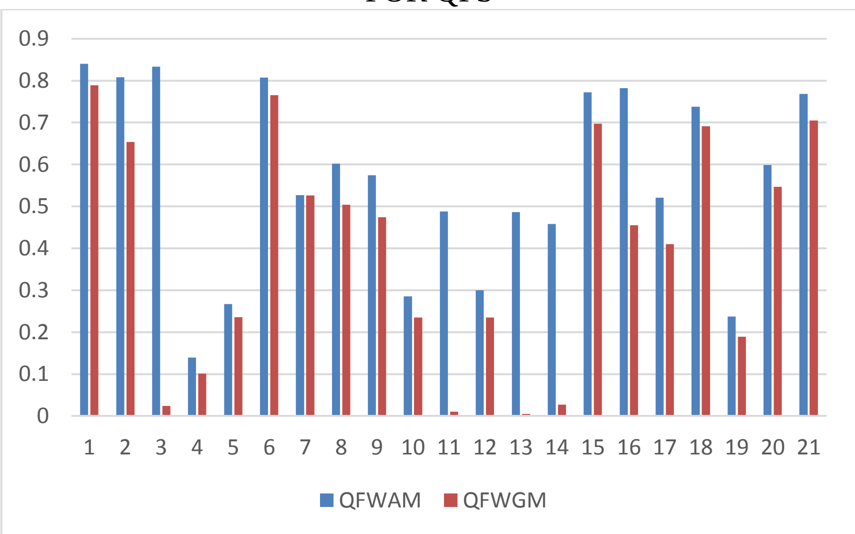

FIG 2. COMPARISON WITH ARITHMETIC AND GEOMETRIC OPERATORS FOR QFS

5.3. COMPARISON ANALYSIS OF EXPECTED AND EXISTING METHODS

From Table XVII , We notice that the ranking order of alternatives differs when different methods are used. However, the best alternative is obtained from the comparison of the Fematean Fuzzy Numbers (FFN)and Quad Fuzzy Numbers (QFN). For instance, the best alternative predicted in the proposed and existing methods is evaluated.Theanalysed problem gives the results favourable for the proposed method QFN AHP in comparison with existing method FFN AHP is $\mathcal{L}_1$.

Alternatives	**QFN- AHP**	**FFN -AHP**
$\mathcal{L}_1$	**0.903**	**0.874**
$\mathcal{L}_2$	0.802	0.626
$\mathcal{L}_3$	0.742	0.023
$\mathcal{L}_4$	0.565	0.501
$\mathcal{L}_5$	0.697	0.560
$\mathcal{L}_6$	0.521	0.423
$\mathcal{L}_7$	0.501	0.498
$\mathcal{L}_8$	0.624	0.404
$\mathcal{L}_9$	0.698	0.474
$\mathcal{L}_{10}$	0.312	0.235
$\mathcal{L}_{11}$	0.263	0.010
$\mathcal{L}_{12}$	0.369	0.112
$\mathcal{L}_{13}$	0.133	0.005
$\mathcal{L}_{14}$	0.445	0.027
$\mathcal{L}_{15}$	0.516	0.397
$\mathcal{L}_{16}$	0.734	0.455
$\mathcal{L}_{17}$	0.214	0.110
$\mathcal{L}_{18}$	0.526	0.198
$\mathcal{L}_{19}$	0.687	0.156
$\mathcal{L}_{20}$	0.469	0.298
$\mathcal{L}_{21}$	0.609	0.508
$\mathcal{L}_{22}$	0.466	0.369
$\mathcal{L}_{23}$	0.541	0.505
$\mathcal{L}_{24}$	0.396	0.368
$\mathcal{L}_{25}$	0.689	0.478
$\mathcal{L}_{26}$	0.686	0.183
$\mathcal{L}_{27}$	0.236	0.058
$\mathcal{L}_{28}$	0.602	0.487
$\mathcal{L}_{29}$	0.056	0.029
$\mathcal{L}_{30}$	0.452	0.119

Table XVI.Comparison between FFN AHP and QFN AHP

METHODS	RANKING ORDER
QFN -AHP	$\mathcal{L}_1 > L_2 > \mathcal{L}_3 > \mathcal{L}_{16} > \mathcal{L}_9 > \mathcal{L}_5 > L_{25} > \mathcal{L}_{19} > \mathcal{L}_{26} > \mathcal{L}_8$ $\mathcal{L}_{21} > \mathcal{L}_{29} > L_4 > \mathcal{L}_{23} > \mathcal{L}_{18} > \mathcal{L}_6 > \mathcal{L}_{15} > L_7 > \mathcal{L}_{20} > \mathcal{L}_{22}$ $\mathcal{L}_{30} > L_{14} > L_{24} > \mathcal{L}_{12} > \mathcal{L}_{10} > \mathcal{L}_{11} > \mathcal{L}_{27} > L_{17} > \mathcal{L}_{13} > \mathcal{L}_{28}$

FFN -AHP	$\mathcal{L}_1 > L_2 > \mathcal{L}_5 > \mathcal{L}_{21} > \mathcal{L}_{23} > \mathcal{L}_4 > L_7 > \mathcal{L}_{28} > \mathcal{L}_{25} > \mathcal{L}_9$ $\mathcal{L}_{16} > L_6 > \mathcal{L}_8 > \mathcal{L}_{15} > \mathcal{L}_{22} > \mathcal{L}_{24} > L_{20} > \mathcal{L}_{10} > \mathcal{L}_{18} > \mathcal{L}_{26}$ $\mathcal{L}_{19} > L_{30} > \mathcal{L}_{12} > \mathcal{L}_{17} > \mathcal{L}_{27} > \mathcal{L}_{29} > L_{14} > \mathcal{L}_3 > \mathcal{L}_{11} > \mathcal{L}_{13}$

TABLE XVII.PROPOSED AND EXISTING METHODS

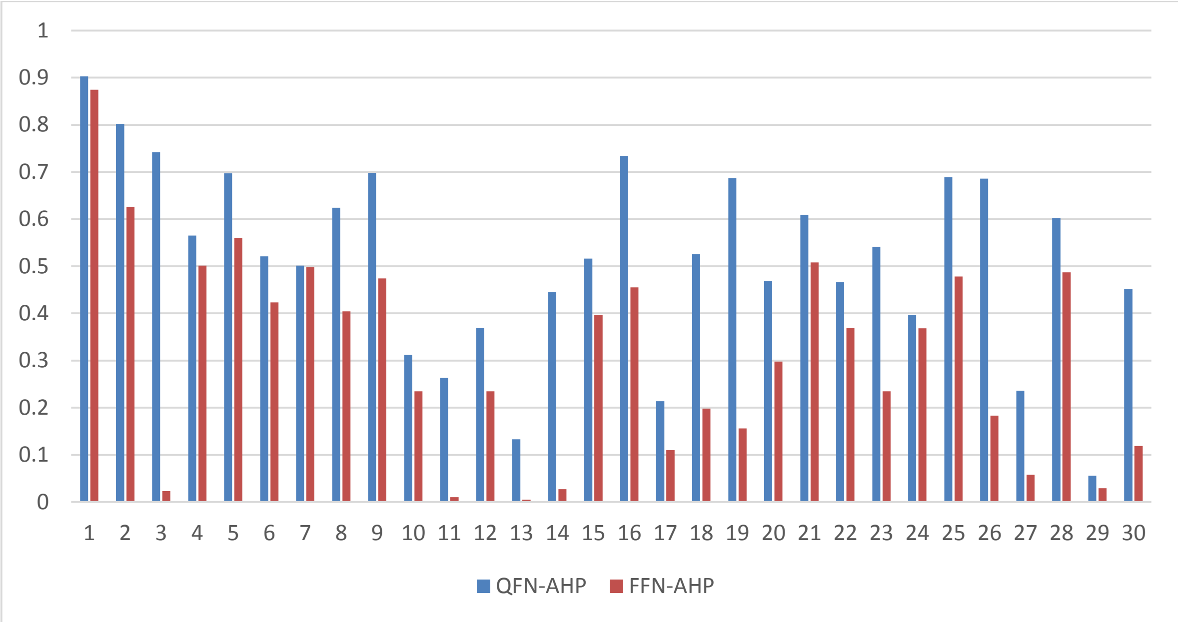

FIG 3. COMPARISON WITH ANALYTIC HIERARCHY PROCESSFOR QFN AND FFN

From Fig 3,We observe that the rankings of all alternatives using existing and proposed methods differ. To delevop that the graphs of methods QFN-AHP and FFN-AHP are monotonically increasing & decreasing between the alternatives,$\mathcal{L}_1$to$\mathcal{L}_{30}$.Therefore, the proposed methodare seems to be more stable.

The above graph indicates the comparison for QFN and FFN that makes this problem more efficient to analyse the best crop variety.This also satisfies the primary condition for the Quad Fuzzy set with membership and Non membership grades.

6. RESULTS &DISCUSSION

From the comparison result above, the best selection of crop variety under the Quad Fuzzy number is estimated to be Co 09004-Amritha and it is to focused to give more productivity of Sugarcane. The implementation of QFN and FFN in combination with MCDM methods represents to be powerful tool for solving problems related to the selection of suitable crop in agriculture. The decision makers can illustrate the real-life application in terms of formulated methods. With different crop varieties ,to achieve a clear solution, criteria must be prioritized while taking into account various constraints.Most importantly, a process of opting season and varieties, Morphological Characters, Crop Management ,Improved Technologies ,Fertilizer Management and Water Management plays an important role in the crop sustainability.

More conditions which are the backbone of the growth are climatic reasons, soil fertility, irrigation methods. The measures for the sugarcane cultivation are listed.

- Cultivate high yielding varieties
- Initiate the correct time for planting
- Decrease the duration with increase in yield

- Organic cultivation to be encouraged
- Agricultural research must be strengthened
- Checks on the harvesting periods
- Farmers interests to be considered

The application stands to help the decision makers to justify and to improve the growth of sugar cane yield in Tamil Nadu. Pre Harvest Practices can also be implemented in the process, With the goal of sustainability, evaluation should be done not only considering a single alternative based on multiple criteria, but also considering multiple scenarios based on multiple criteria.

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