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MULTIVARIABLE I-PD COMPENSATION OF HELICOPTER SYSTEM

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Abstract

A multivariable proportional integral derivative (I-PD) controller, for a 2 DOF helicopter system based on linear quadratic regulator is designed. The design problem of the multivariable controller is translated into a state feedback control that addresses the provided minimal state space representations (A,B,C) of the plant, and the gains of the state feedback control are attained via an optimal LQR design. Next, the PID gains are recovered from the state feedback gain.

Keywords— LQR, 2 dof multivariable controller,2 DOF helicopter, PID.

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I. INTRODUCTION

Proportional-integral-derivative (PID) controllers are used frequently in industry because they have worked well in the past. As many practical system are MIMO in nature, the PID controllers are mostly designed via (i) decentralized control (ii) decoupler based PID control. However, if the system is heavily coupled or the interaction is significant the compensated system frequently suffers from a lack of resilience and performance. Because of the addition of the decoupler, the overall compensator becomes more complex.

Designing a multivariable PID controller, a 2 DOF helicopter system is considered [1], from which the systems dynamics and state space model is found. A multi-input multi-output system's linear quadratic regulator (LQR) approach [2] has been utilised in attempts to create coupled or MIMO PID controllers for multivariable PID controllers. Similar to the characteristics of a helicopter, the twin rotor multiple-input and multiple-output (MIMO) system (TRMS) is a two-input, two-output, highly non-linear and unstable system. By developing a decentralised controller, [3] one desirable objective for controlling such a system is attained. If the off-diagonal terms are high, it affects the output performance of the system. The MIMO system is approximated as an advantage of independent SISO systems. For this purpose, first design the PID controller then transform it to static output feedback system[4].There are also some results available in [5-11]. In summary, the method calls for either a state observer, a particular plant model, or a plant that has a certain state space description. In order to compute the state feedback gain by use of a centralised controller that is modelled after a linear quadratic regulator (LQR).

II. 2 DOF HELICOPTER

A standard laboratory model for the theoretical study of helicopter controls and the verification of the control algorithm is the two-degrees-of-freedom (DOF) helicopter system. It possesses certain similarities to the features of a helicopter, such as the fact that it is mounted on a stationary base and has two propellers that are powered by two DC motors. As indicated in figure, the forward propeller is responsible for elevating the nose of the helicopter about the pitch axis, while the rear propeller is responsible for controlling the movement of the helicopter laterally around the yaw axis.-1. The movement of the helicopter in a side-to-side direction is controlled by the rear propeller, which rotates about the yaw axis. The front propeller is responsible for elevating the nose of the helicopter on the pitch axis. The position of the controller's pitch and yaw angles will be controlled by it. This is the controller's primary function.

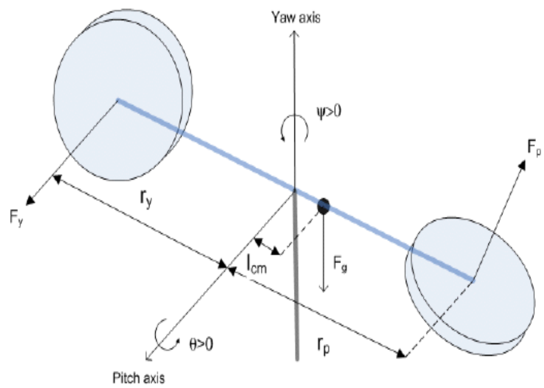

Figure 1: Dynamics of heliopter system

Though it is a multi input multi output system it is difficult to control, so to design the controller the dynamics of the system is needed. Using the Eulers-lagrange formula, the nonlinear equations of the motion of the system is found, and then linearised about the equilibrium point to get the linear state space model of the system.

$$(\partial \dot{p}) = A(\partial p) + B(\partial u)$$
$$(\partial \dot{q}) = C(\partial q) + D(\partial u)$$

III. TRANSFER FUNCTION OF THE MODEL

The linearised system can be written as

$$\dot{p}(t) = A(t)\,p(t) + B(t)u(t)$$
$$q(t) = C(t)\,p(t) + D(t)u(t)$$

Where we can get the values of A, B, C, D matrices from [1]. Table 1 listed below the main parameters associated with the model.

Table 1

Symbol	Description	Values
B_P	It represent the viscoelastic damping equal along the pitch axis	0.8
B_y	It represent the viscoelastic damping about the yaw axis	0.318
J_{eq_p}	Moment of inertia (MI) about the pitch axis	0.0384
J_{eq_y}	Total MI about yaw axis	0.0432
m_{heli}	Total moving mass of the helicopter	1.3872
l_{cm}	Center of mass length along helicopter	0.0186
K_{PP}	Thrust force constant of yaw motor/propeller	0.204
K_{PY}	Thrust torque constant acting on pitch axis	0.0068
K_{yy}	Thrust torque constant acting on yaw axis	0.072

The correspondence helicopter state space matrices are

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\dfrac{B_P}{J_{eq\_p} + m_{heli} l_{cm}^{\;2}} & 0 \\ 0 & 0 & 0 & \dfrac{-B_y}{J_{eq\_y} + m_{heli} l_{cm}^{\;2}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{K_{PP}}{J_{eq\_p} + m_{hel} l_{cm}^{\;2}} & \dfrac{K_{py}}{J_{eq\_p} + m_{hel} l_{cm}^{\;2}} \\ \dfrac{K_{yp}}{J_{eq\_y} + m_{heli} l_{cm}^{\;2}} & \dfrac{K_{YY}}{J_{eq\_y} + m_{hel} l_{cm}^{\;2}} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The transfer function of plant matrix can be calculated as follows,

$$P = C(sI - A)^{-1} B + D$$

$$= \begin{bmatrix} \dfrac{2.3613s^2 + 8.2343s}{s^4 + 12.7475s^3 + 32.2925s^2} & \dfrac{0.2401s^2 + 8.2343s}{s^4 + 12.7475s^3 + 32.2925s^2} \\ \dfrac{0.0787s^2 + 0.2744s}{s^4 + 12.7475s^3 + 32.2925s^2} & \dfrac{0.7894s^2 + 7.3101s}{s^4 + 12.7475s^3 + 32.2925s^2} \end{bmatrix}$$

It can be seen that from the denominator part of the transfer function that two poles are at origin. Thus the system is an unstable system. It also obvious that, there is a strong relationship between the input of the main rotor (u_1) and the yaw angle (ϕ).

IV. DESIGN OBJECTIVE

Objective is to design the controller on the basis of the linear model. The system's nonlinear equations are then linearized about the operational point. The internal workings of the MIMO PID controller are shown in Figure 2.

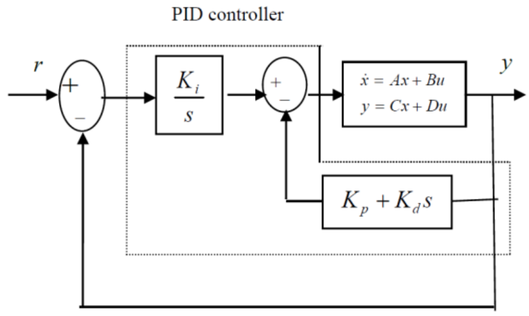

Fig.2. MIMO PID controller

Consider the linear time invariant plant

$$\dot{p}(t) = A(t)\,p(t) + B(t)u(t)$$
$$q(t) = C(t)\,p(t) + D(t)u(t) \qquad (1)$$

The system's control input equation is written as $u = K_p q + K_i \int_0^t (r-t)dt + K_d \frac{dq}{dt}$ (2)

Where, u $\in$ R^m denotes control input, p $\in R^n$ denotes state vector, and q $\in R^n$ denotes output vector in fig 1. r $\in R^p$ is the reference vector and K_p, K_i and K_d $\in R^{m \times p}$ are the gains that need to be designed into the matrices.

V. PROBLEM FORMULATION

To calculate the gain values, we transfer the PID control problem to static output feedback control by introducing a new state variable. This new state variable, $\xi = \int_0^t (r-q)dt$ in conjunction with system (1), produces an augumentated system, which is presented in the following equation.

$$\bar{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}$$

$$\bar{E} = \begin{bmatrix} 0 \\ I \end{bmatrix}, \qquad Z = \begin{bmatrix} p \\ \xi \end{bmatrix},$$

Then by substituting $q = cp$ in equation (2) we get,

$$u = K_p Cp + K_i \xi + K_d C\dot{p} \qquad (3)$$

Now by substituting $\dot{p} = Ap + Bu$ in (3) we have,

$$u = (I - K_d CB)^{-1}\left[\left(K_p C + K_p Cp\right)p + K_i \xi\right]$$

From which we obtain the PID parameters as

$$K_i = (I - K_d CB)\bar{K}_i,$$
$$K_p = (I - K_d CB)\bar{K}_p,$$
$$K_d = \bar{K}_d\left(I + CB\bar{K}_d\right)^{-1} \qquad (4)$$

The information presented above makes it clear that translation from PID to control to state feedback control is feasible when both (I- KdCB) and (I+ KdCB) are invertible. This is necessary for the transformation to take place. It is the necessary condition as well as the sufficient one.

Consider the Linear time invariant system

$$\dot{z} = \bar{A}z + \bar{B}u + \bar{E}r,$$
$$q = \bar{C}z \qquad (5)$$

Linear quadratic method is an easy method to find out the gains of the system. Here we have to find a state feedback control law u=Kz which minimises the quadratic performance cost weighted matrix, which is a positive matrix. The optimal gain vector is obtained by solving the Where Q is state weighted matrix, which is a positive semi definite matrix and R is control algebraic Riccati equation (ARE) for system (5) with r =0.

$$J = \int_0^\infty \left(z^T Q z + u^T R u\right)dt \qquad (6)$$
$$\bar{A}^T P + P\bar{A} - P\bar{B}R^{-1}\bar{B}^T P + Q = 0$$

For $P = P^T \succ 0$, then the optimal gain vectors is $K = -R^{-1}\bar{B}^T P$ (7)

The minimal cost is denoted by $J* = z(0)^T Pz(0)$, where z(0) is the vector representing the beginning state. The MATLAB programme 'lqr' can be used to quickly and easily resolve the issue described above.

Now from equation (4) we get our gains in vector from. As we can see the gains are now in matrix form off-diagonal terms are not zero, we are having enhance tunable parameter which will help to satisfy the better performance than decentralized controller.

VI. SIMULATION RESPONCE

To ensure the stability, the above designed PID block was studied through simulations

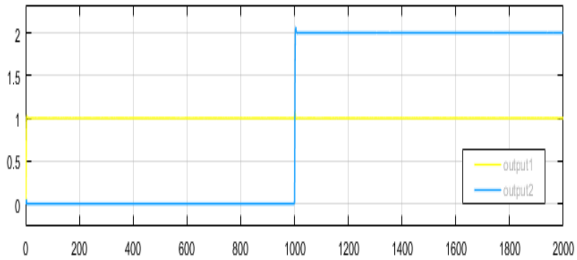

Fig:-output1 is for step input for 1(t) and output 2 is for step input for 2(t-1000).

Here is a example of the robustness response of quadraple tank[2] is shown in fig:-3.

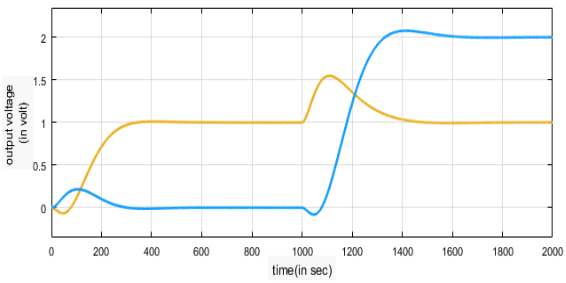

Fig:-3

From the output response of the system we can see that when one output is controlled, another output is not affected which yields decoupling behavior of the proposed control design. On other hand, the decentralized controller fails to guaranty decoupling behavior.

VII. CONCLUSION

The helicopter system will operate to LQR standards, the I-PD controller was developed for it. According to the findings of the simulation, the suggested controller is capable of achieving decoupling in the responses, as well as improved transient and steady-state performance than the decentralized PID controller.

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