An EOQ model for Time dependent deterioration with exponential demand and shortages

D.Samal[1*], M. R. Mishra[2], A. Kalam[3]

ABSTRACT:

In this paper, we develop an inventory model for time dependent deteriorations with exponential demand rate. In this proposed model, shortages are allowed and partially backlogged. The model is explained with the help of numerical examples. Sensitivity analysis of the optimal solution with respect to various parameters of the system is given and results are discussed in details.

Keywords: *Inventory model; Deterioration; Demand and Shortages; Economic order quantity*.

1. Introduction

Inventory models with deteriorating items have received considerably attention in recent years. For controlling inventory, sufficient work has already been accomplished by large number of authors in different areas of real life problems. The main objective of inventory management is to minimize the inventory carrying cost. To meet the future demand, it is very important to determine the optimal stock and optimal time of replenishment of inventory. The one of the important concerns of the inventory management is to decide when and how much to order so that the total cost associated with the inventory system should be minimum. In inventory system deterioration plays an important role. Most of the items deteriorate over time. Hence while determining the optimal inventory policy of that type of products, the effect of deterioration cannot be ignored.

The demand rate is assumed to be a constant in classical inventory models. But demand for physical goods may be time-dependent, price dependent and stock dependent. Goyal (1985) has developed an EOQ model with constant demand rate under the conditions of permissible delay in payments. In his model Goyal ignored the difference between the selling price and purchase cost. Goyal's model was extended by Dave (1985) by assuming the fact that the selling price is necessarily higher than its purchase cost. Aggarwal and Jaggi (1995) considered the inventory model with an exponential deterioration rate under the conditions of permissible delay in payments. Jamal et al. (2000) then further generalized the model to allow for shortages. Hwang and Shinn (1997) developed the model for determining the retailer's optimal price and lot size simultaneously when the supplier permits delay in payments for an order of a product whose demand rate is a function of price elasticity.

[1,2]Department of Mathematics, O P Jindal University, Raigarh, Chhattisgarh, India.
[3]Department of Mathematics, RITE, Bhubaneswar, Odisha, India.
*Corresponding Author: mail_to_debasmita.samal@opju.ac.in

Teng (2002) considered that the selling price is not equal to the purchase price to modify Goyal's model (1985). Huang (2007) investigated the retailer's optimal replenishment policy under permissible delay in payments. Tripathy and Mishra (2011) developed an inventory model on ordering policy for linear deteriorating items for declining demand with permissible delay in payments. Chang et al. (2003) have suggested a model under the conditions that supplier offers trade credit to the buyer if the order quantity is greater than or equal to a pre-determined quantity. Other many related articles can be found by Ouyang et al. (2008), Chang et al. (2003), Chung and Huang (2009) and Teng et al. (2005). In most of the inventory models, holding cost is known as constant, but holding cost may not always be constant. Several researchers like Vander Veen (1967), Goh (1994), Muhlemaan and Valtis Spanopoulos (1980), Weiss (1982) and Naddor (1966) were generalized EOQ models describing various functions of holding cost.

The characteristic of all of the above research papers is that the unsatisfied demand due to shortagesis completely backlogged. However in reality, demands for foods, medicines, raw materials etc. are usually lost during the shortage period. Montgomery et al. (1973) studied both deterministic and stochastic demand inventory models with a mixture of backorder and lot sales. Park (1982) modified and established the solution of Mak (1987) by incorporating a uniform replenishment rate to determine the optimal production inventory control policies. Papachristos and Skouri (2002) developed a partially backlogged inventory model in which the backlogging rate decreases exponentially as the waiting time increases.

Teng et al. (2003) then extended the fraction of unsatisfied demand back ordered to any decreasing function of the waiting time up to the next replenishment. Teng and Yang (2004) generalized the partial backlogging EOQ model to allow for time-varying purchase cost. Yang (2005) made a comparison among various partial backlogging inventory lot size models for deteriorating items on the basis of maximum profit. Teng et al. (2007) compared two pricing and lot sizing model for deteriorating items with shortages. Yang et al. (2010) developed an inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. Dye et al. (2007) developed inventory and pricing strategies for deteriorating items with shortages. Skouri et al. (2011) proposed an inventory model with general ramp type demand rate, constant deterioration rate, partial backlogging of unsatisfied demand and conditions of permissible delay in payments. Sana (2010) considered that the demand rate is price dependent, the deterioration rate is taken to be time proportion and shortages are allowed. Other related articles on inventory system with partial backlogging and shortages have been performed by Hou (2006), Jagga et al. (2006), Patra et al. (2010), Lin (2012), etc.

Nita Shah (2012) developed a time-proportional deterioration model without shortages and with replenishment policy for items having demands depending on price. Considering selling price dependent demand Sarkar (2013) developed a deteriorating model. For deteriorating items Chowdhury and Ghosh (2014) developed an inventory model with price and stock sensitive demand. Khana et. al. (2017) considering price dependent demand, developed a lot size deteriorating model for imperfect quality items. Uncertainties in some situations is due to fuzziness was primarily introduced by Zadeh, also some strategies for decision making in fuzzy environment was proposed by Zadeh et. al (1970). Pattnaik et al (2021) also worked on linearly deteriorating EOQ model for imperfect items with price dependent demand under different fuzzy environments.

Among the important paper published so far with inventory-level-dependent demand rate, mention may be made of by Gupta and Vrat (1986), Mandal and Phaujdar (1989), baker and Urban (1988) etc. Gupta and Vrat (1986) have discussed a situation where the demand rate has been assumed to dependent on the order quantity whereas, Mandal and Phaujdar (1989) have discussed an inventory level. Baker and Urban (1988) have analyzed a similar situation assuming the demand rate to be dependent on the on-hand inventory i according to the relation $R(i) = \alpha i^{\beta}$ where $\alpha > 0, 0 < \beta < 1$. Sahu et al (2007) have analysed a similar situation, assuming the demand rate to be depend on the on-hand inventory i according to the relation $R(i) = \alpha e^{-\beta i}$ where $\alpha > 0, 0 < \beta < 1$. Again Samal, Mishra and Kalam (2022) have analysed two situations, assuming the demand rate to be depend on the on-hand inventory i without shortage, according to the relation $R(i) = \alpha i^{\beta}$, $i \geq S_0$ and $R(i) = \alpha e^{-\beta i}$, $0 \leq i \leq S_0, \alpha > 0, \quad 0 < \beta < 1$.

The present inventory model makes an attempt to study the situation in which (1) Deterioration rate is time dependent; (2) shortages are partial backlogging; (3) demand rate is exponential time-dependent. Several results have been obtained. An optimal solution of the total cost is discussed for various partial backlogging issues. Finally, the sensitivity analysis is given to validate the proposed model.

2. Assumptions:

The following assumptions are being made throughout the manuscript:

a. The demand rate $D = D(t)$ of an item is time dependent i.e. $D(t) = \beta e^{\alpha t}$, where β and α are constants such that, $\beta > 0$, $0 < \alpha < 1$.

b. The lead time is zero.

c. The replenishment rate is infinite and instantaneous.

d. Shortages are allowed and partially backlogged.

e. The deterioration rate is time dependent i.e. $\theta t, 0 < \theta < 1$.

3. Notations:

The following notations are used throughout the paper

$D = D(t) = \beta e^{\alpha t}$, demand rate of the items

T : Cycle time
T_1 : The time at which $I(t) = 0$
A : Set up cost

C_h : The inventory holding cost per unit per unit of time

C_s : The shortage cost per unit per unit of time

C_d : The cost of each deteriorated items

$I(t)$: The on hand inventory level at time t over the period $[0, T]$

$C(T_1, T)$: The total cost per year

DC : The deterioration cost

HC : The holding cost

SC : The shortage cost.

4. Mathematical Formulation

The inventory level varies with time due to both demands and deterioration of material. At $t = T_1$ the inventory level achieves zero, after which shortages are allowed during the time interval $[T_1, T]$ and all of the demand during shortages period $[T_1, T]$ are partially backlogged. The inventory levels of the model are given by the following differential equation

$$\frac{dI(t)}{dt} + \theta t I(t) = -\beta e^{\alpha t}, \; 0 \le t \le T_1 \tag{1}$$

and

$$\frac{dI(t)}{dt} = -\beta, \; T_1 \le t \le T \tag{2}$$

The solution of Eq. (1) and Eq. (2) with the condition $I(T_1) = 0$ is given by

$$I(t) = -\beta\left\{t + \frac{\alpha t^2}{2} - \frac{\theta t^3}{3} - T_1 - \frac{\alpha T_1^2}{2} - \frac{\theta T_1^3}{6} + \frac{\theta t^2 T_1}{2}\right\}, 0 \le t \le T_1 \tag{3}$$

$$I(t) = \beta(T_1 - t), T_1 \le t \le T \tag{4}$$

The amount of deteriorated items during $[0, T_1]$ is given by

$$\frac{\beta\theta T_1^3}{6} \tag{5}$$

The Deterioration cost is given by:

$$DC = \frac{\beta C_d \theta T_1^3}{6} \tag{6}$$

The holding cost during $[0, T_1]$ is

$$HC = \beta C_h\left\{\frac{T_1^2}{2} + \frac{\alpha T_1^3}{3} + \frac{\theta T_1^4}{12}\right\} \tag{7}$$

The shortage cost during $[T_1, T]$ is given by:

$$SC = \frac{\beta C_s}{2}(T - T_1)^2 \tag{8}$$

Therefore, the total cost per year is given by

$$C(T_1, T) = \frac{1}{T}\left[\beta C_h\left\{\frac{T_1^2}{2} + \frac{\alpha T_1^3}{3} + \frac{\theta T_1^4}{12}\right\} + \frac{\beta C_d \theta T_1^3}{6} + \frac{\beta C_s}{2}(T - T_1)^2 + 2A\right] \tag{9}$$

Solving $\frac{\partial C(T_1, T)}{\partial T} = 0$ and $\frac{\partial C(T_1, T)}{\partial T_1} = 0$ and neglecting the higher terms we got

$$T = \sqrt{\frac{(C_h + C_S)4A}{\beta C_h C_s}} \quad \text{and}$$

$$T_1 = \sqrt{\frac{4C_s A}{(C_h + C_S)\beta C_h}}$$

5. Numerical Example and Sensitivity analysis

We provide the numerical example to illustrate the inventory model with the following data: the demand D = 1000 for t = 0 i.e. initial demand, the deterioration rate θ = 0.1, the holding cost C_h = 1.0 per item per unit of time, the deterioration cost C_d = 2.0 per item; the shortage cost C_s = 3.5 per item per unit of time; the set up cost A = 20; then we obtain T = 0.320713, T_1 = 0.249444 and $C(T,T_1)$ = 266.2806 which validate $T_1 < T$. We have performed sensitivity analysis by changing parameters β, A, c_h, c_d and c_s keeping the remaining parameters at their original values. The original values are taken from above example. The corresponding changes in the cycle time T, time T_1 at which $I(T_1) = 0$ and total cost per year are exhibited in the following Tables 1 to 5.

Table 1: Variation in β keeping all the parameters same as in numerical example

β	T	T_1	$C(T, T_1)$
1000	0.320713	0.249444	266.2806
1100	0.305788	0.237835	278.4416
1200	0.29277	0.22771	290.0625
1300	0.281284	0.218777	301.2094
1400	0.271052	0.210819	311.936
1500	0.261861	0.20367	322.2864
1600	0.253546	0.197203	332.2978
1700	0.245976	0.191315	342.0014
1800	0.239046	0.185924	351.4239
1900	0.23267	0.180966	360.5885
2000	0.226779	0.176383	369.5152

Table 2 Variation in A keeping all the parameters same as in numerical example

A	T	T_1	$C(T, T_1)$
20	0.320713	0.249444	266.2806
25	0.358569	0.278887	299.9772
30	0.392792	0.305505	330.862
35	0.424264	0.329983	359.618
40	0.453557	0.352767	386.6904
45	0.48107	0.374166	412.388
50	0.507093	0.394405	436.9357
55	0.531843	0.413656	460.5033
60	0.555492	0.432049	483.2225
65	0.578174	0.449691	505.1982
70	0.6	0.466667	526.5152

Table 3 Variation in C_h keeping all the parameters same as in numerical example

C_h	T	T_1	$C(T, T_1)$
1	0.320713	0.249444	266.2806
2	0.250713	0.159545	329.6128
3	0.222539	0.119829	366.837
4	0.20702	0.096609	391.8869
5	0.197122	0.081168	410.0495
6	0.190238	0.070088	423.878
7	0.185164	0.061721	434.7827
8	0.181265	0.055168	443.6137
9	0.178174	0.049889	450.9173
10	0.175662	0.045542	457.0617

Table 4 Variation in C_d keeping all the parameters same as in numerical example

C_d	T	T_1	$C(T, T_1)$
2	0.320713	0.249444	266.2806
2.2	0.352785	0.274388	270.6939
2.4	0.384856	0.299333	277.8169
2.6	0.416928	0.324277	286.8219
2.8	0.448999	0.349221	297.4087
3	0.48107	0.374166	309.3605
3.2	0.513142	0.39911	322.5174
3.4	0.545213	0.424055	336.7598
3.6	0.577284	0.448999	351.9975
3.8	0.609356	0.473943	368.1613
4	0.641427	0.498888	385.1986

Table 5 Variation in C_s keeping all the parameters same as in numerical example

C_s	T	T_1	$C(T, T_1)$
1	0.4	0.2	206.9333
1.5	0.365148	0.219089	229.0868
2	0.34641	0.23094	243.2935
2.5	0.334664	0.239046	253.2352
3	0.326599	0.244949	260.5995
3.5	0.320713	0.249444	266.2806
4	0.316228	0.252982	270.7994
4.5	0.312694	0.255841	274.4809
5	0.309839	0.258199	277.539
5.5	0.307482	0.260177	280.12
6	0.305505	0.261861	282.3279

The observations found from the Table 1 to Table 5 are summed up as follows:

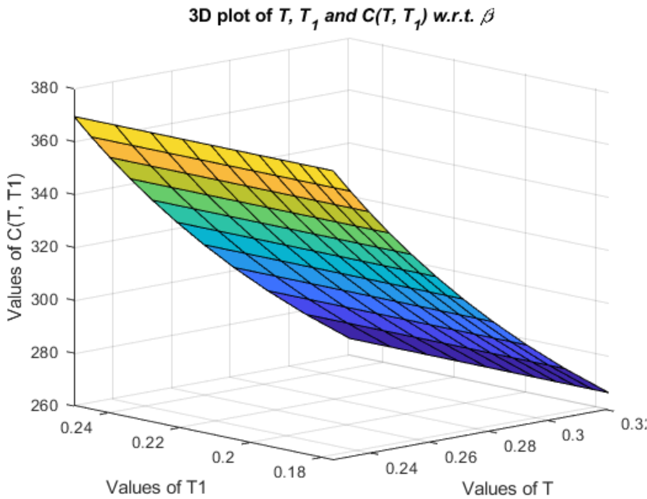
From Table 1 it can be easily seen that: Increase in the values of β results, decrease in the values of T and T_1 but increase in the values of total cost $C(T,T_1)$ keeping all parameters same.

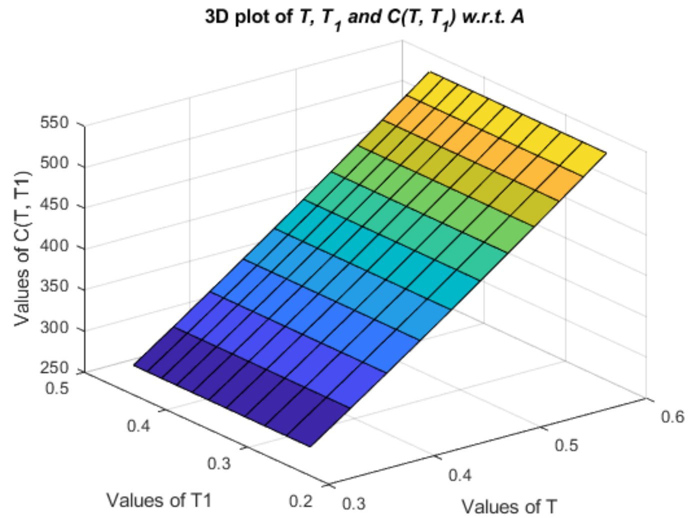
From Table 2 it can be easily seen that: Increase in the values of ordering cost A results, increase in the values of T and T_1 total cost $C(T,T_1)$, keeping all parameters same.

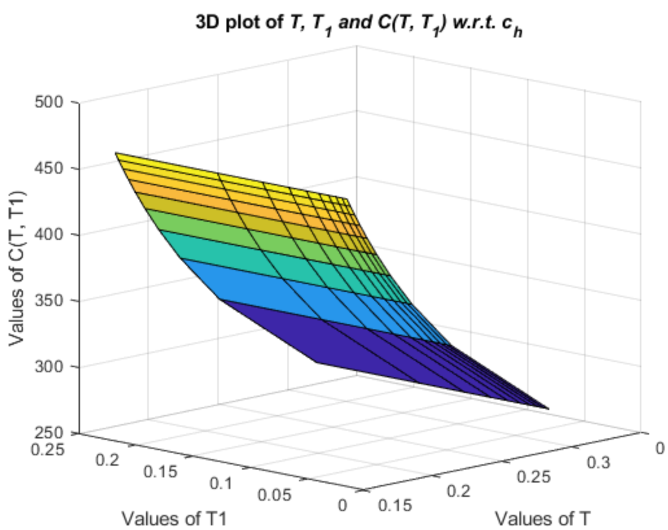
From Table 3 it can be easily seen that: Increase in the values of holding cost C_h results, decrease in the values of T and T_1 but increase in the values of total cost $C(T,T_1)$ keeping all parameters same.

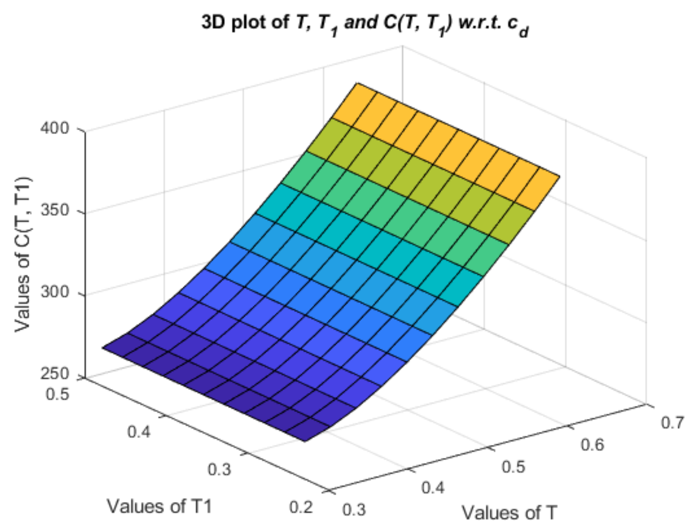
From Table 4 it can be easily seen that: Increase in the values of deterioration cost C_d results, decrease in the values of T and T_1 but increase in the values of total cost $C(T,T_1)$ keeping all parameters same.

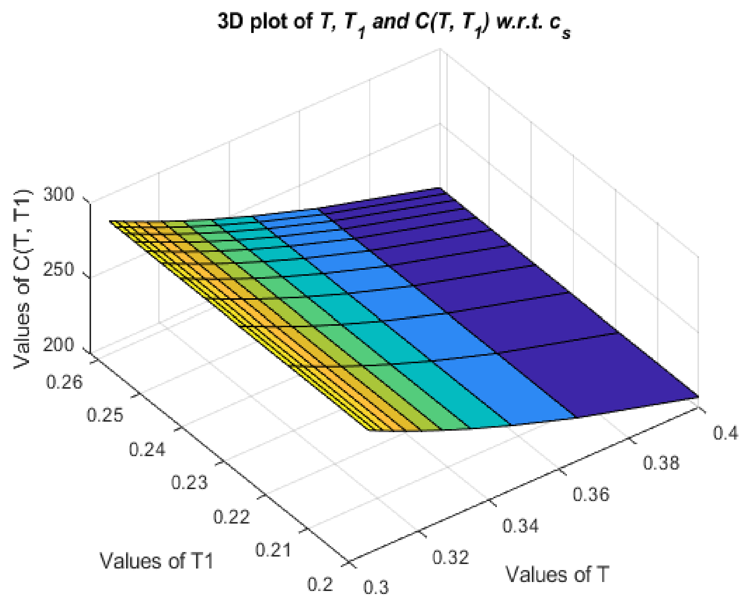
From Table 5 it can be easily seen that: Increase in the values of shortage cost C_s results, slight decrease in the values of T and slight increase in the values of T_1 but increase in the values of total cost $C(T,T_1)$ keeping all parameters same.


3D plot of T, T1 and C(T, T1) w.r.t. β
Values of C(T, T1)
Values of T1
Values of T


3D plot of T, T1 and C(T, T1) w.r.t. A
Values of C(T, T1)
Values of T1
Values of T


3D plot of T, T1 and C(T, T1) w.r.t. ch
Values of C(T, T1)
Values of T1
Values of T


3D plot of T, T1 and C(T, T1) w.r.t. cd
Values of C(T, T1)
Values of T1
Values of T



6. Conclusion and future research

In most of the deterministic EOQ model with shortages for permissible items, demand is either constant or induced by pricing decision. In this paper demand rate is taken as exponential time dependent, the deterioration is time dependent. We also have some important results based on optimal solution. From managerial point of view, sensitivity analysis is quite sensitive with respect to variation of parameters.

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