COMPUTATIONAL ANALYSIS OF UNRELIABLE M^X/M/1 MULTIPLE WORKING VACATION QUEUEING SYSTEM UNDER HETEROGENEOUS ARRIVAL

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ABSTRACT

This paper studies bulk arrival queuing system with breakdown and heterogeneous arrival during multiple working vacations. The heterogeneous arrival of customers follows a Poisson distribution and served under exponential distribution. The system studies performance measures using a probability generating function. Moreover, the decomposition property and the expected system size also discussed.

KEYWORDS

Batch arrival, Breakdown, Heterogeneous arrival, Multiple working vacations, Probability generating function(PGF), Stochastic decomposition Property.

INTRODUCTION

Classical queuing theory focuses on the study of queueing system. It provides a framework of queueing model based on customer arrivals and customers waiting time in a queue before being served. Arrival rate is varied with certain state. It is referred as heterogeneous. For instance, in call centre, healthcare stream, retail store, in restaurants customer arrivals is varying throughout the day. In queueing, vacations are characterized as either only one vacations or multiple vacations.

In vacations queuing system, the server completely suspends service during vacation, while in WV process, the server works at slower rate rather than not being use .Single server queuing system provides service to single customer for duration t, sometimes arrivals occur in a group which is known as batch arrival.

Shogan 1979, [13] studied the concept of M/M/1 with arrival depending on server breakdowns. Choudhary and Templeton,1983 [1] and Medhi 1984[11], investigated batch arrival queueing model. Doshi, B,1986 [2] studied on queueing systems with vacations. The concept of a single server with infinite number of customers, was introduced by Erlang in the year 1909[3]. In 2002, Servi L. D. and Finn S.G. [12], established the theory of vacations models. Krishna Kumar and Pavai Madheswari ,2005 [8], studied Markovian queue with M/M/2 servers and MV's. Wang et al. ,2007 [15] discussed the batch arrival model with multiple vacations and server breakdowns. Julia K and A begum ,2010 [6] analysed the M /M/1queueing system that operates with working vacations periods.

Jain. M & Jain A 2010 [4] discussed some kinds of server breakdowns with working vacation and Stochastic decomposition property. Madhu Jain, Anamika Jain,2010 [10] dealt with a M/M/1/ working vacations queuing model with numerous kind of server unreliability. Julia Rose Mary et al.,2016, [7] studied the bulk arrival queuing system along with server unreliability and MWV. Uma and Manikandan ,2017[14] analysed M^x/M/1 queueing system with triphasic varied service required vacations and balking. Jan –Kees van Ommeren et al.,2020 [5] analysed bulk service multiserver queues with heterogeneous server capacity. Liu mei and Alexander Dudin ,2022[9] discussed the multiserver retrial queue with heterogeneous servers.

Many authors contributed, the vacations queueing model with batch arrivals. Hear, we derive system measures of performance of unreliable M^x/M/1 multiple working vacations queueing system under heterogeneous arrivals. By solving the Chapman-Kolmogorov balancing equations for the steady state system, it is possible to derive the probability generating function to the system length. The system measures of performance are also obtained using probability generating function and stochastic decomposition is also derived.

METHODOLOGY

Consider a bulk arrival queuing system under MWV and breakdown where the customers enter the system according to Poisson distribution with positive number of customers in certain arrival rate of RV's 'X' and it has threshold value of $k < \infty$ with the probability g_k .

$$\text{i.e., } P(X = k) = g_k \quad , k \geq 1$$

Let λ denote composite of entire batch. The average moments of RV's X with PGF are denoted by $X(z)$, E(X) and E(X^2).

$$\text{i.e., } \quad X(z) = \sum_{k=1}^{\infty} g_k z^k \,, E(X^k) = \sum_{n=1}^{\infty} g_n \, n^k$$

During a normal busy period, server provides service for all customers with exponential distribution under parameter μ.At any time the system is vacant ,the server leaves the system ,which follows exponential distribution under parameter ϖ. When customers enters a queue , the system immediately begin service otherwise, it will take another vacation consecutively, until the new customer arrive. While working vacations period occurs the service time takes the parameter μ_v ,which follows exponential distribution.

Heterogeneous arrival refers to customers entering the queueing system with different arrival rate. This means that the arrival process is not uniform or homogeneous, and there are variations among the arrivals. Each of the arrivals during working vacations, regular busy periods and breakdowns periods will be λ_0, λ_1 and λ_2 respectively.

In our daily lives, many queueing situations arise. The server may suddenly fail, while in working progress. In such case, instantly a service providing system is dispatched for restore (breakdown) to resolve. The resolving time and service time are independent to each other. A server tends to destruction at any cost while performing the service with Poisson arrival rate α. When breakdowns occur, the system follows exponential distribution with parameterβ. These kind of service providing server goes on till the system become null. Under these criteria the mentioned model is analysed.

CLASSIFICATION OF THE MODEL

Assume $N_s(t)$ as the amount of customers in the network at duration t

$$M(t)=\begin{cases} 0 \; the\ system\ is\ in\ a\ working\ vacation\ \ period\ at\ time\ t \\ 1 \quad\quad the\ system\ is\ in\ regular\ busy\ period\ at\ time\ t \\ 2 \; the\ system\ is\ in\ breakdown\ \ period\ at\ time\ t \end{cases}$$

Then $\{N_s(t), M(t)\}$ is a Markov Process

Let $Q_n(t) = Pr\{N_s(t) = n\,; M(t) = 0\}, 0\ \le n < \infty$

$P_n(t) = Pr\{N_s(t) = n\,; M(t) = 1\}, 1 \le n < \infty$

$B_n(t) = Pr\{N_s(t) = n, M(t) = 2\}\ ,2 \le n < \infty$

It represents the system length of probability at duration t.

In steady state model,

$P_n = \lim_{t\to\infty} P_n(t)$ and $Q_n = \lim_{t\to\infty} Q_n\,(t)$ and $B_n = \lim_{t\to\infty} B_n(t)$ exists

Then P_n 's , Q_n's and B_n's which satisfies the steady state equations are given by

$\lambda_0 Q_0 = \mu_v Q_1 + \mu P_1$----------(1)

$(\lambda_0 + \varpi + \mu_v)Q_n = \lambda_0 \sum_{k=1}^{n} Q_{n-k} g_k + \mu_v Q_{n+1}, n \ge 1$ ------------(2)

$(\lambda_1 + \mu + \alpha)P_1 = \mu P_2 + \varpi Q_1 + \beta B_1$ ---------------(3)

$(\lambda_1 + \mu + \alpha)P_n = \beta B_n + \lambda_1 \sum_{k=1}^{n-1} P_{n-k} g_k + \mu P_{n+1} + \varpi Q_n\,, n \ge 2$---(4)

$(\lambda_2 + \beta)B_1 = \alpha P_1$ --------------------(5)

$(\lambda_2 + \beta)B_n = \alpha P_n + \lambda_2 \sum_{k=1}^{n-1} B_{n-k} g_k\,, n \ge 2$-------------(6)

The partial probability generating function are defined as follows

$Q(z) = \sum_{n=0}^{\infty} Q_n z^n$,$P(z) = \sum_{n=1}^{\infty} P_n z^n$ and $B(z) = \sum_{n=1}^{\infty} B_n z^n$

In equation (2) multiplying by $z^n$and summing up for $n \ge 1$,then adding with equation (1) and using Roche's theorem , we obtain

$Q(z) = \frac{\mu_v(z-z_1)Q_0}{z_1(\lambda_0 z(1-X(z))+\mu_v(z-1)+\varpi z)}$-------------(7)

In equation (3) multiplying by z and equation (4) with z^n, when $n \ge 2$ then summing up on both sides, we find

$$P(z) = \frac{\lambda_0 z \mu_v Q_0}{z_1}\left[\frac{(z-1)z_1(X(z_1)-1)-(z_1-1)z(X(z)-1)}{\lambda_0 z\big(1-X(z)\big)+\mu_v(z-1)+\varpi z}\right]$$

$$\left\{\frac{\lambda_2\big(1-X(z)\big)+\beta}{\big[\lambda_1 z\big(1-X(z)\big)+\mu(z-1)+\alpha z\big]\lambda_2\big(1-X(z)\big)+\beta\big[\lambda_1 z\big(1-X(z)\big)+\mu(z-1)\big]}\right\} - - - - - - - - (8)$$

Multiplying equation (5) by z and equation (6) multiplying by $z^n$and summing up for $n \ge 2$,then adding with equation (5) ,we get

$B(z) = \frac{\alpha P(z)}{\lambda_2\big(1-X(z)\big)+\beta} - - - - - - -(9)$

Then, the total Probability generating function $P_{MWV}^{BDHr}(z)$ is obtained by adding $Q(z), P(z)$ and $B(z)$

i.e, $$P_{MWV}^{BDHr}(z) = \frac{\mu_v Q_0\{(z-1)[\lambda_0 z z_1(X(z_1)-X(z))+\mu(z-z_1)]+(1-X(z))z(z-z_1)(\lambda_1-\lambda_0)\}}{z_1[\lambda_0 z(1-X(z))+\mu_z(z-1)+\varpi z][\lambda_1 z(1-X(z))+\mu(z-1)]}$$

$$\left[\frac{\lambda_2(1-X(z))+\beta+\frac{\alpha z\{(z-1)(\lambda_0 z_1(X(z_1)-X(z)))+(1-X(z))(z-z_1)(\lambda_2-\lambda_0)\}}{(z-1)(\lambda_0 z z_1(X(z_1)-X(z)))+\mu(z-z_1)+(1-X(z))z(z-z_1)(\lambda_1-\lambda_0)}}{\lambda_2(1-X(z))+\beta+\frac{\alpha\lambda_2 z(1-X(z))}{\lambda_1 z(1-X(z))+\mu(z-1)}}\right] - (10)$$

Using normalizing condition $P_{MWV}^{BDHr}(1) = 1$ with $X(1) = 1, X'(1) = E(X)$, we find

$$Q_0 = \frac{\varpi z_1(\beta\mu(1-\rho)-\alpha\lambda_2 E(X))}{\mu_v[(\alpha+\beta)(\lambda_0 z_1(X(z_1)-1))-E(X)(1-z_1)(\lambda_2\alpha+\lambda_1\beta-(\alpha+\beta)\lambda_0)]+\beta\mu(1-z_1)} - - - - (11)$$

By substituting Q_0 in equation (10) the PGF is

$$P_{MWV}^{BDHr}(z) = \frac{\mu(1-\rho)}{\lambda_1 z(1-X(z))+\mu(z-1)}\left(\frac{\frac{(z-1)[\lambda_0 z z_1(X(z_1)-X(z))+\mu(z-z_1)]+(1-X(z))\,z\,(z-z_1)(\lambda_1-\lambda_0)}{\lambda_0 z(1-X(z))+\mu_v(z-1)+\varpi z}}{\frac{\lambda_0 z_1(X(z_1)-1)+\mu(1-z_1)-E(X)(1-z_1)(\lambda_1-\lambda_0)}{\varpi}}\right)$$

$$\left[\frac{\lambda_2(1-X(z))+\beta+\frac{\alpha z\{(z-1)(\lambda_0 z_1(X(z_1)-X(z)))+(1-X(z))(z-z_1)(\lambda_2-\lambda_0)\}}{(z-1)(\lambda_0 z z_1(X(z_1)-X(z)))+\mu(z-z_1)+(1-X(z))z(z-z_1)(\lambda_1-\lambda_0)}}{\lambda_2(1-X(z))+\beta+\frac{\alpha\lambda_2 z(1-X(z))}{\lambda_1 z(1-X(z))+\mu(z-1)}}\right]$$

$$\frac{1-\frac{\alpha\rho}{\beta(1-\rho)}}{1+\frac{\alpha(\lambda_0 z_1(X(z_1)-1)-E(X)(1-z_1)(\lambda_2-\lambda_0))}{\beta(\lambda_0 z_1(X(z_1)-1)+\mu(1-z_1)-E(X)(1-z_1)(\lambda_1-\lambda_0))}}$$

Therefore ,the total PGF can also be written as

DECOMPOSITION PROPERTY

To decompose the probability generating function of unreliable $M^x/M/1$ multiple working vacations queueing system under heterogeneous arrival in to the product of two random variables. In that one of which is

$$P_{M^x/M/1/MWV}^{BD} = \frac{\mu(1-\rho)}{\lambda_1 z(1-X(z))+\mu(z-1)}\left(\frac{\frac{(z-1)[\lambda_0 z z_1(X(z_1)-X(z))+\mu(z-z_1)]+(1-X(z))\,z\,(z-z_1)(\lambda_1-\lambda_0)}{\lambda_0 z(1-X(z))+\mu_v(z-1)+\varpi z}}{\frac{\lambda_0 z_1(X(z_1)-1)+\mu(1-z_1)-E(X)(1-z_1)(\lambda_1-\lambda_0)}{\varpi}}\right)$$

Coincide with batch arrival multiple working vacation queueing system as mentioned in [7] and the other one is

$$\left[\frac{\lambda_2(1-X(z))+\beta+\frac{\alpha z\left\{(z-1)\left(\lambda_0 z_1(X(z_1)-X(z))\right)+(1-X(z))(z-z_1)(\lambda_2-\lambda_0)\right\}}{(z-1)\left(\lambda_0 z z_1(X(z_1)-X(z))\right)+\mu(z-z_1)+(1-X(z))z(z-z_1)(\lambda_1-\lambda_0)}}{\lambda_2(1-X(z))+\beta+\frac{\alpha\lambda_2 z(1-X(z))}{\lambda_1 z(1-X(z))+\mu(z-1)}}\right]$$

$$\frac{1-\frac{\alpha\rho}{\beta(1-\rho)}}{1+\frac{\alpha(\lambda_0 z_1(X(z_1)-1)-E(X)(1-z_1)(\lambda_2-\lambda_0))}{\beta(\lambda_0 z_1(X(z_1)-1)+\mu(1-z_1)-E(X)(1-z_1)(\lambda_1-\lambda_0))}}$$

This decomposition allows to analyse the classical $M^x/M/1$ queueing model and the additional queue length separately. Which can be useful for performance analysis.

EXPECTED SYSTEM SIZE

Now, the average system length of the model is written as

$$L = \frac{d}{dz}\left(P_{MWV}^{BDHr}(z)\right) \; at \; z = 1$$

$$= \frac{1-\frac{\alpha\rho}{\beta(1-\rho)}}{1+\frac{\alpha(\lambda_0 z_1(X(z_1)-1)-E(X)(1-z_1)(\lambda_2-\lambda_0))}{\beta(\lambda_0 z_1(X(z_1)-1)+\mu(1-z_1)-E(X)(1-z_1)(\lambda_1-\lambda_0))}}$$

$$\Bigg\{\left(1+\frac{\alpha D}{\beta C}\right)\left(\frac{1}{\dot{C}}\right)\left[\frac{\lambda_1 C\left(E(X^2)+E(X)\right)}{2\mu(1-\rho)}+\frac{C(\lambda_0 E(X)-\mu_v)}{\varpi}+\frac{\left(\mu-\lambda_0 E(X)\right)}{1}\right]$$
$$+\left(\frac{\rho\mu}{\beta C^2}\right)\left(\frac{1}{\dot{C}}\right)\left[C^3+\alpha C\left[z_1-\frac{D}{\lambda_1 E(X)}\right](D-C)+\alpha DC\left(\frac{C}{\beta}-\frac{1}{\rho}\right)-C\right]$$
$$+\left(\frac{E(X)(1-z_1)(\lambda_1-\lambda_0)}{\dot{C}}\right)\left[1+\frac{\alpha(\lambda_2-\lambda_0)}{\beta(\lambda_1-\lambda_0)}\right]\left(\frac{\lambda_1\left(E(X^2)+E(X)\right)}{2\mu(1-\rho)}\right.$$
$$\left.+\frac{\mu_v-\lambda_0 E(X)}{\varpi}\right)+\left[1+\frac{\alpha(\lambda_2-\lambda_0)}{\beta(\lambda_1-\lambda_0)}\right]\left(\frac{\lambda_1-\lambda_0}{\dot{C}}\right)\left(\frac{E(X^2)(z_1-1)-z_1 E(X)}{2}\right)$$
$$+\frac{\alpha\lambda_2\left(E(X)\right)^2(1-z_1)(\lambda_1-\lambda_0)}{\beta^2\dot{C}}\Bigg\}$$

where $\dot{C} = C - E(X)(1-z_1)(\lambda_1-\lambda_0)$, $C = \mu(1-z_1)+\lambda_0 z_1(X(z_1)-1)$,

and $D = \lambda_0 z_1(X(z_1)-1)$

Hence, the average length of the described model is computed.

CONCLUSION

The computational analysis of unreliable $M^x/M/1$ multiple working vacations queueing system under heterogeneous arrival are studied. The Stochastic decomposition property is verified using Probability generating function, and system performances are also obtained. Further, this model research may be extended to multiple breakdowns in order to enhance system performances.

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