SUM OF POWER n DIVISOR CORDIAL LABELING FOR SUBDIVISION GRAPHS

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Abstract:

A sum of Power n divisor cordial labeling of a graph G with vertex set V is a bijection $f: V \to \{1,2,3,\dots,|V(G)|\}$ such that an edge uv is assigned the label 1 if 2 divides $(f(u)+f(v))^n$ and 0 otherwise. The number of edges labeled with 0 and the number of edges labeled with 1 differ atmost 1. A graph with a sum of power n divisor cordial labeling is called a sum of power n divisor cordial graph. We establish in this paper that subdivision of some standard graphs are sum of power n divisor cordial graphs.

Keywords: Sum of power n, subdivision, path, star, cycle, tree, ladder

Introduction:

Let $G = (V, E)$ be a (p, q) graph with $p = |V(G)|$ vertices and $q = |E(G)|$ edges, where $V(G)$ and $E(G)$ denote the vertex set and edge set of the graph. In this paper, we consider the graphs which are simple, finite and undirected. For graph theoretic terminology and notations we refer to Harary [2]. The concept of graph labeling was introduced by Rosa [10] in 1967. A detailed survey of graph labeling is available in Gallian [1]. The concept of sum divisor cordial labeling has been introduced by Lourduswamy et al [6]. Jaslin Melbha and Preetha lal [3] introduced the concept of sum square divisor cordial labeling. Dr. L. Pandiaselvi and Dr. K. Palani [8] are proved cycle related graphs. R. Ponraj [9] introduced 4- Remainder

Cordial of some tree related graphs. Kathireasan [5] has proved that the subdivision of ladder graphs are graceful labeling.

1. Preliminaries

Definition 1.1. A sum of Power n divisor cordial labeling of a graph G with vertex set V is a bijection $f: V \to \{1,2,3,\ldots,|V(G)|\}$ such that an edge uv is assigned the label 1 if 2 divides $(f(u)+f(v))^n$ and 0 otherwise. The number of edges labeled with 0 and the number of edges labeled with 1 differ atmost 1. A graph with a sum of power n divisor cordial labeling is called a sum of power n divisor cordial graph.

Definition 1.2. A subdivision graph $S(G)$ of a graph G is a graph that can be obtained from G by subdividing each edge of G exactly once.

Definition 1.3. $Sp(P_m, K_{1,n})$ is a graph in which the root of the star $K_{1,n}$ is attached at one end of the path P_m.

Definition 1.4. $P_n \otimes S_m$ is a graph obtained from a path P_n by attaching root of a star S_m at every pendent vertex of P_n.

Definition 1.5. A snail S_n $(n \geq 4)$ is obtained from $P_n = \alpha_1, \alpha_2, \ldots, \alpha_n$ by adding two parallel edges between α_i and α_{n-i+1} for $i = 1,2,\ldots,\left\lfloor\frac{n}{2}\right\rfloor$.

Definition 1.6. A tortoise T_n $(n > 4)$ is obtained from path $P_n = \alpha_1, \alpha_2, \ldots, \alpha_n$ by attaching one edges between α_i and α_{n-i+1} for $i = 1,2,\ldots,\left\lfloor\frac{n-1}{2}\right\rfloor$.

Definition 1.7. A Bistar graph is the graph obtained by joining the centre vertices of two copies of $K_{1,n}$ by an edge and it is denoted by $B_{m,n}$.

Definition 1.8. A slanting ladder graph SL_n is the graph obtained from two paths $\alpha_1, \alpha_2, \ldots, \alpha_n$ and $\beta_1, \beta_2, \ldots, \beta_n$ by joining each α_i with $\beta_{i+1}, 1 \leq i \leq n-1$.

Definition 1.9. An open triangular ladder $O(TL_n), n \geq 2$ is obtained from an open ladder $O(L_n)$ by adding the edges $\alpha_i\beta_{i+1}, 1 \leq i \leq n-1$.

2. Main Results

Theorem 2.1. The subdivision graph $SS_p(P_n, K_{1,n})$ is sum of power n divisor cordial graph if n is odd.

Proof: Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be the vertices of the path P_n and $\beta_1, \beta_2, \ldots, \beta_n$ be the vertices of the star $K_{1,n}$. Now the root of the star $K_{1,n}$ is attached at one end of the path P_n. Thus, the resultant graph is $S_p(P_n, K_{1,n})$. Let $\{\gamma_i : 1 \leq i \leq n\}$ be the new vertices which subdivide the edges $\alpha_i\alpha_{i+1}$ and let $\{\delta_i : 1 \le i \le n\}$ be the new vertices which subdivide the edges $\alpha_n\beta_i\,; 1 \le i \le n$. Define a function $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ by $f(\gamma_i) = i\,; 1 \le i \le n-1$,n$f(\delta_i) = n - 1 + i\,;$ $1 \le i \le n$,$f(\alpha_i) = \delta_n + 2i - 1; 1 \le i \le n$,$f(\beta_i) = \delta_n + 2i;$ $1 \le i \le n$. Then the induced edge labels are $f^*(\alpha_{2i}\gamma_{2i}) = 1; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$, $f^*(\gamma_{2i}\alpha_{2i+1}) = 1;$ $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$,$f^*(\alpha_{2i-1}\gamma_{2i-1}) = 0\,;$ $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$, $f^*(\gamma_{2i-1}\alpha_{2i}) = 0\,; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$, $f^*(\alpha_n\delta_{2i-1}) = 0\,; 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$, $f^*(\alpha_n\delta_{2i}) = 1\,; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$, $f^*(\delta_{2i-1}\beta_{2i-1}) = 1\,; 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$, $f^*(\delta_{2i}\beta_{2i}) = 0\,;$ $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$. We observe that, $e_f(0) = 2n$ and $e_f(1) = 2n$. Thus $|e_f(0) - e_f(1)| \le 1$. Hence the graph $SS_p(P_n, K_{1,n})$ is sum of power n divisor cordial graph.

Example 2.2. The graph $SS_p(P_5, K_{1,5})$ is sum of power n divisor cordial graph is shown below.

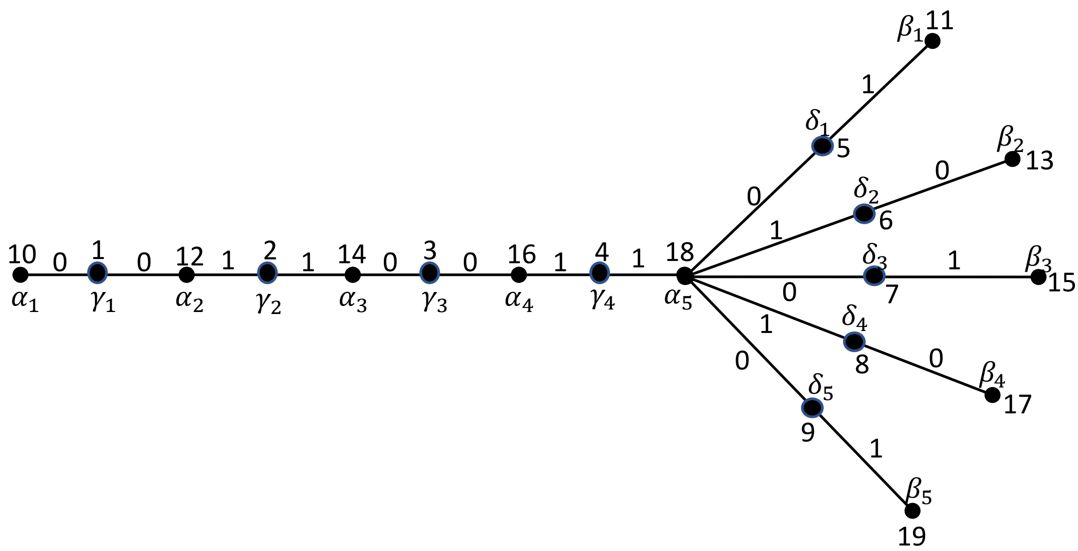

Figure 2.1.

Theorem 2.3. The subdivision graph $S(P_n \otimes S_m)$; n, m are even is sum of power n divisor cordial graph.

Proof: Let $\{A_i : 1 \le i \le n\}$ be the vertex of the path P_n and let $\{A'_i: 1 \le i \le n-1\}$ be the vertices of P_n. Which subdivide the edges $\alpha_i\alpha_{i+1}$ and let $\{B_i, C_i, D_i, \dots, Z_i : 1 \le i \le m\}$ be the vertices of S_m and let $\{b_i, c_i, d_i, \dots, z_i\,; 1 \le i \le m\}$ be the vertices of the star which subdivide the edges $\{A_1B_i, A_2C_i, A_3D_i, \dots, A_nZ_i : 1 \le i \le m\}$. Define a function

$f: V(G) \to \{1, 2, \dots, |V(G)|\}$ by $f(b_i) = i\,; 1 \le i \le m$, $f(c_i) = b_m + i\,;$ $1 \le i \le m$,..., $f(z_i) = y_m + i\,; 1 \le i \le m$, $f(B_i) = \alpha_1 + ni\,; 1 \le i \le m$, $f(C_i) = \alpha_2 + ni\,; 1 \le i \le m$, . . . , $f(Z_i) = \alpha_n + ni\,; 1 \le i \le m$, $f(A_i) = z_m + i\,; 1 \le i \le m$, $f(A'_i) = Z_m + i\,; 1 \le i \le n-1$. Then the induced edge labels are $f^*(A_1 b_{2i-1}) = 1\,; 1 \le i \le \frac{m}{2}$, $f^*(A_2 c_{2i}) = 1\,; 1 \le i \le \frac{m}{2}$, . . . , $f^*(A_n z_{2i}) = 1\,; 1 \le i \le \frac{m}{2}$, $f^*(b_{2i-1} B_{2i-1}) = 1\,; 1 \le i \le \frac{m}{2}$, $f^*(c_{2i} C_{2i}) = 1\,;$ $1 \le i \le \frac{m}{2}$, ... , $f^*(z_{2i} Z_{2i}) = 1\,; 1 \le i \le m-2$, $f^*(A_i A'_i) = 1\,;$ $1 \le i \le n-1$,$f^*(A_1 b_{2i}) = 0\,;$ $1 \le i \le \frac{m}{2}$, $f^*(A_2 c_{2i-1}) = 0\,; 1 \le i \le \frac{m}{2}$, ..., $f^*(A_n z_{2i-1}) = 0\,; 1 \le i \le \frac{m}{2}$, $f^*(b_{2i} B_{2i}) = 0\,; 1 \le i \le \frac{m}{2}$, $f^*(c_{2i-1} C_{2i-1}) = 0\,;$ $1 \le i \le \frac{m}{2}$, . . . , $f^*(z_{2i} Z_{2i}) = 0\,; 1 \le i \le \frac{m}{2}$, $f^*(A'_i A_{i+1}) = 0\,; 1 \le i \le \frac{m}{2}$. We observe that $e_f(0) = \frac{nm}{2} + n - 1$ and $e_f(1) = \frac{nm}{2} + n - 1$. Thus $|e_f(0) - e_f(1)| \le 1$. Hence the graph $S(P_n \otimes S_m)$ is sum of power n divisor cordial graph.

Example 2.4. The graph $S(P_4 \otimes S_4)$ is sum of power n divisor cordial graph is shown below.

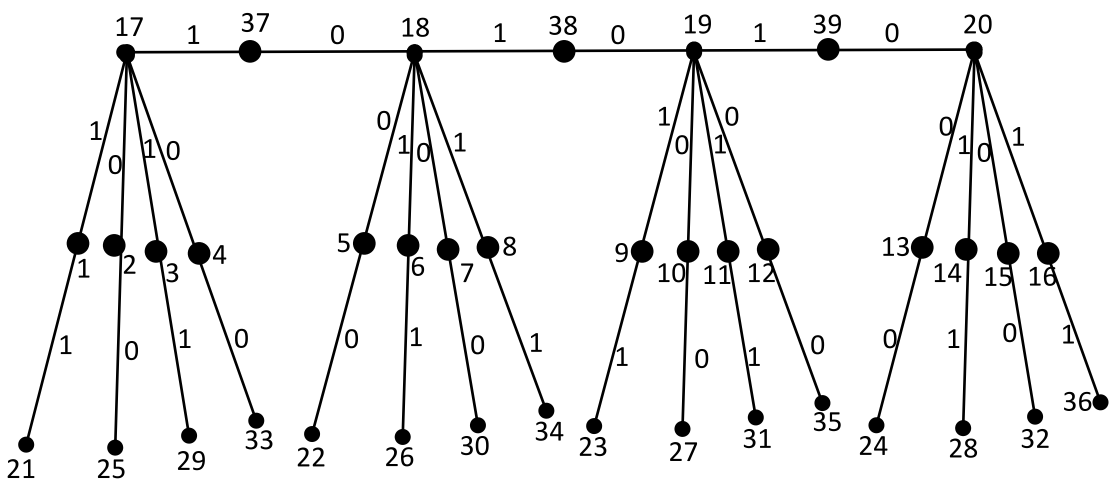

Figure 2.2.

Theorem 2.5. The subdivision graph of the snail graph $S(S_n)$ is a sum of power n divisor cordial graph.

Proof. Let $\{\alpha_i : 1 \le i \le n\}$ be the vertices of the snail graph S_n and let $\{A_i : 1 \le i \le n-1\}$ be the vertices which subdividing the edges $\alpha_i \alpha_{i+1}\,;$ $1 \le i \le n-1$. Let $E(G) = \{\alpha_i \alpha_{i+1}\,; 1 \le i \le n-1\} \cup \Big\{2(\alpha_i \alpha_{n-i+1}\,; 1 \le i \le$

$\left\lfloor\frac{n}{2}\right\rfloor\}$. Let $\{B_i : 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor\}$ be the vertices which subdivide the upper part of the edges of the snail graph and let $\{C_i : 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor\}$ be the vertices which subdivide the lower part of the edges of the snail graph. Define a function $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ by $f(\alpha_i) = i\,; 1 \le i \le n$, $f(A_i) = n + i\,;$ $1 \le i \le n-1$, $f(B_i) = A_n + i\,; 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$, $f(C_i) = B_n + i\,; 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$. Then the induced edge labels are

Case (i). For odd number of vertices;

$f^*(\alpha_i A_i) = 0; 1 \le i \le n-1$, $f^*(A_i \alpha_{i+1}) = 1\,; 1 \le i \le n-1$, $f^*(\alpha_i B_i) = 0\,;$ $1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$, $f^*(B_i \alpha_{n-i+1}) = 0\,; 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$, $f^*(\alpha_i C_i) = 1\,; 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$, $f^*(C_i \alpha_{n-i+1}) = 1\,; 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$.

Case (i). For even number of vertices;

$f^*(\alpha_i A_i) = 1; 1 \le i \le n-1$, $f^*(A_i \alpha_{i+1}) = 0\,; 1 \le i \le n-1$, $f^*(\alpha_i B_i) = 0\,;$ $1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$, $f^*(B_i \alpha_{n-i+1}) = 1\,; 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$, $f^*(\alpha_i C_i) = 1\,; 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$, $f^*(C_i \alpha_{n-i+1}) = 0\,; 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$.

From both the cases, we observe that $e_f(0) = 2n - 1$ and $e_f(1) = 2n - 1$. Thus $|e_f(0) - e_f(1)| \le 1$**.** Hence the graph $S(S_n)$ is sum of power n divisor cordial graph.

Example 2.6. The graph $S(S_5)$ is sum of power n divisor cordial graph is shown below.

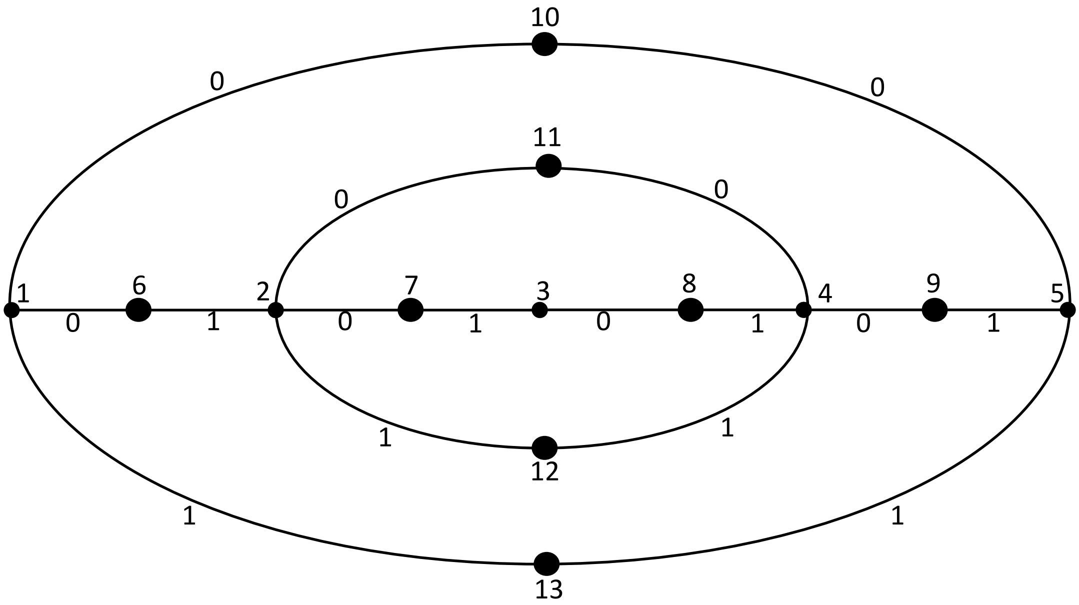

Figure 2.3.

Theorem 2.7. The subdivision graph of the tortoise graph $S(T_n)$ is a sum of power n divisor cordial graph, if $n > 4$ and n is even.

Proof. Let $V(G) = \{\alpha_i : 1 \le i \le n\}$ be the vertices of the tortoise graph T_n and let $E(G) = \{\alpha_i\alpha_{i+1} ; 1 \le i \le n-1\} \cup \left\{(\alpha_i\alpha_{n-i+1} ; 1 \le i \le \left\lfloor\frac{n-1}{2}\right\rfloor\right\}$ be the edges of the tortoise graph T_n. Let $\{A_i : 1 \le i \le n-1\}$ be the vertices which subdividing the edges $\alpha_i\alpha_{i+1} ; 1 \le i \le n-1$ and let $\{B_i : 1 \le i \le \left\lfloor\frac{n-1}{2}\right\rfloor\}$ be the vertices which subdivide the edges $\alpha_i\alpha_{n-i+1} ; 1 \le i \le \left\lfloor\frac{n-1}{2}\right\rfloor$. Define a function $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ by $f(\alpha_i) = i ; 1 \le i \le n$, $f(A_i) = n + i ;$ $1 \le i \le n-1$, $f(B_i) = A_n + i ; 1 \le i \le \left\lfloor\frac{n-1}{2}\right\rfloor$. Then the induced edge labels are $f^*(\alpha_i A_i) = 1; 1 \le i \le n-1$, $f^*(A_i\alpha_{i+1}) = 0 ; 1 \le i \le n-1$, $f^*(\alpha_i B_i) = 0 ;$ $1 \le i \le \left\lfloor\frac{n-1}{2}\right\rfloor$, $f^*(B_i\alpha_{n-i+1}) = 1 ; 1 \le i \le \left\lfloor\frac{n-1}{2}\right\rfloor$. We observe that $e_f(0) = 3n - 3$ and $e_f(1) = 3n - 3$. Thus $|e_f(0) - e_f(1)| \le 1$**.** Hence the graph $S(T_n)$ is sum of power n divisor cordial graph, if $n > 4$ and n is even.

Example 2.8. The graph $S(T_6)$ is sum of power n divisor cordial graph is shown below.

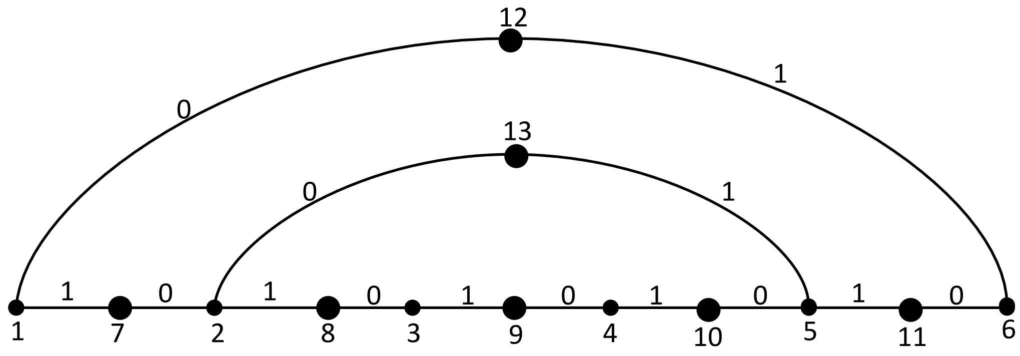

Figure 2.4.

Theorem 2.9. Subdivision of the bistar graph $S(B_{m,n})$ is sum of power n divisor cordial graph.

Proof. Let $G = S(B_{m,n})$. Let α, β be the vertices of K_2. Join m pendant vertices $\alpha_1, \alpha_2, \dots, \alpha_m$ to the one end of K_2 and join n pendant vertices $\beta_1, \beta_2, \dots, \beta_n$ to the other end of K_2. The resultant graph is the bistar $B_{m,n}$. Let $a_1, a_2, \dots, a_m$ be the subdividing vertices which subdivide the edges $\alpha\alpha_i$; $1 \le i \le m$. Let $b_1, b_2, \dots, b_n$ be the subdividing vertices which subdivide the edges $\beta\beta_i$; $1 \le i \le n$ and let c be the vertex which subdivide $\alpha\beta$. Then the resultant graph is $G = S(B_{m,n})$. Define a function $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ by $f(\alpha_i) = 2i$; $1 \le i \le m$, $f(a_i) = 2i - 1$; $1 \le i \le m$, $f(\alpha) = \alpha_m + 1$, $f(b_i) = \alpha + 2i - 1$; $1 \le i \le n$, $f(\beta_i) = \alpha + 2i$; $1 \le i \le n$, $f(\beta) = \beta_n + 1$, $f(c) = \beta + 1$. Then the induced edge labels are $f^*(\alpha a_i) = 1$; $1 \le i \le m$, $f^*(a_i\alpha_i) = 0$; $1 \le i \le m$, $f^*(\beta b_i) = 1$; $1 \le i \le n$, $f^*(b_i\beta_i) = 0$; $1 \le i \le n$, $f^*(\alpha c) = 1$, $f^*(c\beta) = 0$. We observe that $e_f(0) = m + n + 1$ and $e_f(1) = m + n + 1$. Thus $|e_f(0) - e_f(1)| \le 1$**.** Hence the graph $S(B_{m,n})$ is sum of power n divisor cordial graph.

Example 2.10. A sum of power n divisor cordial labeling of $S(B_{6,6})$ is given below.

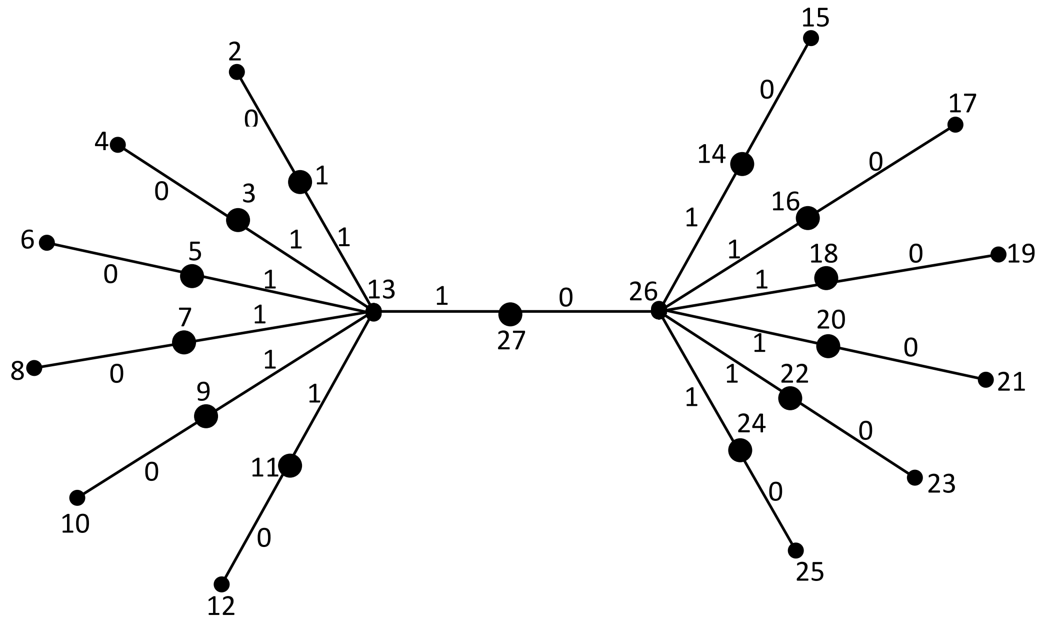

Figure 2.5.

Theorem 2.11. Subdivision of double star graph $K_{1,n,n}$ is sum of power n divisor cordial graph.

Proof: Let $V(G) = \{\alpha, \alpha_1, \alpha_2, \dots, \alpha_n, \beta_1, \beta_2, \dots, \beta_n\}$ be the vertices of the double star $K_{1,n,n}$ and $E(G) = \{\alpha\alpha_i , \alpha_i\beta_i : 1 \le i \le n\}$. Then the resultant graph is $K_{1,n,n}$. Let $\{a_i ; 1 \le i \le n\}$ be the subdividing vertices which subdivide the edges $\alpha\alpha_{i+1} ; 1 \le i \le n$ and let $\{b_i ; 1 \le i \le n\}$ be the subdividing vertices which subdivide the edges $\alpha_i\alpha_i; 1 \le i \le n$. Then the resultant graph is $G = S(K_{1,n,n})$. Define a function $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ by $f(\beta_i) = 2i - 1 ; 1 \le i \le n$, $f(b_i) = 2i ; 1 \le i \le n$, $f(a_i) = b_n + 2i - 1 ; 1 \le i \le n$, $f(\alpha_i) = b_n + 2i$; $1 \le i \le n$, $f(\alpha) = \alpha_n + 1$. Then the induced edge labels are $f^*(\alpha a_i) = 1$; $1 \le i \le n$, $f^*(a_i\alpha_i) = 0 ; 1 \le i \le n$, $f^*(\alpha_i b_i) = 1 ; 1 \le i \le n$, $f^*(b_i\beta_i) = 0$; $1 \le i \le n$. We observe that $e_f(0) = 2n$ and $e_f(1) = 2n$. Thus $|e_f(0) - e_f(1)| \le 1$. Hence the graph $S(K_{1,n,n})$ is sum of power n divisor cordial graph.

Example 2.12. A sum of power n divisor cordial labeling of $S(K_{1,7,7})$ is given below.

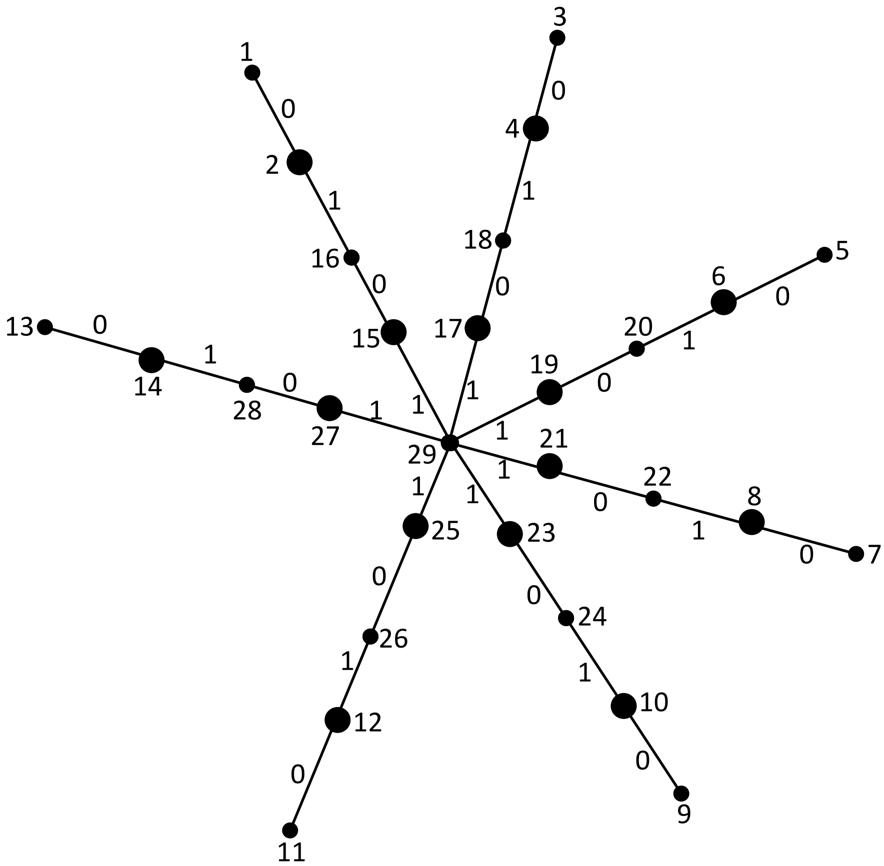

Figure 2.6.

Theorem 2.13. The Subdivision graph of an open triangular ladder graph $O(TL_n), n \geq 2$ is sum of power n divisor cordial graph.

Proof: Let $G = S(O(TL_n))$. Let the vertices be $\{\alpha_i\, ; 1 \leq i \leq n\}$ and $\{\beta_i\, ; 1 \leq i \leq n\}$. Let $\{a_i\, ; 1 \leq i \leq n-1\}$ be the vertices which subdivide the edges $\alpha_i \alpha_{i+1}\, ; 1 \leq i \leq n-1$, $\{b_i\, ; 1 \leq i \leq n-1\}$ be the vertices which subdivide the edges $\beta_i \beta_{i+1}\, ; 1 \leq i \leq n-1$, $\{c_i\, ; 1 \leq i \leq n-2\}$ be the vertices which subdivide the edges $\alpha_i \beta_i\, ; 2 \leq i \leq n-1$ and $\{d_i\, ; 1 \leq i \leq n-1\}$ be the vertices which subdivide the edges $\alpha_i \beta_{i+1}\, ; 1 \leq i \leq n-1$. Define a function $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ as follows;

Case (i). For odd number of n

$f(\alpha_i) = i\, ; 1 \leq i \leq n$, $f(a_i) = n + i\, ; 1 \leq i \leq n-1$, $f(\beta_i) = a_{n-1} + i\, ;$ $1 \leq i \leq n$, $f(b_i) = \beta_n + i\, ; 1 \leq i \leq n-1$, $f(c_i) = b_{n-1} + 2i; 1 \leq i \leq n-2$, $f(d_i) = b_{n-1} + 2i - 1\, ; 1 \leq i \leq n-1$. Then the induced edge labels are $f^*(\alpha_i a_i) = 0\, ; 1 \leq i \leq n-1$, $f^*(\beta_i b_i) = 0; 1 \leq i \leq n-1$, $f^*(\alpha_{2i+1} c_{2i}) = 0;$ $1 \leq i \leq \left\lfloor \frac{n-2}{2} \right\rfloor$, $f^*(\alpha_{2i} c_{2i}) = 0\, ; 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$, $f^*(c_{2i-1} \beta_{2i}) = 0; 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$,

$f^*(d_{2i}\beta_{2i+1}) = 0\,; 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor$, $f^*(b_i\alpha_{i+1}) = 1; 1 \le i \le n-1$, $f^*(b_i\beta_{i+1}) = 1;$ $1 \le i \le n-1$, $f^*(\alpha_{2i-1}d_{2i-1}) = 1\,; 1 \le i \le \left\lceil\frac{n}{2}\right\rceil$, $f^*(d_{2i-1}\beta_{2i}) = 1\,;$ $1 \le i \le \left\lceil\frac{n}{2}\right\rceil$, $f^*(\alpha_{2i}c_{2i-1}) = 1; 1 \le i \le \left\lfloor\frac{n-1}{2}\right\rfloor$, $f^*(c_{2i}\beta_{2i+1}) = 1; 1 \le i \le \left\lceil\frac{n-2}{2}\right\rceil$.

Case (ii). For even number of n

$f(\alpha_i) = i\,; 1 \le i \le n-1$, $f(a_i) = n-1+i\,; 1 \le i \le n-2$, $f(\beta_i) = a_{n-2} + i\,; 1 \le i \le n-1$, $f(b_i) = \beta_{n-1} + i : 1 \le i \le n-2$, $f(c_i) = b_{n-1} + 2i;$ $1 \le i \le n-1$, $f(d_i) = b_{n-1} + 2i - 1; 1 \le i \le n-2$, $f(\alpha_n) = c_{n-2} + 1$, $f(a_{n-1}) = \alpha_n + 1$, $f(\beta_n) = a_{n-1} + 1$, $f(b_{n-1}) = \beta_n + 1$, $f(d_{n-1}) = b_{n-1} + 1$.

Then the induced edge labels are $f^*(\alpha_i a_i) = 0\,; 1 \le i \le n-1$, $f^*(a_{n-1}\alpha_n) = 0$, $f^*(\beta_i b_i) = 0; 1 \le i \le n-2$, $f^*(b_{n-1}\beta_n) = 0$, $f^*(c_{2i-1}\beta_{2i}) = 0; 1 \le i \le \frac{n}{2}$, $f^*(\beta_{2i+1}c_{2i}) = 0; 1 \le i \le \frac{n}{2}$, $f^*(\alpha_{2i}d_{2i}) = 0\,; 1 \le i \le \left\lfloor\frac{n-1}{2}\right\rfloor$, $f^*(a_{n-1}\alpha_n) = 0$, $f^*(\alpha_{2i}c_{2i}) = 0\,; 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil$, $f^*(a_i\alpha_{i+1}) = 1; 1 \le i \le n-2$, $f^*(b_i\beta_{i+1}) = 1;$ $1 \le i \le n-1$, $f^*(\beta_{n-1}b_{n-1}) = 1$, $f^*(\alpha_{2i}c_{2i-1}) = 1; 1 \le i \le \frac{n}{2}$, $f^*(c_{2i}\beta_{2i+1}) = 1; 1 \le i \le \frac{n}{2}$, $f^*(\alpha_{2i-1}d_{2i-1}) = 1\,; 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil$, $f^*(d_{2i-1}\beta_{2i}) = 1\,; 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil$.

From both the cases, we get, $\left|e_f(0)\right| = 4n-4$ and $|e_f(1)| = 4n-4$. Thus $|e_f(0) - e_f(1)| \le 1$. Hence the graph $S(O(TL_n))$ is sum of power n divisor cordial graph.

Example 2.14. A sum of power n divisor cordial labeling of $S(O(TL_5))$ is shown below.

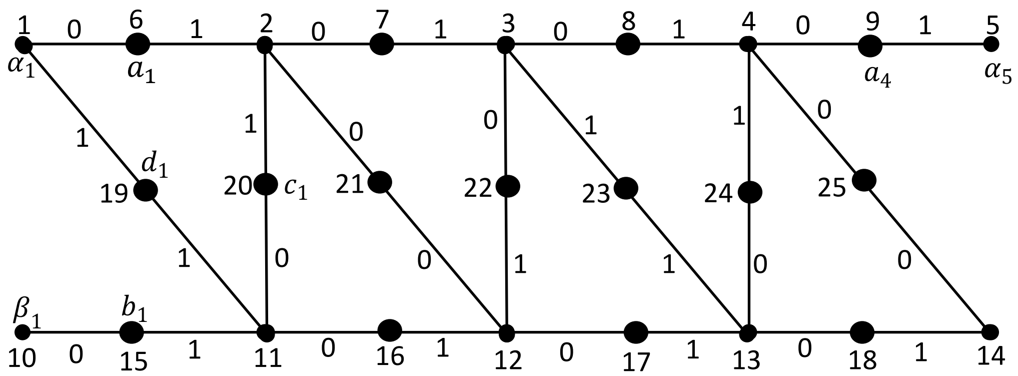

Figure 2.7.

Theorem 2.15. The Subdivision graph of the slanting ladder graph SL_n is sum of power n divisor cordial graph.

Proof: Let $G = S(SL_n)$. Let the vertices be $\{\alpha_i\,; 1 \le i \le n\}$ and $\{\beta_i\,; 1 \le i \le n\}$. Let $\{a_i\,; 1 \le i \le n-1\}$ be the vertices which subdivide the edges $\alpha_i\alpha_{i+1}$; $1 \le i \le n-1$, $\{b_i\,; 1 \le i \le n-1\}$ be the vertices which subdivide the edges $\beta_i\beta_{i+1}\,; 1 \le i \le n-1$ and $\{c_i\,; 1 \le i \le n\}$ be the vertices which subdivide the edges $\alpha_i\beta_{i+1}\,; 1 \le i \le n-1$. Define a function $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ as follows;

Case (i). For even number of n

$f(\alpha_i) = i\,; 1 \le i \le n, f(\beta_i) = n + i\,; 1 \le i \le n, f(a_i) = \beta_n + i; 1 \le i \le n-1$, $f(c_i) = a_n + i : 1 \le i \le n-1$, $f(b_i) = c_n + i : 1 \le i \le n-1$. Then the induced edge labels are $f^*\left(a_i\alpha_{i+\_1}\right) = 0\,; 1 \le i \le n-1$, $f^*(\alpha_i c_i) = 0$; $1 \le i \le n-1$, $f^*(b_i\beta_{i+1}) = 0; 1 \le i \le n-1$, $f^*(\alpha_i a_i) = 1; 1 \le i \le n-1$, $f^*(\beta_i b_i) = 1; 1 \le i \le n-1, f^*(c_i\beta_{i+1}) = 1; 1 \le i \le n-1$.

Case (ii). For odd number of n

$f(\alpha_i) = i\,; 1 \le i \le n-1, f(\beta_i) = n - 1 + i\,; 1 \le i \le n-1, f(a_i) = \beta_{n-1} + i$; $1 \le i \le n-2$, $f(c_i) = a_{n-2} + i : 1 \le i \le n-2$, $f(b_i) = c_{n-2} + i$; $1 \le i \le n-2$, $f(a_{n-1}) = b_{n-2} + 1$, $f(\alpha_n) = a_{n-1} + 1$, $f(\beta_n) = \alpha_n + 1$, $f(c_{n-1}) = \beta_n + 1$, $f(b_{n-1}) = c_{n-1} + 1$. Then the induced edge labels are $f^*(\alpha_i a_i) = 1\,; 1 \le i \le n-1$, $f^*(\beta_i b_i) = 1; 1 \le i \le n-1$, $f^*(b_{n-1}\beta_n) = 1$, $f^*(c_i\beta_{i+1}) = 1; 1 \le i \le n-2$, $f^*(a_i\alpha_{i+1}) = 0; 1 \le i \le n-1$, $f^*(\alpha_i c_i) = 0$; $1 \le i \le n-1, f^*(b_i\beta_{i+1}) = 0; 1 \le i \le n-2, f^*(c_{n-1}\beta_n) = 0$.

From both the cases, we get, $\left|e_f(0)\right| = 3n - 3$ and $|e_f(1)| = 3n - 3$. Thus $|e_f(0) - e_f(1)| \le 1$. Hence the graph $S(SL_n)$ is sum of power n divisor cordial graph.

Example 2.16. A sum of power n divisor cordial labeling of $S(SL_4)$ is shown below.

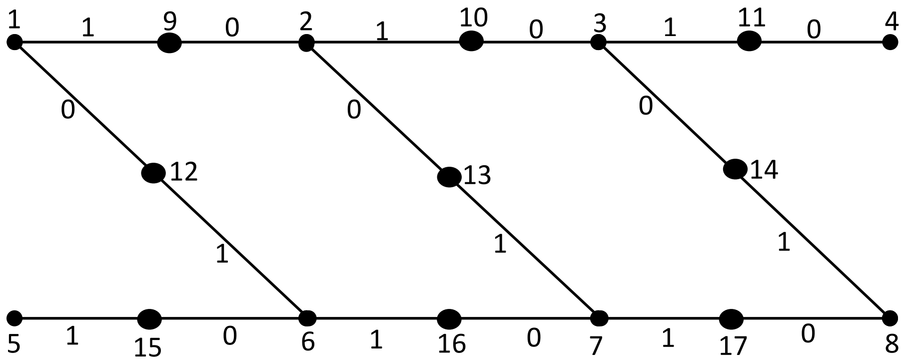

Figure 2.8.

Conclusion:

The study of labeled graph is important due to its diversified applications. In this paper, we found some new results for subdivision of path, star, cycle, tree and ladder related graphs are sum of power n divisor cordial graphs.

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