

Abstract

Nanomaterials are known to be foundation pillars for creating materials that incorporate several functional components. So, it is crucial to assess the physico-chemical characteristics of the moieties, which can be carried out using various molecular descriptors. Topological descriptors are numerical values derived from a graph that are able to describe the topology of the graph and are typically graph invariants. Despite the conventional degree-based descriptors, a novel version based on the temperature of a vertex has recently gained interest. In this study, some of the significant temperature indices like hyper temperature indices, connectivity temperature indices, general temperature indices, *GA* and *F*-temperature indices of the carbon nanosheet $\nu C5C7(\mathbf{x}, \mathbf{y})$ are formulated.

Keywords: Nanomaterials, Pent-Heptagonal Nanosheet, Molecular Descriptors, Hyper temperature indices, Connectivity temperature indices, General temperature indices, *GA* and *F*-temperature Indices.

1 Introduction

Numerous graph-theoretical approaches for analyzing and predicting physicochemical, ecological, and biomedical aspects of molecules have been emerged in recent years. A graph constructed from the structure of the molecule adequately portrays the topology of the molecule and hence it is not unexpected that graph-theoretical techniques have been successful in understanding physiological and biological aspects of several classes of chemical molecules. The goal of developing a topological index is to provide a numerical value to each chemical structure while being as selective and feasible. These indices can be used to categorize structures and predict chemical and biological properties [1].

As the theoretical foundation of a sophisticated information network, molecular descriptors, which are intimately linked to the idea of molecular structure, play a crucial role in scientific studies including QSAR and QSPR analysis. Quantitative Structure-Activity Relationships (QSARs) are the end result of a process that begins with a clear overview of molecular structures and ends with certain interpretation, speculation, and prediction on the behavior of molecules in various circumstances under study. In fact, molecular descriptors are used to model a variety of chemical features in several scientific domains. These theories include quantum chemistry, information theory, organic chemistry, graph theory to name a few [2].

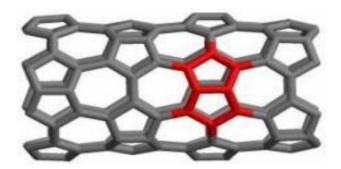


Figure 1: Chemical framework of the Nanostructure $\nu C5C7(\mathbf{x}, \mathbf{y})$

. On a variety of subjects connected to bio-informatics and pharmacogenomics, the degreebased indices have been utilized to construct and analyze the mathematical features of the real-world networks [3]. Kulli [4] has published some new topological techniques called Temperature indices, which are variations on degree-based descriptors.

The molecular-scale graphitic carbon sheets known as carbon nanosheets have exceptional qualities. They exhibit extraordinary electrical properties, are among the sturdiest and strongest fibers ever discovered, among many other distinctive qualities. These factors have led to an enormous amount of research and commercial interest in nanostructures, with thousands of papers on them being published annually [5-9]. However, the development of commercial applications has been relatively slow, primarily due to the high production costs of better quality nanotubes. We will examine the pent heptagonal carbon nanosheet $\nu C5C7(\mathbf{x}, \mathbf{y})$ in this paper. A molecular diagram of the structure can be seen in Figure 1.

This study evaluates a wide range of the fundamental temperature indicators of the nanosheet. These descriptors are effective alternatives for extensive, time-consuming laboratory analysis and are crucial to chemical graph theory. Some interesting research on temperature indices may be found in [10-14]. Some recent works in the topological descriptors can be seen in [15-18].

2 Methodology

Chemical graphs are representations of molecules where atoms are indicated by vertices and chemical bonds by edges of a graph. A topological descriptor is a numerical value related with a network which can describe the graph's underlying properties [7]. We are considering simply connected definite graphs throughout the study. Let \mathfrak{S} denotes such a graph and let the set of vertices and edges be denoted by $\mathcal{V}(\mathfrak{S})$ and $E(\mathfrak{S})$ accordingly.

Fajtlowicz proposed the temperature of a vertex $t \in \mathcal{V}(\mathfrak{S})$ of a graph \mathfrak{S} in [19] as follows;

$$\tau(t) = \frac{d_{\mathfrak{S}}(t)}{n - d_{\mathfrak{S}}(t)},$$

where *n* denotes the cardinality of the vertex set and $d_{\mathfrak{S}}(t)$ is the valency of $t \in \mathcal{V}(\mathfrak{S})$.

The work on various degree-based structural descriptors served as an inspiration for Kulli's [4] formulation on various temperature indices. Table 1 lists some of the fundamental temperature indices.

T 1				
Indices	Expressions			
First Hyper Temperature Index	$H\tau_1(\mathfrak{S}) = \sum_{e=tv \in E(\mathfrak{S})} (\tau(t) + \tau(v))^2$			
Second Hyper Temperature Index	$H\tau_2(\mathfrak{S}) = \sum_{e=tv \in E(\mathfrak{S})} (\tau(t) \times \tau(v))^2$			
Sum Connectivity Temperature Index	$S\tau(\mathfrak{S}) = \sum_{e=tv\in E(\mathfrak{S})} \frac{1}{\sqrt{\tau(t) + \tau(v)}}$			
Product Connectivity Temperature Index	$P\tau(\mathfrak{S}) = \sum \frac{1}{1}$			
Reciprocal Product Connectivity Temperature Index	$\frac{\sum_{e=tv\in E(\mathfrak{S})}\sqrt{\tau(t)\times\tau(v)}}{RP\tau(\mathfrak{S}) = \sum_{e=tv\in E(\mathfrak{S})}\sqrt{\tau(t)\times\tau(v)}}$			
Arithmetic-Geometric Temperature Index	$AG\tau(\mathfrak{S}) = \sum_{e=tv \in E(\mathfrak{S})} \frac{\tau(t) + \tau(v)}{2\sqrt{\tau(t) \times \tau(v)}}$			
General First Temperature Index	$\tau_1^a(\mathfrak{S}) = \sum_{e=tv \in E(\mathfrak{S})} (\tau(t) + \tau(v))^a$			
General Second Temperature Index	$\tau_2^a(\mathfrak{S}) = \sum_{e=tv \in E(\mathfrak{S})} (\tau(t) \times \tau(v))^a$			

F-Temperature Index	$F\tau(\mathfrak{S}) = \sum_{e=tv \in E(\mathfrak{S})} \left(\tau(t)^2 + \tau(v)^2\right)$		
General Temperature Index	$\tau_a(\mathfrak{S}) = \sum_{e = tv \in E(\mathfrak{S})} \left(\tau(t)^a + \tau(v)^a \right)$		

Table 1: Temperature Indices for a graph \mathfrak{S}

Mathematical Analysis of the Nanosheet $\nu C5C7(x, y)$

The primary objective is to analyse the growth model of the Carbon sheet $\nu C5C7(\mathbf{x}, \mathbf{y})$. Figure 2 depicts the two dimensional interlayer of the structure. For the generalization, we consider the vertical pentagon as the base code. Hence, for the structure $\nu C5C7(\mathbf{x}, \mathbf{y})$, \mathbf{x} and \mathbf{y} represent the pentagon count of the 2-pentagon sets in vertical and horizontal directions respectively. The cardinalities of the vertex and edge sets are $16\mathbf{x}\mathbf{y} + 2\mathbf{x} + 5\mathbf{y}$ and $24\mathbf{x}\mathbf{y} + 4\mathbf{y}$, accordingly.

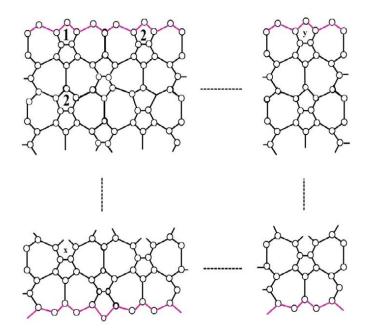


Figure 2: Two dimensional lattice framework of $\nu C5C7(\mathbf{x}, \mathbf{y})$

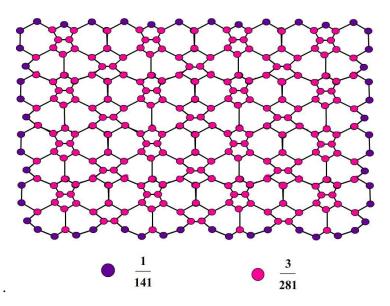


Figure 3: An illustration of vertex temperatures of $\nu C5C7(4, 4)$.

. For the graph $\nu C5C7(\mathbf{x}, \mathbf{y})$, denoted by \mathfrak{S} with a vertex set \mathcal{V} , the cardinality of its vertex set is denoted by $|\mathcal{V}(\mathfrak{S})|$. Now, for each $t \in \mathcal{V}(\mathfrak{S})$ the vertex temperatures can be obtained using the following expression;

$$\tau(t) = \frac{d_{\mathfrak{S}}(t)}{|\mathcal{V}(\mathfrak{S})| - d_{\mathfrak{S}}(t)}$$

Since the degrees of the structure $\nu C5C7(\mathbf{x}, \mathbf{y})$ can be either 2 or 3, the vertex temperature degrees will be of the form $\left(\frac{2}{|\mathcal{V}(\mathfrak{S})|-2}\right)$ and $\left(\frac{3}{|\mathcal{V}(\mathfrak{S})|-3}\right)$. The edge partition table based on the vertex temperatures is shown in Table 2. An illustration of vertex temperatures of $\nu C5C7(\mathbf{4}, \mathbf{4})$ is shown in Figure 3.

$(\tau(t), \tau(v))$	Number of edges			
$(\frac{2}{2x+5y+16xy-2},\frac{2}{2x+5y+16xy-2})$	2x + 2y + 4			
$(\frac{2}{2x+5y+16xy-2},\frac{3}{2x+5y+16xy-3})$	8x + 10y - 8			
$(\frac{3}{2x+5y+16xy-3},\frac{3}{2x+5y+16xy-3})$	24xy - 10x - 8y + 4			

Table 2: Vertex temperature edge partitioning of $\nu C5C7(\mathbf{x}, \mathbf{y})$

3 Main Results

This section deals with the expressions of temperature indices obtained for the carbon nanosheet $\nu C5C7(\mathbf{x}, \mathbf{y})$ denoted as \mathfrak{S} . We use the edge partition obtained in Table 2 and the formulae for temperature indices to obtain the results.

3.1 Hyper Temperature Indices

The following theorem gives the expressions for the first and second hyper temperature indices of the carbon nanosheet $\nu C5C7(\mathbf{x}, \mathbf{y})$

Theorem 1. For $\mathbf{x}, \mathbf{y} \ge 1$,

i)
$$H\tau_1(\nu C5C7(\mathbf{x}, \mathbf{y})) =$$

 $(8x + 10y - 8)(\frac{2}{2x + 5y + 16xy - 2} + \frac{3}{2x + 5y + 16xy - 3})^2 + \frac{16(2x + 2y + 4)}{(2x + 5y + 16xy - 2)^2} - \frac{36(10x + 8y - 24xy - 4)}{(2x + 5y + 16xy - 3)^2}$

ii)
$$H\tau_2(\nu C5C7(\mathbf{x}, \mathbf{y})) =$$

$$\frac{16(2x+2y+4)}{(2x+5y+16xy-2)^4} - \frac{81(10x+8y-24xy-4)}{(2x+5y+16xy-3)^4} + \frac{36(8x+10y-8)}{((2x+5y+16xy-2)(2x+5y+16xy-3))^2}$$

Proof.

First hyper temperature index,
$$H\tau_1(\mathfrak{S}) = \sum_{e=tv\in E(\mathfrak{S})} (\tau(t) + \tau(v))^2$$

$$= (2x + 2y + 4)\left(\frac{2}{2x + 5y + 16xy - 2} + \frac{2}{2x + 5y + 16xy - 2}\right)^{2} + (8x + 10y - 8) \times \left(\frac{2}{2x + 5y + 16xy - 2} + \frac{3}{2x + 5y + 16xy - 3}\right)^{2} + (24xy - 10x - 8y + 4) \times \left(\frac{3}{2x + 5y + 16xy - 3} + \frac{3}{2x + 5y + 16xy - 3}\right)^{2}$$

Second hyper temperature index,
$$H\tau_2(\mathfrak{S}) = \sum_{e=tv \in E(\mathfrak{S})} (\tau(t) \times \tau(v))^2$$

$$= (2x + 2y + 4)\left(\frac{2}{2x + 5y + 16xy - 2} \times \frac{2}{2x + 5y + 16xy - 2}\right)^{2} + (8x + 10y - 8)$$

$$\left(\frac{2}{2x + 5y + 16xy - 2} \times \frac{3}{2x + 5y + 16xy - 3}\right)^{2} + (24xy - 10x - 8y + 4) \times$$

$$\left(\frac{3}{2x + 5y + 16xy - 3} \times \frac{3}{2x + 5y + 16xy - 3}\right)^{2}$$

On simplifying the above equations, we obtain the results.

3.2 Temperature Connectivity Indices

The following theorem gives the expressions for the connectivity indices like sum connectivity, product connectivity, reciprocal connectivity temperature indices of the carbon nanosheet $vC5C7(\mathbf{x}, \mathbf{y})$

Theorem 2. For $x, y \ge 1$,

$$i) S\tau(vC5C7(\mathbf{x}, \mathbf{y})) = (x + y + 2)\sqrt{2x + 5y + 16xy - 2} + \frac{(8x + 10y - 8)}{\sqrt{\frac{2}{2x + 5y + 16xy - 2} + \frac{3}{2x + 5y + 16xy - 3}}} - \frac{\sqrt{6}(10x + 8y - 24xy - 4)\sqrt{2x + 5y + 16xy - 3}}{6} - \frac{\sqrt{6}(10x + 8y - 24xy - 4)(2x + 5y + 16xy - 3)}{3} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}{6} - \frac{(10x + 8y - 24xy - 4)\sqrt{\frac{1}{(2x + 5y + 16xy - 3)^2}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}}{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}}} + \frac{\sqrt{6}(8x + 10y - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}} + \frac{\sqrt{6}(8x + 10x - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}}} + \frac{\sqrt{6}(8x + 10x - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}}} + \frac{\sqrt{6}(8x + 10x - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}}} + \frac{\sqrt{6}(8x + 10x - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}}} + \frac{\sqrt{6}(8x + 10x - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}}} + \frac{\sqrt{6}(8x + 10x - 8)\sqrt{\frac{1}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy$$

Proof.

Sum connectivity temperature index, $S\tau(\mathfrak{S}) = \sum_{e=tv\in E(\mathfrak{S})} \frac{1}{\sqrt{\tau(t) + \tau(v)}}$

$$= \frac{2x+2y+4}{\sqrt{\frac{2}{2x+5y+16xy-2} + \frac{2}{2x+5y+16xy-2}}} + \frac{8x+10y-8}{\sqrt{\frac{2}{2x+5y+16xy-2} + \frac{3}{2x+5y+16xy-3}}} + \frac{24xy-10x-8y+4}{\sqrt{\frac{3}{2x+5y+16xy-3} + \frac{3}{2x+5y+16xy-3}}}$$

Product connectivity temperature index,
$$P\tau(\mathfrak{S}) = \sum_{e=tv\in E(\mathfrak{S})} \frac{1}{\sqrt{\tau(t) \times \tau(v)}}$$
$$= \frac{2x+2y+4}{\sqrt{\frac{2}{2x+5y+16xy-2}} \times \frac{2}{\sqrt{\frac{2}{2x+5y+16xy-2}} \times \frac{3}{\sqrt{\frac{2}{2x+5y+16xy-3}}}} + \frac{24xy-10x-8y+4}{\sqrt{\frac{3}{2x+5y+16xy-3}} \times \frac{3}{2x+5y+16xy-3}}$$

Reciprocal product connectivity temperature index,

$$RP\tau(\mathfrak{S}) = \sum_{e=tv\in E(\mathfrak{S})} \sqrt{\tau(t) \times \tau(v)}$$
$$= (2x+2y+4) \sqrt{\frac{2}{2x+5y+16xy-2} \times \frac{2}{2x+5y+16xy-2}}$$
$$+ (8x+10y-8) \sqrt{\frac{2}{2x+5y+16xy-2} \times \frac{3}{2x+5y+16xy-3}} + (24xy-10x-8y+4) \times \sqrt{\frac{3}{2x+5y+16xy-3} \times \frac{3}{2x+5y+16xy-3}}}$$

On simplifying the above equations, we obtain the results.

3.3 General Temperature Indices

General temperature indices involve general first, general second and the general temperature indices. The results obtained for the above indices are formulated in the theorem given below.

Theorem 3 For $x, y \ge 1$,

i)
$$\tau_1^a(\nu C5C7(\mathbf{x}, \mathbf{y})) =$$

 $(8x + 10y - 8)(\frac{2}{2x + 5y + 16xy - 2} + \frac{3}{2x + 5y + 16xy - 3})^a - \frac{6^a(10x + 8y - 24xy - 4)}{(2x + 5y + 16xy - 3)^a} + \frac{2^{2a}(2x + 2y + 4)}{(2x + 5y + 16xy - 2)^a}$
 $ii) \tau_2^a(\nu C5C7(\mathbf{x}, \mathbf{y})) = (8x + 10y - 8)(\frac{6}{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)})^a + (2x + 2y + 4)(\frac{4}{(2x + 5y + 16xy - 2)^2})^a - (10x + 8y - 24xy - 4)(\frac{9}{(2x + 5y + 16xy - 3)^2})^a$
 $iii) \tau_a(\nu C5C7(\mathbf{x}, \mathbf{y})) = (8x + 10y - 8)((\frac{2}{2x + 5y + 16xy - 2})^a + (\frac{3}{2x + 5y + 16xy - 3})^a) + 2(2x + 2y + 4)(\frac{2}{2x + 5y + 16xy - 2})^a - 2(10x + 8y - 24xy - 4)(\frac{3}{2x + 5y + 16xy - 3})^a$

Proof.

General first temperature Index,
$$\tau_1^a(\mathfrak{S}) = \sum_{e=tv\in E(\mathfrak{S})} (\tau(t) + \tau(v))^a$$

$$= (2x + 2y + 4)\left(\frac{2}{2x + 5y + 16xy - 2} + \frac{2}{2x + 5y + 16xy - 2}\right)^{a} + (8x + 10y - 8) \times \left(\frac{2}{2x + 5y + 16xy - 2} + \frac{3}{2x + 5y + 16xy - 3}\right)^{a} + (24xy - 10x - 8y + 4) \times \left(\frac{3}{2x + 5y + 16xy - 3} + \frac{3}{2x + 5y + 16xy - 3}\right)^{a}$$

General second temperature Index, $\tau_2^a(\mathfrak{S}) = \sum_{e=tv \in E(\mathfrak{S})} (\tau(t) \times \tau(v))^a$

$$= (2x + 2y + 4)\left(\frac{2}{2x + 5y + 16xy - 2} \times \frac{2}{2x + 5y + 16xy - 2}\right)^{a} + (8x + 10y - 8) \times \left(\frac{2}{2x + 5y + 16xy - 2} \times \frac{3}{2x + 5y + 16xy - 3}\right)^{a} + (24xy - 10x - 8y + 4) \times \left(\frac{3}{2x + 5y + 16xy - 3} \times \frac{3}{2x + 5y + 16xy - 3}\right)^{a}$$

General temperature index,
$$\tau_a(\mathfrak{S}) = \sum_{e=tv \in E(\mathfrak{S})} (\tau(t)^a + \tau(v)^a)$$

$$= (2x + 2y + 4)\left(\left(\frac{2}{2x + 5y + 16xy - 2}\right)^{a} + \left(\frac{2}{2x + 5y + 16xy - 2}\right)^{a}\right) + (8x + 10y - 8) \times \left(\left(\frac{2}{2x + 5y + 16xy - 2}\right)^{a} + \left(\frac{3}{2x + 5y + 16xy - 3}\right)^{a}\right) + (24xy - 10x - 8y + 4) \times \left(\left(\frac{3}{2x + 5y + 16xy - 3}\right)^{a} + \left(\frac{3}{2x + 5y + 16xy - 3}\right)^{a}\right)$$

On simplifying the above equations, we obtain the results.

3.4 GA and F-temperature indices

Theorem 4. For $x, y \ge 1$,

The results obtained after evaluating F-temperature index and arithmetic-geometric temperature index of the carbon nanosheet $\nu C5C7(\mathbf{x}, \mathbf{y})$ is dealt in the following theorem.

$$i)AG\tau(\nu\mathcal{C}5\mathcal{C}7(\mathbf{x},\mathbf{y})) = (24xy - 8x - 6y + 8) + \left(\frac{\sqrt{6}(8x + 10y - 8)(\frac{2}{2x + 5y + 16xy - 2} + \frac{3}{2x + 5y + 16xy - 3})}{\frac{12}{\sqrt{(2x + 5y + 16xy - 2)(2x + 5y + 16xy - 3)}}}\right)$$
$$ii)F\tau(\nu\mathcal{C}5\mathcal{C}7(\mathbf{x},\mathbf{y})) = (8x + 10y - 8)(\frac{4}{(2x + 5y + 16xy - 2)^2} + \frac{9}{(2x + 5y + 16xy - 3)^2}) + \frac{8(2x + 2y + 4)}{(2x + 5y + 16xy - 2)^2} - \frac{18(10x + 8y - 24xy - 4)}{(2x + 5y + 16xy - 3)^2}$$

Proof.

Arithmetic – geometric temperature index,
$$AG\tau(\mathfrak{S}) = \sum_{e=tv\in E(\mathfrak{S})} \frac{\tau(t) + \tau(v)}{2\sqrt{\tau(t) \times \tau(v)}}$$
$$= \frac{(2x+2y+4)(\frac{2}{2x+5y+16xy-2} + \frac{2}{2x+5y+16xy-2})}{2\sqrt{\frac{2}{2x+5y+16xy-2} \times \frac{2}{2x+5y+16xy-2}}} + \frac{(8x+10y-8)(\frac{2}{2x+5y+16xy-2} + \frac{3}{2x+5y+16xy-3})}{2\sqrt{\frac{2}{2x+5y+16xy-2} \times \frac{3}{2x+5y+16xy-3}}}$$
$$+ \frac{(24xy-10x-8y+4)(\frac{3}{2x+5y+16xy-3} + \frac{3}{2x+5y+16xy-3})}{2\sqrt{\frac{3}{2x+5y+16xy-3} \times \frac{3}{2x+5y+16xy-3}}}$$
$$F - temperature index, \quad F\tau(\mathfrak{S}) = \sum_{e=tv\in E(\mathfrak{S})} (\tau(t)^2 + \tau(v)^2)$$
$$= (2x+2y+4)((\frac{2}{2x+5y+16xy-2})^2 + (\frac{2}{2x+5y+16xy-2})^2) + (8x+10y-8)$$
$$((\frac{2}{2x+5y+16xy-2})^2 + (\frac{3}{2x+5y+16xy-3})^2) + (24xy-10x-8y+4) \times$$
$$((\frac{3}{2x+5y+16xy-3})^2 + (\frac{3}{2x+5y+16xy-3})^2)$$

On simplifying the above equations, we obtain the results.

4 Comparative Study of Various Temperature Indices

A graphical comparative analysis of various temperature indices have been carried out in this section. The graphs are drawn using MATLAB. This visual interpretation of the indices will help to comprehend the growth trends and relationship between different indices and also assist in conducting various experiments involving the same. The comparison graphs can be seen in Figure 4. Table 3 and Table 4 shows the growth pattern of the numerical values where a is taken to be 1.

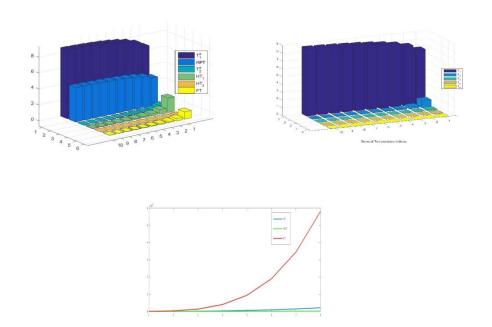


Figure 4: Graphical comparison of various temperature indices

х, у	$H\tau_1$	$H\tau_2$	Sτ	Ρτ	RPτ	AGτ	$ au_1^a$	$ au_2^a$	Fτ
1	1.7916	7.76×10^{-3}	56.7809	234.3326	3.4571	28.2590	6.9761	0.4404	0.9108
2	0.5659	2×10^{-4}	387.3110	2919.0179	3.7842	104.6158	7.6084	0.1401	0.2856
3	0.2809	2.22×10^{-5}	1226.8569	13319.6415	3.9637	228.9776	7.9570	0.0697	0.1413
4	0.1683	4.53×10^{-6}	2810.5981	39773.6562	4.07241	401.3430	8.1681	0.0418	0.0845
5	0.1121	1.3×10^{-6}	5373.6938	93690.5159	4.1447	621.7103	8.3087	0.0279	0.0563
6	0.0801	4.58×10^{-7}	9151.2983	189551.674	4.1961	890.0786	8.4089	0.0199	0.0401
7	0.0601	1.89×10^{-7}	14378.5640	344910.5836	4.2346	1206.4477	8.4838	0.0149	0.0301
8	0.0467	8.81×10^{-8}	21290.6427	580392.698	4.2645	1570.8173	8.5418	0.0116	0.0234
9	0.0374	4.46×10^{-8}	30122.6860	919695.4703	4.2883	1983.1871	8.5882	0.0093	0.0187
10	0.0306	2.42×10^{-8}	41109.8452	1389588.354	4.3078	2443.5573	8.6260	0.0076	0.0153

Table 3: Computed numerical values for various temperature indices.

х, у	$ au_1$	$ au_2$	τ ₃	$ au_4$	$ au_5$
1	6.9761	0.9108	1.2370×10^{-1}	1.732×10^{-3}	2.4×10^{-3}
2	7.6084	0.2856	1.0930×10^{-2}	4.2429×10^{-4}	1.6630×10^{-5}
3	7.9570	0.1413	2.5446×10^{-3}	4.6222×10^{-5}	8.4492×10^{-7}
4	8.1681	0.0845	8.8404×10^{-4}	9.3052×10^{-6}	9.8401×10^{-8}
5	8.3087	0.0563	3.8454×10^{-4}	2.6406×10^{-6}	1.8201×10^{-8}
6	8.4089	0.0401	1.9338×10^{-4}	9.3461×10 ⁻⁷	4.5307×10^{-9}
7	8.4838	0.0301	1.0765×10^{-5}	3.8603×10^{-7}	1.3879×10^{-9}
8	8.5418	0.0234	6.4611× 10 ⁻⁵	1.7874×10^{-7}	4.9559×10^{-10}
9	8.5882	0.0187	4.1097×10^{-5}	9.0370× 10 ⁻⁸	1.9912×10^{-10}
10	8.6260	0.0153	2.7375×10^{-5}	4.8997×10^{-8}	8.7855×10^{-11}

Table 4: Computed numerical values of general temperature index for different *a* values

5 Conclusion

The expressions for a group of temperature indices, including hyper, connectivity, general, GA and F indices have been derived in this work. Additionally, a comparison study approach using datas and graphs has also been devised. These formulae will be useful to researchers working with Pent-heptagonal nanostructures. Also, It has been discovered that structural descriptors may be employed as a prediction tool in QSPR/QSAR investigations.

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