A STUDY ON SOME RESULTS OF ANTI – FUZZY NORMAL SUBNEARRINGS

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Abstract

In this paper, we define some basic definition, Near-ring, Subnearring, Normal Subnearring and prove some properties of Anti-fuzzy normal subnearring.

1. Introduction

The concept of fuzzy sets was initiated by Zadeh 1965 to respect / manipulate data and information possessing non-statistical uncertainties. It was specifically designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to many problems.

The first publication in fuzzy subset theory by Zadeh (1965) and then by Goguen (1967, 1969) show the intention of the authors to generalize the classical set. In classical set theory, a subset A of a set X can be defined by its characteristic function $\chi_A : X \rightarrow \{0,1\}$ is defined by $\chi_A(X) = 0$, if $x \notin A$ and $\chi_A(X) = 1$, if $x \in A$.

2. Preliminaries

We list here the basic definitions are given which will be used in this paper.

Definition 2.1 A non-empty set R together with two binary operations denoted by $+$ and $.$ are called addition and multiplication which satisfy the following axioms are called a nearing.

(i) $(R, +)$ is a group,

(ii) $.$ is an associative binary operation on R,

(iii) $a.(b+c) = a.b + a.c$ and $(a+b).c = a.c + b.c$ for all $a, b, c \in R$.

Example 2.1 $(Z, +, .)$ is nearing under the usual addition and multiplication, where Z is an integer.

Definition 2.2 A non-empty subset S of a nearing $(R, +, .)$ is called a subnearring if S itself is a nearing under the same operation as in R.

Example 2.2 $(2Z, +, .)$ is a subring of $(Z, +, .)$, where Z is an integer.

Definition 2.3 Let $(R, +, .)$ and $(R', +, .)$ be any two nearrings. Then the function $f : R \rightarrow R'$ is called a nearring homomorphism if it satisfies the following axioms:

(i) $f(x+y) = f(x) + f(y)$,

(ii) $f(xy) = f(x)f(y)$, for all x and y in R.

Definition 2.4 Let $(R, +, .)$ and $(R', +, .)$ be any two nearrings. Then the function $f: R \to R'$ is called a nearing anti-homomorphism if, it satisfies the following axioms:

(i) $f(x+y) = f(x) + f(y)$,

(ii) $f(xy) = f(x)f(y)$, for all x and y in R.

Definition 2.5 Let R be a nearring. An anti-fuzzy subnearring A of R is said to be a anti-fuzzy normal subnearring (AFNSNR) of R if it compliestic below conditions:

(i) $\mu_A(x+y) = \mu_A(y+x)$,

(ii) $\mu_A(xy) = \mu_A(yx)$, for all x and y in R.

3. Properties of Anti-Fuzzy Normal Subnearrings

Theorem 1. *Let $(R, +, .)$ be a nearring. If A and B are two anti-fuzzy normal subnearrings of R. Then their union $A \cup B$ is an anti-fuzzy normal subnearring of R.*

Proof of Theorem 1

Let x and $y \in R$.

Let $A = \{< x, \mu_A(x) > / x \in R\}$ and $B = \{< x, \mu_B(x) > / x \in R\}$ be anti-fuzzy normal subnearrings of a nearring R.

Let $C = A \cup B$ and $C = \{< x, \mu_C(x) > / x \in R\}$.

Then, Clearly C is an anti-fuzzy subnearring of a nearring R, since A and B are two anti-fuzzy subnearrings of a nearring R.

And,

(i) $\mu_C(x+y) = \max\{\mu_A(x+y), \mu_B(x+y)\},$

$= \max\{\mu_A(y+x), \mu_B(y+x)\}$

$= \mu_C(y+x),$ for all x and y in R.

$\therefore \mu_C(x+y) = \mu_C(y+x),$ for all x and y in R.

(ii) $\mu_C(xy) = \max\{\mu_A(xy), \mu_B(xy)\},$

$= \max\{\mu_A(yx), \mu_B(yx)\}$

$= \mu_C(yx),$ for all x and y in R.

$\therefore \mu_C(xy) = \mu_C(yx),$ for all x and y in R.

Hence $A \cup B$ is an anti-fuzzy normal subnearring of a nearing R.

The proof of Theorem 1 is complete.

Theorem 2. *Let R be a nearring. The union of a family of anti-fuzzy normal subnearrings of R is an anti-fuzzy normal subnearrings of R.*

Proof of Theorem 2

Let $\{A_i\}_{i\in I}$ be a family of ani-fuzzy normal subnearrings of a nearring R and let $A = \bigcup_{i\in I} A_i$.

Then for x and y in R.

Clearly the union of a family of anti-fuzzy subnearrings of a nearring R is an anti-fuzzy subnearring of a nearring R.

(i) $$\mu_A(x+y) = \operatorname*{Sup}_{i\in I} \mu_{A_i}(x+y)$$

$$= \operatorname*{Sup}_{i\in I} \mu_{A_i}(y+x)$$

$\therefore \mu_A(x+y) = \mu_A(y+x)$, for all x and y in R.

(ii) $$\mu_A(xy) = \operatorname*{Sup}_{i\in I} \mu_{A_i}(xy)$$

$$= \operatorname*{Sup}_{i\in I} \mu_{A_i}(yx)$$

$\therefore \mu_A(xy) = \mu_A(yx)$, for all x and y in R.

Hence the union of a family of anti-fuzzy normal subnearrings of a nearring R is an anti-fuzzy normal subnearring of a nearring R.

The proof of Theorem 2 is complete.

Theorem 3. *Let* A be an anti-fuzzy normal subnearring of a nearring R. Then for any y in R we have $\mu_A(y+x-y) = \mu_A(-y+x+y)$, for every x in R.

Proof of Theorem 3

Let A be an anti-fuzzy normal subnearring of a nearring R.

For any y in R we have,

$$\mu_A(y+x-y) = \mu_A(x)$$

$$= \mu_A(x+y-y)$$

$$= \mu_A(-y+x+y).$$

$$\therefore \mu_A(y+x-y) = \mu_A(-y+x+y).$$

The proof of Theorem 3 is complete.

Theorem 4. *Let* A and B be anti-fuzzy subnearring of the nearrings G and H, respectively. If A and B be anti-fuzzy normal subnearring, then $A \times B$ is an anti-fuzzy normal subnearring of $G \times H$.

Proof of Theorem 4

Let A and B be anti-fuzzy normal subnearring of the nearring G and H, respectively.

Clearly $A \times B$ is an anti-fuzzy subnearring of $G \times H$.

Let x_1 and x_2 be in G, y_1 and y_2 be in H.

Then (x_1, y_1) and (x_2, y_2) are in $G \times H$.

Now,

$$\mu_{A\times B}[(x_1, y_1) + (x_2, y_2)] = \mu_{A\times B}(x_1 + x_2, y_1 + y_2)$$

$$= \max\{\mu_A(x_1 + x_2), \mu_B(y_1 + y_2)\}$$

$$= \max\{\mu_A(x_2 + x_1), \mu_B(y_2 + y_1)\}$$

$$= \mu_{A\times B}(x_2 + x_1, y_2 + y_1)$$

$$= \mu_{A\times B}[(x_2, y_2) + (x_1, y_1)]$$

$$\therefore \mu_{A\times B}[(x_1, y_1) + (x_2, y_2)] = \mu_{A\times B}[(x_2, y_2) + (x_1, y_1)].$$

And,

$$\mu_{A\times B}[(x_1, y_1)(x_2, y_2)] = \mu_{A\times B}(x_1 x_2, y_1 y_2)$$

$$= \max\{\mu_A(x_1 x_2), \mu_B(y_1 y_2)\}$$

$$= \max\{\mu_A(x_2 x_1), \mu_B(y_2 y_1)\}$$

$$= \mu_{A\times B}(x_2 x_1, y_2 y_1)$$

$$= \mu_{A\times B}[(x_2, y_2)(x_1, y_1)]$$

$$\therefore \mu_{A\times B}[(x_1, y_1)(x_2, y_2)] = \mu_{A\times B}[(x_2, y_2)(x_1, y_1)].$$

Hence, $A \times B$ is an anti-fuzzy normal subnearring of $G \times H$.

The proof of Theorem 4 is complete.

6. Conclusion

In this paper, we discussed about Near-ring, Subnearring, Anti-fuzzy normal Subnearring and proved some properties.

References

[1] K. T. Atanassov, *Intuitionistic Fuzzy Sets*, Fuzzy Sets and Systems, Vol. 20(1) (1986), 87-96.
[2] K. T. Atanassov, *Intuitionistic Fuzzy Sets Theory and Applications*, Physica-Verlag, A Springer-Verlag company, Bulgaria, (April 1999).
[3] R. Biswas, *Fuzzy Subgroups and Anti-fuzzy Subgroups*, Fuzzy Sets and Systems, Vol. 35 (1990), 121-124.
[4] N. Palaniappan, and R. Muthuraj, *The Homomorphism, Anti-homomorphism of a fuzzy and an Anti-fuzzy Groups*, Varahmihir Journal of Mathematical Sciences, Vol. 4, No. 2 (2004), 387-399.
[5] Salah Abou-Zaid, *On Generalized Characteristic Fuzzy Subgroups of a Finite Group*, Fuzzy Sets and Systems, (1991), 235-241.
[6] L. A. Zadeh, *Fuzzy Sets*, Information and control, Vol. 8 (1965), 338-353.