RADIO EVEN GEOMETRIC MEAN GRACEFUL LABELING ON DEGREE SPLITTING OF SNAKE RELATED GRAPHS

T. MARY SHALINI[1], C. DAVID RAJ[2]

[1]Research Scholar, Reg. No: 18213232092001, St. Jude's College, Thoothoor-629 176.

[2]Assistant Professor, Department of Mathematics, Malankara Catholic College, Mariagiri-629153.

Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012

[1]shofdain@gmail.com

[2]davidrajmccm@gmail.com

Abstract:

A Radio Even Geometric Mean Graceful Labeling of a connected graph G is a bijection μ:V(G) → {2, 4, 6, …, 2|V|} satisfying the condition $d(a, b) + \left\lceil \sqrt{\mu(a)\mu(b)} \right\rceil \geq 1 + diam(G)$ for every $a, b \in V(G)$. A graph which admits radio even geometric mean graceful labeling is called a radio even geometric mean graceful graph. In this paper, we introduce the radio even geometric mean graceful labeling on degree splitting of snake related graphs.

Keywords: labeling, radio even geometric mean graceful labeling, degree splitting of graph.

I. Introduction

Graphs described here is simple, undirected and connected. Let V(G) and E(G) denote the vertex set and edge set of a graph G respectively. A graph labeling is an assignment of integers to the vertices or edges or both based on certain conditions. The concept of radio labeling was introduced by Chartrand et al [1] in 2001. S. Somasundaram and R. Ponraj introduced the notion of mean labeling of graphs [4]. Radio mean labeling was introduced by Ponraj et al [5]. R. Ponraj and S. Somasundram developed the concept of degree splitting of graphs [7]. S. Somasundaram, S.S. Sandhya and S.P. Viji introduced the concept of Geometric mean labeling on Degree splitting graphs [6]. C. David Raj, K. Sunitha and A. Subramanian introduced radio odd mean and even mean labeling of some graphs [8]. David Raj. C, Subramanian, A, and K. Sunitha determined the radio mean labeling of double triangular snake graph and quadrilateral snake graph [9]. Brindha Mary. V. T, C. David Raj and C. Jaya Sekaran investigated even radio mean graceful labeling on degree splitting of snake related graphs [11]. C. David Raj, T.

Mary Shalini and K. Rubin Mary determined radio geometric mean graceful labeling on splitting of wheel related graphs [10]. We refer Gallian for more comprehensive survey [2]. We follow Harary [3] for some standard words, expressions and symbols. The notations DS(G) is the degree splitting of G, $d(u,v)$ is the distance between the vertices u and v and diam(G) is the diameter of G.

Definition 1.1: A triangular snake T_n is obtained from a path $v_1, v_2, \ldots, v_n$ by joining v_i and v_{i+1} to a new vertex u_i, $1 \leq i \leq n-1$. That is, every edge of path is replaced by a triangle C_3.

Definition 1.2: A double triangular snake DT_n is a graph obtained from a path $v_1, v_2, \ldots, v_n$ by joining v_i and v_{i+1} to new vertices u_i and w_i, $1 \leq i \leq n-1$. That is, double triangular snake consists of two triangular snakes that have a common path.

Definition 1.3: A quadrilateral snake Q_n is a graph obtained from a path $v_1, v_2, \ldots, v_n$ by joining v_i and v_{i+1} to new vertices u_i and w_i respectively, $1 \leq i \leq n-1$ and join u_i and w_i. That is, every edge of a path is replaced by a cycle C_4.

II. Main Result

Theorem 2.1: $DS(T_n)$ is a radio even geometric mean graceful graph.

Proof:

Let v_i, $1 \leq i \leq n$ be the vertices of path P_n. Consider a vertex u_i, $1 \leq i \leq n-1$. Join u_i with v_i and v_{i+1}, $1 \leq i \leq n-1$. The graph thus obtained is T_n.

Case 1. $n = 2,3$

Introduce a new vertex v and join it with the vertices of T_n of degree two. The new graph is $DS(T_n)$ whose vertex set is $V = \{u_i, 1 \leq i \leq n-1, v_i, 1 \leq i \leq n\} \cup \{v\}$.Clearly, $diam\big(DS(T_n)\big) = \begin{Bmatrix} 1 \text{ } if \text{ } n = 2 \\ 2 \text{ } if \text{ } n = 3 \end{Bmatrix}$.

Therefore, obviously the radio even geometric mean graceful condition is satisfied for every pair of vertices.

Case 2. $n \geq 4$

Introduce two new vertices u, v and join them with the vertices of T_n of degree two and four respectively. The resultant graph is $DS(T_n)$ whose vertex set is $V = \{u_i, 1 \leq i \leq n-1, v_i, 1 \leq i \leq n\} \cup \{u, v\}$. Clearly, $diam\big(DS(T_n)\big) = 3$.

Define a bijection $\mu: V\big(DS(T_n)\big) \rightarrow \{2, 4, 6, \ldots, 2|V(DS(T_n))|\}$ by

$\mu(u_i) = 2i,\ 1 \le i \le n-1;$

$\mu(v_i) = 2n + 2i,\ 1 \le i \le n;$

$\mu(v) = 4n + 2;$

$\mu(u) = 2n.$

To check the radio even geometric mean graceful condition for μ.

Sub Case (i): Verify the pair $(u_i,\ u_j)$, $1 \le i \le n-2,\ i+1 \le j \le n-1;$

$$d(u_i,\ u_j) + \left\lceil\sqrt{\mu(u_i)\mu(u_j)}\right\rceil = 2 + \left\lceil\sqrt{(2i)(2j)}\right\rceil \ge 4 = 1 + diam(DS(T_n)).$$

Sub Case (ii): Verify the pair $(u_i,\ v_j)$, $1 \le i \le n-1,\ 1 \le j \le n;$

$$d(u_i,\ v_j) + \left\lceil\sqrt{\mu(u_i)\mu(v_j)}\right\rceil \ge 1 + \left\lceil\sqrt{(2i)(2n+2j+2)}\right\rceil \ge 4.$$

Sub Case (iii): Verify the pair $(u_i,\ v)$, $1 \le i \le n-1;$

$$d(u_i,\ v) + \left\lceil\sqrt{\mu(u_i)\mu(v)}\right\rceil = 2 + \left\lceil\sqrt{(2i)(2n+2)}\right\rceil \ge 4.$$

Sub Case (iv): Verify the pair $(u_i,\ u)$, $1 \le i \le n-1;$

$$d(u_i,\ u) + \left\lceil\sqrt{\mu(u_i)\mu(u)}\right\rceil = 1 + \left\lceil\sqrt{(2i)(4n+2)}\right\rceil \ge 4.$$

Sub Case (v): Verify the pair $(v_i,\ v_j)$, $1 \le i \le n-1,\ i+1 \le j \le n;$

$$d(v_i,\ v_j) + \left\lceil\sqrt{\mu(v_i)\mu(v_j)}\right\rceil \ge 1 + \left\lceil\sqrt{(2n+2i+2)(2n+2j+2)}\right\rceil \ge 4.$$

Sub Case (vi): Verify the pair $(v_i,\ v)$, $1 \le i \le n;$

$$d(v_i,\ v) + \left\lceil\sqrt{\mu(v_i)\mu(v)}\right\rceil \ge 1 + \left\lceil\sqrt{(2n+2i+2)(2n+2)}\right\rceil \ge 4.$$

Sub Case (vii): Verify the pair $(v_i,\ u)$, $1 \le i \le n;$

$$d(v_i,\ u) + \left\lceil\sqrt{\mu(v_i)\mu(u)}\right\rceil \ge 1 + \left\lceil\sqrt{(2n+2i+2)(4n+2)}\right\rceil \ge 4.$$

Sub Case (viii): Verify the pair $(u,\ v)$;

$$d(u,\ v) + \left\lceil\sqrt{\mu(u)\mu(v)}\right\rceil = 3 + \left\lceil\sqrt{(4n+2)(2n+2)}\right\rceil \ge 4.$$

Thus all the pair of vertices satisfies the radio even geometric mean graceful condition.

Hence $DS(T_n)$ is a radio even geometric mean graceful graph.

Example 2.2:

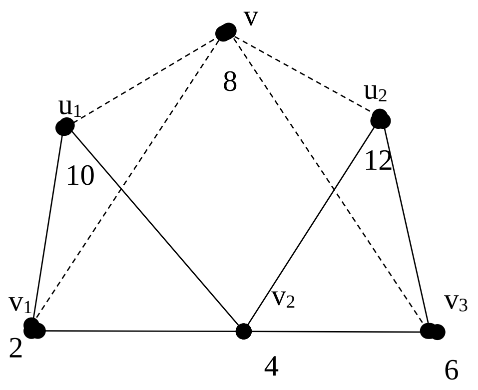
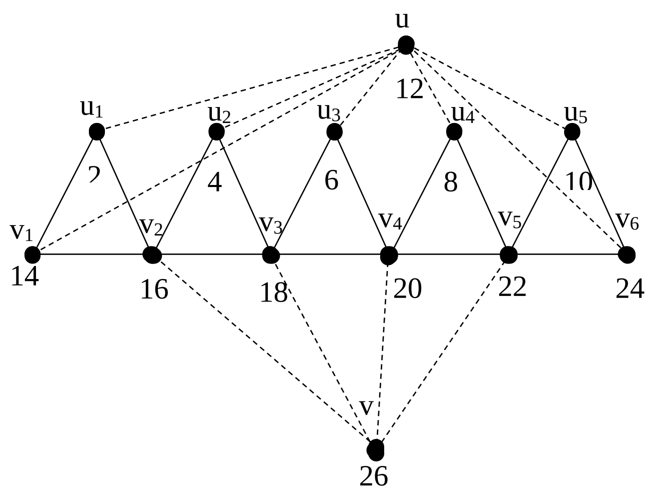


Fig 1. **(a)** Radio Even Geometric Mean Graceful Labeling of $DS(T_3)$.



(b). Radio Even Geometric Mean Graceful Labeling of $DS(T_6)$.

Theorem 2.3: $DS(D(T_n))$ is a radio even geometric mean graceful graph.

Proof:

Let $v_i,\ 1 \le i \le n$ be the vertices of path P_n. Consider the vertices u_i and $w_i,\ ,1 \le i \le n-1$. Join u_i with v_i and w_i with $v_{i+1}, 1 \le i \le n-1$. The graph thus obtained is $D(T_n)$.

Case 1: $n = 2, 3$.

Introduce two new vertices u and w and join them with the vertices of $D(T_n)$ of degree two and three respectively. The new graph thus obtained is $DS(D(T_n))$ whose vertex set is $V(DS(D(T_n))) = \{u_i,\ w_i,\ 1 \le i \le n-1,\ v_i,\ 1 \le i \le n\} \cup \{u,\ w\}$. Clearly, $diam(DS(D(T_n))) = 3$. Therefore, obviously the radio even geometric mean graceful condition is satisfied for every pair of vertices.

Case 2: $n \geq 4$.

Introduce three new vertices u, v and w and join them with the vertices of $D(T_n)$ of degree two, six and three respectively. The resultant graph is $DS(D(T_n))$ whose vertex set is $V(DS(D(T_n))) = \{u_i,\ w_i,\ 1 \le i \le n-1,\ v_i,\ 1 \le i \le n\} \cup \{u,\ v,\ w\}$. Clearly, $diam(DS(D(T_n)) = 3$.

Define a bijection $\mu: V\big(DS(D(T_n))\big) \to \{2, 4, 6, \ldots, 2|V(DS(D(T_n)))|\}$ by

$\mu(u_i) = 2i,\ 1 \le i \le n-1;$

$\mu(w_i) = 2n + 2i,\ 1 \le i \le n-1;$

$\mu(v_i) = 4n + 2i,\ 1 \le i \le n;$

$\mu(v) = 2n;$

$\mu(w) = 4n;$

$\mu(u) = 6n + 2.$

To check the radio even geometric mean graceful condition for μ.

Sub Case (i): Verify the pair $\left(u_i,\ u_j\right)$, $1 \le i \le n-2,\ i+1 \le j \le n-1;$

$$d\left(u_i,\ u_j\right) + \left\lceil\sqrt{\mu(u_i)\mu\left(u_j\right)}\right\rceil = 2 + \left\lceil\sqrt{(2i)2j}\ \right\rceil \ge 4 = 1 + diam(DS(D(T_n))).$$

Sub Case (ii): Verify the pair $\left(u_i, w_j\right)$, $1 \le i, j \le n-1;$

$$d\left(u_i, w_j\right) + \left\lceil\sqrt{\mu(u_i)\mu(w_j)}\ \right\rceil \ge 2 + \left\lceil\sqrt{(2i)(2n+2j)}\ \right\rceil \ge 4.$$

Sub Case (iii): Verify the pair $\left(u_i, v_j\right)$, $1 \le i \le n-1,\ 1 \le j \le n;$

$$d\left(u_i, v_j\right) + \left\lceil\sqrt{\mu(u_i)\mu(v_j)}\ \right\rceil \ge 1 + \left\lceil\sqrt{(2i)(4n+2j)}\ \right\rceil \ge 4.$$

Sub Case (iv): Verify the pair (u_i, v), $1 \le i \le n-1;$

$$d(u_i, v) + \left\lceil\sqrt{\mu(u_i)\mu(v)}\ \right\rceil = 2 + \left\lceil\sqrt{(2i)(2n)}\right\rceil \ge 4.$$

Sub Case (v): Verify the pair (u_i, w), $1 \le i \le n-1;$

$$d(u_i, w) + \left\lceil\sqrt{\mu(u_i)\mu(w)}\ \right\rceil \ge 2 + \left\lceil\sqrt{(2i)(4n)}\right\rceil \ge 4.$$

Sub Case (vi): Verify the pair (u_i, u), $1 \le i \le n-1;$

$$d(u_i, u) + \left\lceil\sqrt{\mu(u_i)\mu(u)}\ \right\rceil = 1 + \left\lceil\sqrt{(2i)(6n+2)}\right\rceil \ge 4.$$

Sub Case (vii): Verify the pair $\left(w_i, v_j\right)$, $1 \le i \le n-1,\ 1 \le j \le n;$

$$d\left(w_i, v_j\right) + \left\lceil\sqrt{\mu(w_i)\mu(v_j)}\ \right\rceil \ge 1 + \left\lceil\sqrt{(2n+2i)(4n+2j)}\ \right\rceil \ge 4.$$

Sub Case (viii): Verify the pair (w_i, v), $1 \le i \le n-1;$

$$d(w_i, v) + \left\lceil \sqrt{\mu(w_i)\mu(v)} \right\rceil = 2 + \left\lceil \sqrt{(2n+2i)(2n)} \right\rceil \geq 4.$$

Sub Case (ix): Verify the pair (w_i, w) , $1 \leq i \leq n-1$;

$$d(w_i, w) + \left\lceil \sqrt{\mu(w_i)\mu(w)} \right\rceil \geq 2 + \left\lceil \sqrt{(2n+2i)(4n)} \right\rceil \geq 4.$$

Sub Case (x): Verify the pair (w_i, u) , $1 \leq i \leq n-1$;

$$d(w_i, u) + \left\lceil \sqrt{\mu(w_i)\mu(u)} \right\rceil = 1 + \left\lceil \sqrt{(2n+2i)(6n+2)} \right\rceil \geq 4.$$

Sub Case (xi): Verify the pair (w_i, w_j) , $1 \leq i \leq n-2,\ i+1 \leq j \leq n-1$;

$$d(w_i, w_j) + \left\lceil \sqrt{\mu(w_i)\mu(w_j)} \right\rceil = 2 + \left\lceil \sqrt{(2n+2i)(2n+2j)} \right\rceil \geq 4.$$

Sub Case (xii): Verify the pair (v_i, v), $1 \leq i \leq n$;

$$d(v_i, v) + \left\lceil \sqrt{\mu(v_i)\mu(v)} \right\rceil \geq 1 + \left\lceil \sqrt{(4n+2i)(2n)} \right\rceil \geq 4.$$

Sub Case (xiii): Verify the pair (v_i, w), $1 \leq i \leq n$;

$$d(v_i, w) + \left\lceil \sqrt{\mu(v_i)\mu(w)} \right\rceil \geq 1 + \left\lceil \sqrt{(4n+2i)(4n)} \right\rceil \geq 4.$$

Sub Case (xiv): Verify the pair (v_i, u), $1 \leq i \leq n$;

$$d(v_i, u) + \left\lceil \sqrt{\mu(v_i)\mu(u)} \right\rceil = 2 + \left\lceil \sqrt{(4n+2i)(6n+2)} \right\rceil \geq 4.$$

Sub Case (xv): Verify the pair (v_i, v_j) , $1 \leq i \leq n-1,\ i+1 \leq j \leq n$;

$$d(v_i, v_j) + \left\lceil \sqrt{\mu(v_i)\mu(v_j)} \right\rceil \geq 1 + \left\lceil \sqrt{(4n+2i)(4n+2j)} \right\rceil \geq 4.$$

Sub Case (xvi): Verify the pair (v, w);

$$d(v, w) + \left\lceil \sqrt{\mu(v)\mu(w)} \right\rceil = 3 + \left\lceil \sqrt{(2n)(4n)} \right\rceil \geq 4.$$

Sub Case (xvii): Verify the pair (v, u);

$$d(v, u) + \left\lceil \sqrt{\mu(v)\mu(u)} \right\rceil = 3 + \left\lceil \sqrt{(2n)(6n+2)} \right\rceil \geq 4.$$

Sub Case (xviii): Verify the pair (w, u);

$$d(w, u) + \left\lceil \sqrt{\mu(w)\mu(u)} \right\rceil = 3 + \left\lceil \sqrt{(4n)(6n+2)} \right\rceil \geq 4.$$

Thus all the pair of vertices satisfies the radio even geometric mean graceful condition.

Hence $DS(D(T_n))$ is a radio even geometric mean graceful graph.

Example 2.4:

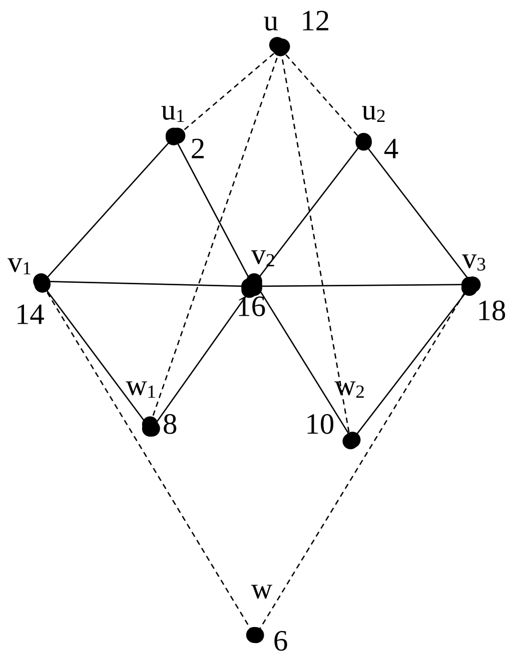
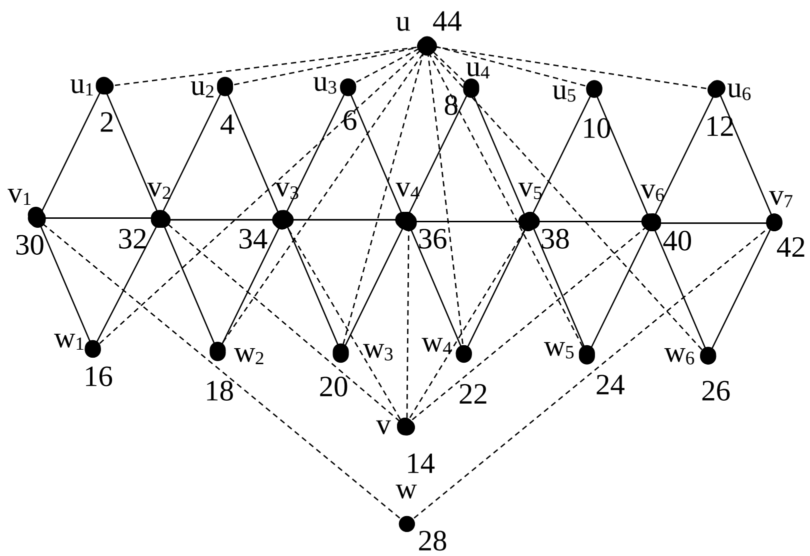


Fig 2. (a) Radio Even Geometric Mean Graceful Labeling of $DS(D(T_3))$.



(b). Radio Even Geometric Mean Graceful Labeling of $DS(D(T_7))$.

Theorem 2.5: $DS(Q_n)$ is a radio even geometric mean graceful graph.

Proof:

Let $v_i,\ 1 \leq i \leq n$ be the vertices of path P_n. Consider the vertices u_i and w_i, $1 \leq i \leq n-1$. Join u_i with v_i and w_i with v_{i+1}, $1 \leq i \leq n-1$. Also join u_i with w_i, $1 \leq i \leq n-1$. The new graph is Q_n.

Case 1: $n = 2, 3$.

Introduce a new vertex u and join it with the vertices of Q_n of degree two. The new graph thus obtained is $DS(Q_n)$ whose vertex set is $V(DS(Q_n)) = \{u_i,\ w_i,\ 1 \leq i \leq n-1,\ v_i,\ 1 \leq i \leq n\} \cup \{u\}$. Clearly, $diam(DS(Q_n)) = 2$. Therefore, obviously the radio even geometric mean graceful condition is satisfied for every pair of vertices.

Case 2: $n \geq 4$.

Introduce two new vertices u, v and join them with the vertices of Q_n of degree two and four respectively. The new graph thus obtained is $DS(Q_n)$ whose vertex set is $V(DS(Q_n)) = \{u_i,\ w_i,\ 1 \leq i \leq n-1,\ v_i,\ 1 \leq i \leq n\} \cup \{u,\ v\}$. Clearly, $diam(DS(Q_n)) = 3$.

Define a bijection $\mu: V\big(DS(Q_n)\big) \rightarrow \{2, 4, 6, \ \ldots., 2|V(DS(Q_n))|\}$ by

$\mu(w_i) = 2i,\ 1 \leq i \leq n-1;$

$\mu(v_i) = 4n + 2i,\ 1 \leq i \leq n;$

$\mu(u_i) = 2n + 2i,\ 1 \leq i \leq n-1;$

$\mu(v) = 2n;$

$\mu(u) = 4n.$

To check the radio even geometric mean graceful condition for μ.

Sub Case (i): Verify the pair $(w_i,\ w_j),\ 1 \leq i \leq n-2,\ i+1 \leq j \leq n-1;$

$$d(w_i, w_j) + \left\lceil \sqrt{\mu(w_i)\mu(w_j)} \right\rceil = 2 + \left\lceil \sqrt{(2i)2j} \right\rceil \geq 4 = 1 + \ diam(DS(Q_n)).$$

Sub Case (ii): Verify the pair $(w_i,\ v),\ 1 \leq i \leq n-1;$

$$d(w_i,\ v) + \left\lceil \sqrt{\mu(w_i)\mu(v)} \right\rceil \geq 2 + \left\lceil \sqrt{(2i)2n} \right\rceil \geq 4.$$

Sub Case (iii): Verify the pair $(w_i,\ u_j),\ 1 \leq i,\ j \leq n-1;$

$$d(w_i,\ u_j) + \left\lceil \sqrt{\mu(w_i)\mu(u_j)} \right\rceil \geq 2 + \left\lceil \sqrt{2i(2n+2j)} \right\rceil \geq 4.$$

Sub Case (iv): Verify the pair $(w_i,\ u),\ 1 \leq i \leq n-1;$

$$d(w_i,\ u)\ +\left\lceil\sqrt{\mu(w_i)\mu(u)}\ \right\rceil\geq 1+\left\lceil\sqrt{2i(4n)}\right\rceil\geq 4.$$

Sub Case (v): Verify the pair $(w_i,\ v_j),\ 1\leq i\leq n-1,\ 1\leq j\leq n$;

$$d(w_i,\ v_j)+\left\lceil\sqrt{\mu(w_i)\mu(v_j)}\right\rceil\geq 1+\left\lceil\sqrt{2i(4n+2j)}\ \right\rceil\geq 4.$$

Sub Case (vi): Verify the pair $(u_i,\ v),\ 1\leq i\leq n-1$;

$$d(u_i,\ v)+\left\lceil\sqrt{\mu(u_i)\mu(v)}\ \right\rceil\geq 2+\left\lceil\sqrt{(2n+2i)(2n)}\right\rceil\geq 4.$$

Sub Case (vii): Verify the pair $(u_i,\ u),\ 1\leq i\leq n-1$;

$$d(u_i,\ u)+\left\lceil\sqrt{\mu(u_i)\mu(u)}\ \right\rceil=1+\left\lceil\sqrt{(2n+2i)(4n)}\right\rceil\geq 4.$$

Sub Case (viii): Verify the pair $(u_i,\ v_j),\ 1\leq i,\ j\leq n-1$;

$$d(u_i,\ v_j)+\left\lceil\sqrt{\mu(u_i)\mu(v_j)}\right\rceil\geq 1+\left\lceil\sqrt{(2n+2i)(4n+2j)}\ \right\rceil\geq 4.$$

Sub Case (ix): Verify the pair $(u_i,\ u_j),\ 1\leq i\leq n-2,\ i+1\leq j\leq n-1$;

$$d(u_i,\ u_j)+\left\lceil\sqrt{\mu(u_i)\mu(u_j)}\ \right\rceil=2+\left\lceil\sqrt{(2n+2i)(2n+2j)}\ \right\rceil\geq 4.$$

Sub Case (x): Verify the pair $(v_i,\ v_j),\ 1\leq i\leq n-1,\ i+1\leq j\leq n$;

$$d(v_i,\ v_j)+\left\lceil\sqrt{\mu(v_i)\mu(v_j)}\ \right\rceil\geq 1+\left\lceil\sqrt{(4n+2i)(4n+2j)}\ \right\rceil\geq 4.$$

Sub Case (xi): Verify the pair $(v_i,\ u),\ 1\leq i\leq n$;

$$d(v_i,\ u)+\left\lceil\sqrt{\mu(v_i)\mu(u)}\ \right\rceil\geq 1+\left\lceil\sqrt{(4n+2i)(4n)}\right\rceil\geq 4.$$

Sub Case (xii): Verify the pair $(v_i,\ v),\ 1\leq i\leq n$;

$$d(v_i,\ v)+\left\lceil\sqrt{\mu(v_i)\mu(v)}\ \right\rceil\geq 1+\left\lceil\sqrt{(4n+2i)(2n)}\right\rceil\geq 4.$$

Sub Case (xiii): Verify the pair $(u,\ v)$;

$$d(u,\ v)+\left\lceil\sqrt{\mu(u)\mu(v)}\right\rceil=3+\left\lceil\sqrt{(2n)(4n)}\right\rceil\geq 4.$$

Thus all the pair of vertices satisfies the radio even geometric mean graceful condition.

Hence $DS(Q_n)$ is a radio even geometric mean graceful graph.

Example 2.6:

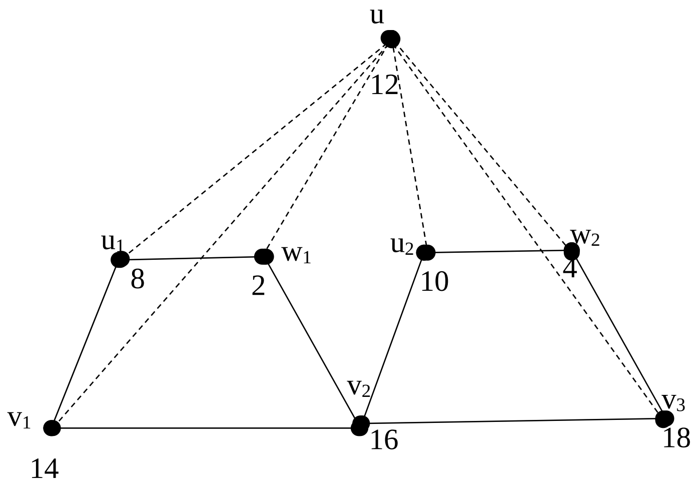


Fig 3. (a) Radio Even Geometric Mean Graceful Labeling of $DS(Q_3)$.

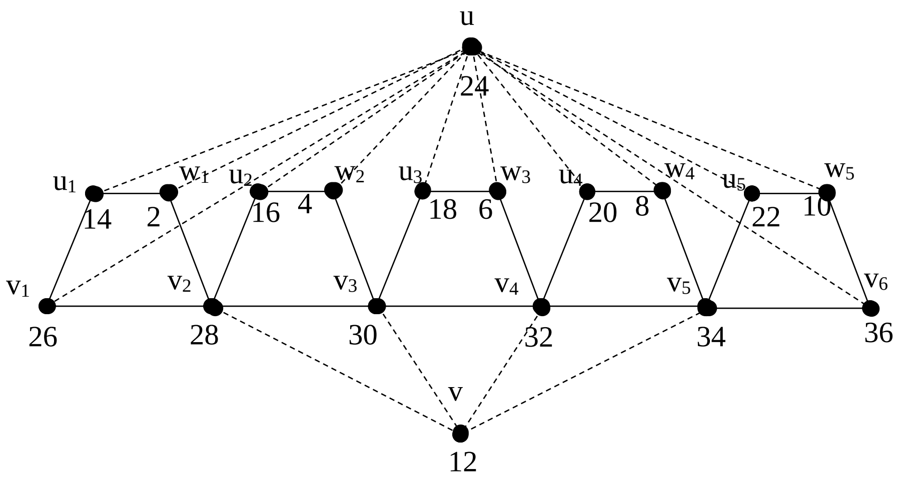


Fig 3. (b) Radio Even Geometric Mean Graceful Labeling of $DS(Q_6)$.

REFERENCES

[1] Gary Chartrand, David Erwin, P. Zhang and Harary. P, Radio labeling of graphs, Bulletin of the Institute of Combinatorics and its Applications.33 (2001), 77-85.

[2] J.A. Gallian, "A dynamic survey of graph labeling", The Electronic Journal of Combinatorics (2019), DS6.

[3] F. Harary, Graph Theory, Narosa publication House Reading, New Delhi, (1988).

[4] S. Somasundaram, R. Ponraj, " Mean labeling of Graphs", National Academy of

Science Letters, 26(2003),210-213.

[5] R. Ponraj, S. Sathish Narayanan and R. Kala, Radio Mean labeling of a graph, AKCE International Journal of graphs and combinatorics 12 (2015), 224-228.

[6] S. Somasundaram, S.S. Sandhya and S.P. Viji, Geometric mean labelling on Degree Splitting graphs, Journal of Discrete Mathematical Sciences and cryptography, 19(2), 305-320, DOI: 10.1080/09720529.2015.1084781.

[7] R. Ponraj and S. Somasundaram, On the degree splitting graph of a graph, NATL.ACAD.SCI. LETT: 27(7-8) (2004), 275-278.

[8] David Raj. C, Sunitha. K and Subramanian. A, 2017, ‘Radio odd mean and even mean labeling of some graphs’, International Journal of Mathematical Archive, Vol. 8, no. 11, pp. 109-115.

[9] C. David Raj, A. Subramanian and K. Sunitha, ‘Radio mean labeling of double triangular snake graph and quadrilateral snake graph’, International Journal of Mathematical Archive-8(11),2017, 109-115.

[10] C. David Raj, T. Mary Shalini and K. Rubin Mary, Radio Geometric Mean Graceful Labeling on degree splitting of wheel related Graphs, Dogo Rangsang Research Journal, Vol 13, Issue-3 (2023).

[11] Brindha Mary. V.T, C. David Raj and C. Jaya Sekaran V.T, ‘Radio Even Mean Graceful labeling on degree splitting of some Graphs’, Malaya Journal of Matematik, Vol. 8, No. 4, 2323-2328.