

A STUDY IN THE FRAME OF REFERENCES IN GENERAL THEORY OF RELATIVITY

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Abstract-

The law would need to be written as follows in the context of the topic we are discussing in relation to this Frame of Reference: The only thing that "rotates" is the water, which has a fixed angular velocity and a level surface. With each departure from this particular condition of motion, the deviation from a plane grows. A paraboloid is also produced when we are at rest. Once more, it makes no difference how the pail rotates. Many physical laws would need to include an additional factor, the angular velocity "" of the Pail relative to a "more suitable" Frame of reference, let's say Earth, if our description of nature were based on the Frame of reference associated with the Pail.

Key words- Rotates, pails, Paraboloide, Frame of references

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DOI:10.48047/ecb/2023.12.si10.0067

INTRODUCTION

Nearly all of the principles of physics relate to how specific objects behave in space through time. Only a location relative to another body suited for that purpose can be used to indicate the position of a body or the location of an event. For instance, the weights' velocities and accelerations in an experiment with Atwood's machine are related to the machine itself, which is ultimately the earth. The gravitational centre of the sun may be used by an astronomer to explain the motion of the planets. All motions can be categorised as relative motions to a certain reference body. We think it is possible to rigidly bind a structure made of rods that extends into space to the reference body, at least conceptually. We characterise any position by three integers, the co-ordinates of that space point, using this conceptual framework as a Cartesian coordinate system in three dimensions. A frame of reference is a term used to describe a conceptual framework that is rigorously attached to a physical body or other clearly defined point. While certain organisations might not be appropriate as reference organisations. The issue of choosing appropriate frames of reference was crucial to the advancement of research even before the theory of relativity was created. Galileo, the founder of post-medieval physics, believed that the choice of the heliocentric frame was so crucial that he was willing to risk being imprisoned or perhaps killed in order to convince his contemporaries to embrace the new frame of reference. In the end, his disagreement with the authorities centred over the choice of reference body.

Later, when Newton presented a thorough analysis of the physics of his period, the heliocentric frame of reference had gained widespread acceptance. Newton thought that more debate was still required. He came up with the well-known Pail experiment to demonstrate that some frames of reference were better suited than others for describing nature: He added water to a pail. He caused the pail to spin around its axis by twisting the rope holding it up. The water's surface went from being flat to becoming paraboloid as it gradually started to rotate. He stopped the pail once the water's rotational speed had reached a certain point. The river slowed down before coming to a complicated stop. At the same moment, the shape of a plane returned to its surface.

MOTIVATION OF THESIS

The explanation provided above is predicated on an Earth-related frame of reference. This is a possible formulation of the law determining the shape of the water's surface. If the water does not rotate, its surface is always flat. As a paraboloid, the surface is unaffected by the motion of the pail when the water rotates. Let's now explain the entire experiment using a frame of reference that is rotating in relation to the Earth at a constant angular velocity equal to the pail's maximum speed. In the beginning, the water's surface is flat, the rope, the pail, and the water "rotate" with a given constant angular velocity in relation to our new frame of reference. Then the rope, and in turn the pail, is "Stopped" and the water progressively "Slows down" as its surface assumes a paraboloid shape. The rope and then the pail are made to "rotate" again relative to our frame of reference (i.e. stopped with respect to Earth) after the water has come to a "Complete rest," its surface still a poroboloid; the water gradually starts to participate in the "rotation" while its surface flattens out. In the end, the entire device is "rotating" with its previous angular velocity and the water's surface is once more a plane. According to this Frame of Reference, the law would need to be written as follows: The only thing that "rotates" is the water, which has a fixed angular velocity and a level surface. With each departure from this particular condition of motion, the deviation from a plane grows. A paraboloid is also produced when we are at rest. Once more, it makes no difference how the pail rotates. What is meant by a "suitable" Frame of reference is very clearly demonstrated by the Newton's Pail experiment. Using any Frame of reference, we are able to define nature's laws and describe it.

When the equations of planetary motion are defined in terms of the heliocentric Frame of Reference rather than the Geometric Frame, they become essentially simpler. That is why, even before Kepler and Newton were successful in formulating the underlying rules, Copernicus and Galileo's account outweighed Ptolemy's. Investigations were conducted to identify the impact of this option in a mathematical form once it was obvious that the choice of a Frame of references determines the form of a law of nature.. By contrasting a particular body's motion with that of a mass point that is not being affected by any forces, we can determine whether the body is "accelerated" or "un-accelerated." However, the terms "at rest" and "in uniform motion" have no definite meaning; whether a body is "at rest" or "in uniform motion" relies entirely on the inertial system employed to describe it. The Principle of Relativity is the term used to describe the idea that all inertial systems are comparable when used to describe nature...

When Maxwell created the electromagnetic field equations, these equations appeared to be incompatible with the theory of relativity because, according to this theory, electromagnetic waves should propagate in empty space at a universal, constant velocity, or "C," of about 3×10^{10} cm/sec. However, it appeared that this could not be true with respect to both of the two different inertial systems, which were moving relative to one another. The definitions of "absolute rest" and "absolute motion" may both be applied to the same frame of reference, where the speed of electromagnetic radiation would be the same in all directions. Many experimenters made arduous efforts to identify this frame of reference and calculate the earth's speed relative to it.

However, none of these efforts were successful. Contrarily, every experiment appeared to support the idea that the law of relativity also applied to the laws of electrodynamics and mechanics. In a new hypothesis he put forth, H. A. Lorentz acknowledged the existence of a privileged frame of reference while also explaining why it was impossible to identify it through experimentation. But he had to make a lot of assumptions that no imaginable experiment could have verified. This theory was not entirely convincing in this regard. Only a reform of our core concepts of space and time, according to Einstein, could break the deadlock between theory and experiment. The principle of relativity was then applied to all of physics after this adjustment. Today, this is referred to as special theory of relativity. It proves that all inertial systems are fundamentally equivalent. Their dominant position among all imaginable frames of reference is fully preserved. By analysing this privileged position, the so-called General theory of relativity was able to provide a new theory of gravitation.

GENERAL RELATIVITY

A Review of litrature: One of Einstein's biggest contributions to physics was his theory of relativity, which revolutionised how people thought about and approached the creation of physical laws and the cosmos. In actuality, the theory of relativity is based on the fundamental idea that the world is a (3+1)-dimensional differentiable manifold when time and regular 3dimensional space are united to form space time [11, 43]. Space time "events" are identified by their X_0), where the zeroth component denotes the "time" allocated to the specific event and the other three components denote the space values. A few years ago, scientists started to become quite interested in Einstein's theory of general relativity. Brans and Dicks [2], Bergamann [3], Wagoner [51], Nordtvedt [28], and Sen and Dunn [46] have proposed new relativistic theories of gravity. Their predictions are contrasted with those of the more established theories using the observational data and available experimental results.

A thorough investigation into what they refer to as "metric theories of gravitation" has been conducted by Throne and Will [47]. These are the theories that can be expressed in terms of Riemannian spacetime geometry, maybe with additional structure. The Riemannian linear connection of space-time is used to derive the total stress-energy tensor of matter, which is believed to obey a differential conservation equation..

The field equations of General Relativity are a set of coupled, nonlinear differential equations, which are extremely, difficult to solve analytical except in cases of extreme symmetry. Certainly, the universe as a whole has an extremely complex structure which presents a formidable obstacle to attempts to model astrophysical phenomena. Freidmann [13, 14], Robertson [34,40] and Walker [52,53], were the first to introduce certain assumptions about the large scale structures of the universe which put the General Relativistic considerations into a manageable form.

The six basic assumption underlying most General Relativistic models of the universe are defined (which are refer to as GRC 1-6 [41]:

GRC 1: A Riemannian manifold of 4–dimensions (space time) is taken to describe the universe as a whole.

GRC2: A set of coordinate patches which cover the entire manifold where any point or event is labelled by $4 - \text{topple } x^a$; where $\alpha = 0, 1, 2, 3$.

GRC3: A matrix form on the manifold is used to calculate the separation of events ds^2 in space-time; where $ds^2 = g_{\alpha\beta} dx^{\alpha} dx^{\beta}$ and repeated indices are summed. Further, the matrix is such that, regardless of the co-ordinate system used, a transformation can be found, which puts the metric into the form (-1, 1, 1, 1). This gives us the locally Lorentzian structure of space-time.

GRC4: The field equations of the metric satisfy (with cosmological constant)

GRC5: An energy momentum tensor $\Sigma_{\alpha\beta}$ exists which describes the energy-momentum content of the universe as a whole.

GRC6: The motion of test particles is described by the geodesic equation:

 $d^{2}x^{\alpha}/ds^{2} + \Gamma^{\alpha}{}_{\beta r} dx^{\beta}/ds dx^{r}/ds = 0....(2)$

GRC7: The covariant derivative or the connection can be defined in a coordinate free manner. Its

independence of the coordinates chosen is then settled once and for all.

(1)
$$\nabla_{\mathbf{X}_1} + \mathbf{X}_2 \mathbf{Y} = \nabla_{\mathbf{X}_1} \mathbf{Y} + \nabla_{\mathbf{X}_2} \mathbf{Y}$$

(2) $\nabla_{\mathbf{X}} (\mathbf{Y}_1 + \mathbf{Y}_2) = \nabla_{\mathbf{X}} \mathbf{Y}_1 + \nabla_{\mathbf{X}} \mathbf{Y}_2$

In addition to the seven assumptions above, the primary models of interest are those, which satisfy additional criteria, the two socalled "cosmological assumption". These two statements are added to make the models consistent with moderndays astrophysical observations. Although these assumptions are not necessary in the universe's early history, they appear to be supported today by the evidence of observations. The so-called cosmological assumptions (which we refer to as CA1 and CA2)[40] are:

CA1:- The universal is homogeneous in space. The way that matter is distributed across the universe seems to be universal. There are no apparent favoured regions of matter concentration in the visible cosmos, and stars, galaxies, inter stellar dust and gas, etc., appear to be dispersed uniformly throughout. The universal can be simulated using a "perfect" fluid with density, temperature T, and four constant speeds U, as long as the values of space are not smaller than 109 light years.

CA2:- The universe is isotopic in space. Regardless of the direction of measurement, the distribution of matter and the measurement of different physical quantities (such background temperature) have some significance. For example, the isotropy of the background microwave radiation temperature of about 3^{0} K, is isotopic to within 0.2%[42]. This data along with number counts of galaxies, which show no preferred direction provided another simplification, which enable one to further eliminate the number of possible models of the universe.

In general relativity the question of singularity is much discussed problem. Penrose [30], Hawking [17] and Geroch [15] have shown that the occurrence of space-time singularities is a general Prediction of the theory and not just the consequence of the symmetry of the models. Modifying instant's equations of general relativity has been one of the techniques followed to avoid space-time singularities. Recently Trautman [48] has proposed that spin and torsion may avert gravitational singularities by considering a friedman type of universe in the frame work of Einstein-Carton theory and obtaining a minimum radius R_0 at T=0.

In general theory of relativity given by Einstein mass has a dominant role but not the spin, the density of energy momentum is the source of curvature. To introduce torsion and relating it to spin one can obtain an interesting link between the theory of gravitation and the special theory of relativity. The Einstein-Cartan theory introduces torsion and links it to the density of the intrinsic angular momentum to restore the connection between mass and spin. At long finally, the similarity between mass and spin extends to the idea of equivalency. The world line of a spineless test particle travelling under the influence of gravitational forces only depends on its initial position and velocity, not on its mass, according to the underlying concept. The velocity of spin is also dependent on the initial data, but not on the size of the particle's spin.

In actuality, Minankowski provided the fundamental framework for the theory of relativity's four-dimensional space-time continuum. Minankowski developed a novel idea of a four-dimensional space-time continuum in 1908 using the special theory of relativity and Riemann's four-dimensional geometry. This idea may be viewed as a geometrical interpretation of the special theory. The Minankowski space-time continuum is presented as follows:

$$ds^{2} = - dx^{2} - dy^{2} - dz^{2} + c^{2}dt^{2}; (-2 \text{ signature})$$

= +dx² + dy² + dz² - c²dt²; (+2 signature)

The modified form of the above equation in tensor form is

Where ∇x^i is the difference in the ith co-ordinate value between events P and Q i.e.

Here η_{ij} is a 4x4 matrix called the flat space Minkowski matrix tensor, which has the form (with-2 signature),

$$\eta_{ij} = \begin{vmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} \dots \dots \dots \dots \dots (6)$$

This space time interval form the basis of special relativity, i.e. for any two observers moving with constant velocity, with respect to each other, the observation of two events by each observer gives the same value of ds^2 even thought each observer may assign different value of x^i to each event as measured in his co-ordinate system[27,43]

In general relativity the metric tensor g_{ij} replace the Minkowski metric η_{ij} and is allowed to vary as function of the geometry of space–time. Thus derivatives of function, vector or tensor fields take on a more complicated form as term must be added to compensate for the curvature of space time

[1,27]. Thus one may define the covariant derivative of a vector field. \bar{y} along vector... as;

$$\nabla_{\bar{X}} \bar{y} = y_{,j}^{1} x^{j} Ei....(7)$$

Where

$$y^{i}_{;j} = y^{i}_{,j} + \Gamma^{i}_{jk}y^{k}$$
.....(8)

The Riemann curvature tensor, which play a major role in the theory, measures the noncommutatively of covariant derivatives in time:

For space time to be flat requires that the left hand side of equation (9) equal zero for all events on the space time topology if one performs the operation shown in equation (9) above, one derives for the form of the Riemann tensor.

In our development of theory, a distinction must be made between holonomic (or an holonomic) frame of reference .It is clear that the observer ,whether embedded in a flat minkowski space time or a curve topology, must make reference to a local coordinate frame in describing the motion of "events" as they occur. In a locally Lorentzian frame [27], the three space axis are chosen such that they are orthogonal simple Cartesian co-ordinate. The time axes x are chosen such that it is orthogonal to these three axes in four dimensional space-times. A single object trace out a "world-line" in fourdimensional space time, where the time line t is a parameterized by the local proper time. This world line may be thought of as a function of the proper time. The unit vector along this time line or world line may be formed by taking the derivative of the function with respect to the proper time. The tangent vector so formed is the E⁴ unit vector. Thus , one can erect an ortho normal set of unit vectors or tetrad (meaning "group of four") at event on the observer's world-line. The simplest unit vectors set up by above process may be defined as the operation.

$$E_i = \frac{\partial}{\partial x^i}$$
.....(11)

Forming the commutator of any two unit vectors (Lie Derivatives), it can be seen that if the unit vectors are defined as in equation (11),the commutator must vanish because of the property of partial differentiation. Thus:

$$\begin{bmatrix} E_A E_B \end{bmatrix} = E_A E_B - E_B E_A = \frac{\partial}{\partial x^A} \frac{\partial}{\partial x^B} + \frac{\partial}{\partial x^B} \frac{\partial}{\partial x^A} = 0....(12)$$

Such a set of tetrads is said to constitute a coordinated or holonomic co-ordinate system. However ,let it be assumed for a moment that one has chosen a tetrad such that not all commutators vanish. let it be further assumed that the tetrad chosen is represented as some linear combination of the holonomic basis shown in equation (11). Thus, one has a new set of tetrads $[E_a]$ defined as:

Here, h_a^B is the linear transformation connecting the tetrad set E_a to the holonomic tetrad defined in equation (11). one may use the convention that upper case letters indicate a holonomic basis and lower case indicates an holonomic basis. It can be shown that the lie derivative.

must be represented as a linear combination of tetrads. Thus, in general, One has

$$\label{eq:eq:expansion} [E_a, E_b] = C^d_{ab} E_d. \ldots \ldots (14)$$
 and thus

$$[E_a, E_b] = -[E_a, E_b] = C_{ab}^d E_d....(15)$$

Where the value C_{ab}^d are called commutation coefficients or "the object of anholonomity." It can be seen from equation (14) and (15) that

This property of the tetrads becomes important in general relativity calculations because of the role the tetrads play in the definition of the affine connections. It can be shown [27] that if the commutator of the tetrads does not vanish, then the affine connection has the form,

$$\Gamma^{i}_{jk} = \frac{1}{2} g^{ll} (g_{lj,k} + g_{lk, j} - g_{jk,l}) + \frac{1}{2} (C^{i}_{jk} + C^{i}_{kj} - C^{i}_{jk})$$
.....(17)

The first term on the right hand side of equation (17) is called the christoffel symbol of the second kind. If the co-ordinate system chosen is holonomic (all commutation coefficients are zero), the affine connection reduces to the christoffel form. However, if a non-holonomic basis is chosen, the second term in equation (17) must be added. As educated in equation (16), the connection coefficients have the property of being anti symmetric in the indices a and b. Indeed one can show using equation (17) and subtracting a like term but reversing the indices j and k, that

It is a at this point that the fundamental difference between the Einstein theory of relativity and the Einstein-Cartan theory can be made.

In the standard theory of relativity the affine connection is always symmetric in the lower two indices as long as one choice to work in a holonomic reference frame. If one choice to work in an holonomic frame, the additional term in equation(17) can cause the affine connection to become anti- symmetric in the lower to indices. However, this asymmetric is only an artefact of the co-ordinate system chosen.

The asymmetry can be removed simply by transforming the co-ordinate frame to one which is holonomic. This property of asymmetry in affine connection form one of the basic tenants of the Einstein–Cartan theory. Here, one assumes that the affine connection has an asymmetric part (called "torsion") which is not a result of the an holonomic frame chosen. In effect, one finds that torsion is a real property of Einstein-Carton space-time, it cannot be "transformed" away [16].

The form of Riemann tensor shown in equation (10) in thus good only for holonomic co-ordinate systems. If one performed the covariant derivative operation shown in equation (9), a new form of the Riemann tensor which is valid in holonomic or non-holonomic co-ordinate system is derived. The form of the Riemann tensor remains the same regardless of whether one works in ordinary general relativity or Eienstien-Cartan theory. The new form of the Riemann tensor is given by:

So far, we've talked about the characteristics of space-time curvature as well as the tensor properties and terms required to carry out computations in a curved space-time. General relativity also links the curvature to the energy-Momentum content of space-time. There is no inherent reason for the two concepts—curvature and energy—to be related; this connection is only a hypothesis. However, Einstein was able to create a tensor link between curvature and energy in his search for a set of gravitational field equations that could be used to predict purely gravitational occurrences at the microscopic scale. One would think that a relationship connecting curvature and energy might take the form

Where T would be a fourth-rank tensor to the energy-momentum content of space-time, R the Riemann tensor, and K a constant .if one takes the divergence of the side of equation (20), the right

side must vanish because of the conservation of energy-momentums

It turns out that the Riemann tensor's divergence is not zero. In direct opposition to experiment, this would indicate that energy conservation is invalid in gravitational phenomena. Additionally, it is difficult to create a fourth-rank divergence-free stress momentum tensor. The stress energy momentum content of space-time has been discovered to be a second Rank divergence less tensor from several different areas of physics, including electromagnetism and fluid dynamics. One may remedy the above problem by taking the trace of the i and k indices of the Riemann tensor (thus forming the 2nd rank Ricci tensor), thus,

$$\mathbf{R}_{ij} = \mathbf{R}^{1}_{i1j} + \mathbf{R}^{2}_{i2j} + \mathbf{R}^{3}_{i3j} + \mathbf{R}^{4}_{i4j}$$
.....(21)

And then forming the so-called curvature scalar R by raising theindices and taking the trace i and j.

One thus form a divergence less, second rank tensor G_{ij} called the Einstein tensor as a combination of the Ricci tensor, metric tensor, and curvature scalar to be;

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R....(23)$$

The Einstein tensor G_{ij} is related to a second rank SEMT (stress energy momentum tensor) T_{ij} by the following equation.

$$G_{ij} = KT_{ij}$$
....(24)

The formulation turns out to be much easier to work from as compared to one of the form Equation (20). A SEMT can be more easily constructed as a 2nd rank tensor and terms added corresponding to fluid shear, momentum, electromagnetic field and so forth. Also the Einstein tensor constants derivatives of the metric tensor g_{ii} no higher than second order, an important consideration when trying to solve the equation formed from equation (24). the constant K connecting the Einstein tensor to the SEMT is arrived at by solving equation (24) for case of weak gravitational field, and placing the requirement that equation reduces to the Newtonian the gravitational equation (Poisson's equation) in the limit as curvature goes to zero. From the form of the resulting solution, it becomes apparent that the metric tensor components can be interpreted as the "potentials" of the gravitational fields, with the g₄₄ component corresponding to the ordinary Newtonian gravitational potential as:

$$\varphi = -\frac{(g_{44}+1)c^2}{2}....(25)$$

Where ϕ is a solution to Poisson's equation: $\nabla^2 \phi$ =4 π Gp where ρ is the mass density.

If one considers the case of a perfect fluid, (of considerable interest in cosmological models because of its simplicity) the SEMT takes the following simple form:

$$T_{ij} = (\rho + P)u_iu_j + g_{ijp}....(26)$$

Where ρ is the fluid density, P is the fluid pressure and u is the four velocity of the fluid such that $u_i u^i = -1$

If we consider a charged perfect fluid, the energy momentum tensor T_{ij} splits up into two parts viz. T_{ij} and E_{ij} for matter and charges respectively i.e.

$$T_{j}^{i}=T_{j}^{i}+E_{j}^{i}$$
....(27)

The field equation (known as Einstein-Maxwell field equation) Governing the energy momentum tensor of charge perfect fluid will be discussed in next chapter.

Comparison of Einstein Model with actual Universes:

The most unsatisfactory feature of the Einstein Model as a basis for the cosmology of the actual universe is, that it provides no reason to expect any systematic shift in the wave length of light from distant object .in the actual universe, however, the work of Hubble and Humason shows a definite redshift in the light from the nebulae which increase with the distance. This is of course the main consideration which will lead us to prefer nonstatic to static models of the universe as a basis for actual cosmology.

The most unsatisfactory feature of the Einstein Model is its correspondence with a universe which could actually contain a finite concentration of uniformly distributed matter. In this respect it gives us a cosmology which is superior to the provide by the de-sitter model. This advantage is gained only at the expense of introducing the extra cosmological term a g_{ij} into Einstein's original field equation which is a device similar to the modification in Poisson's equation proposed in order to permit a inform static distribution of matter in flat space of the Newtonian theory.

CONCLUSION

The analytical solution of Einstein's field equation for a static anisotropic fluid sphere is described in the thesis "Some Studies in General Theory of Relativity" by making the assumption that spacetime is conformally flat with a selected energy *Eur. Chem. Bull.* 2023, 12(Special Issue 10), 584 - 591 density. In Einstein's general theory of relativity, mass predominates but not spin; the source of curvature is the density of energy momentum. One can gain an intriguing connection between the special theory of relativity and the theory of gravitation by relating tension to spin. Numerous physical parameters can be determined because this model is both physically sound and Singularity-free.

By adopting a suitable type of mass density, the Einstein-Maxwell field equation for a static, uniformly flat, charged, ideal fluid sphere may be solved. A highly nonlinear system of equations and a sparse number of exact solutions have been used to solve the Einstein Maxwell field equation in the presence of matter and charge. As the topic of selfenergy can be answered, it is anticipated that the precise solutions of the field equations in general relativity for extended charged distribution will be helpful in the study of quantum field theory in a Reimannin manifold.

REFERENCES:

- 1. Adler, R.J. Bazin, M. and Schiffer, M.M. (1975): Introduction to general relativity, Second Edition, McGrow Hill.
- 2. Brans, C. and Dicker, R,H.(1961):Mach's principle and Relativistic Theory of Gravitation , Phy,Rev.124,925.
- 3. Bergmann, P.G.(1968)Comments on the Scalar tensor Theory, Int. j. Theo. Phys., <u>1</u>, 25.
- 4. Cartan, E. (1922): C.R. Acad .Sci. (Paris), <u>174</u>, 593
- 5. Cartan, E. (1922,24): Ann. Ec. Norm. Sup., <u>40</u>, 325, <u>41</u>, 1.
- 6. Cartan, E. (1925): Ann. Ec. Norm. Sup., <u>42</u>, .17.
- 7. Costa de Beauregard, O. (1942):C.R. Acad Sci. (Paris), <u>214</u>, 904.
- 8. Costa de Beauregard, O. J. Math. Pure and Appl., <u>22</u>, 85, (1943).
- 9. Costa de Beauregard, O. (1964): Phy. Rev., <u>134</u>, B471.
- 10.De Sitter, W. (1917): Mon. Not. Roy, Aston.Soc., 78, 3.
- (a) Einstein, R. (1979) Letters on Absolute Parallelism (E.Cartanand A. Einstein 1929-32), Princeton Univ., Press.
- 11.Einstein, A. (1905): Annalen der Physik, <u>17</u>, 891.
- 12.(a). Einstein, A. (1915): Preuss, Akadniss, Berlin, Sitzber, 778, 799, 831, and 844.
- Ehlers, J. Rosenblum, A. Goldberg, J.N. (1973) Havas, P. (1976), Comments on gravitational radiation damping and energy loss in binary system, Astrophys. J. 208, 1, 77,Sec.1.1.
- 14.Friedmann, A. A. (1922): Z Phys., 10, 377.
- 15.Friedmann, A. A. (1924): Z Phys., 21, 326.

- 16.Geroch, R. P. (1966): Phys, Rev. Lett., 17, 446.
- 17.Geroch, R. P. (1980): Int. J. Theory Phys, <u>19</u>, 573.
- 18.Hawking, S. W. (1966): Proc. Roy. Soc. (London), <u>A295</u>, 490.
- 19.Hehl, F. W. (1973): G. R. G. 4, 333.
- 20.Hehl, F. W. et. al.(1976): Rev. Mod. Phys., <u>48</u>, 393.
- 21.Kibble, T. W. B.(1961): J.Math. Phys., 2212.
- 22.Kannar, J. (1995): G. R. G. 37, 23.
- 23.Kerlick, G. D. (1975): Spin and torsion in general relativity.
- 24.Kushowic, B. (1976): Acta Cosmologica, <u>3</u>, 109.
- 25.Kramer, D., Stephani, H. Herlt, E. (1980): Exact solutions of Einstein's
- 26.field equation, Cambridge Univ. Press, Pages 200,209.
- 27.Kerr, R. P. (1963): Phys. Rev., Lett., <u>11</u>, 237.
- 28. Kinersley, W. (1975): Recent progress in exact solution, in :Shaviv, G.
- 29.and Rosen, J(Ed.). General Relativity and Gravitation (Proceedings of G. R. 7, Tel-Aviv, 1974), Wiley, New York, London). See 1.1, 1.4, 11.3, 24.1, 30.3.
- 30. Misner, C. W., Throne, K. S., Wheeler, J. A. (1973): Gravitation, W. H. Freeman and company, San Francisco.
- 31.Nordtvedt, K. Jr., (1970): Post-Newtoninan metric for a general class of scalar-tensor gravitational theories and observational consequences, Ap. J.<u>161</u>, 1059.
- 32.Nordstrom, G. (1918): Proc. Kon. Ned. Aked. Wet. <u>20</u>, 1238.
- 33.Penrose, R. (1965): Gravitational Collape and space-time singularities, Phys., Rev. Lett., <u>14</u>, 57.
- 34. Papapetrou, A. (1949): Phil. Mag., <u>40</u>, 937.
- 35.Parasanna, A. R. (1974): Einstein-Cartan theory ortho geometrasation of spin, preprint, (1974).
- 36.Reissner, H. (1916): Ann. Phys.(Germany) <u>50</u>, 106.
- 37.Robertson, H. P. (1928): Phil. Mag., <u>5</u> 835.
- 38.Robertson, H. P. (1929): Proc. Nat. Acad. Sci.; <u>15</u> 822.
- 39.Robertson, H. P. (1933):Rev. Mod. Phys., <u>5.</u>62.
- 40. Robertson, H. P. (1935): Astrophys. J.; 82, 284.
- 41.Robertson, H. P. (1936): Astrophys. J.; <u>83</u>, 187.