SOLUTION OF SOME MHD FLOW PROBLEM HELP OF HOMOTOPY PERTURBATION METHOD

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Abstract: The problem of magneto–hydrodynamic (MHD) flow along a nonlinear stretching sheet in corporation with a transverse magnetic field is analyzed. The governing equations are transformed into nonlinear ordinary differential equations and solved by adopting the homotopy perturbation method. Homotopy perturbation analysis has been found to be an excellent agreement with the method used by this paper's authors.

Keywords: Stretching sheet, MHD, Homotopy perturbation method, Magnetic field.

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1. INTRODUCTION:

Today, there is a strong need to investigate magnetohydrodynamic flow in order to solve problems that relate to its use at all levels of the applied physics and technological sciences. The MHD induced flow has its own set of problems, depending on the situation. Flows between shrinking sheets or stretching sheets could be involved in some situations. As a result, there are both shrinking and expanding flow problems that require appropriate solutions. More than a dozen researchers examined the flows caused by sheet stretching, in addition to looking at their behaviour when flowing over diminishing surfaces. The stretching of the sheet has a material impact on boundary layer flow and heat transfer, which is an important use in chemical engineering with regard to polymer technology. The quality of chemical technology products is determined by the characteristics of heat transfer over a stretched surface.

In recent years, emphasis has been placed on the study of nonlinear fluid behavior. Different kinds of salt solutions, molten polymers, lubricants, bio fluids and etc. are the most commonly used examples of nonlinear fluids. The importance of MHD flow studies is growing as a consequence of their relevance and applicability to solving the associated problems, in particular with regard to nonlinear stretching sheets. Apart from their application in the different areas, several prominent scientists have studied flow models used over a stretched sheet to determine whether they are suitable for industrial production processes involving manufacture of glass fibers, plastic items, fabrication of metals etc. Taking the MHD flows over a nonlinear stretching sheet, some works are being pursued by Hayat et al. [4], Eerdunbuhe [3], Cortell [1], Kai et al. [11], Ullah et al. [14] etc.

While solving certain MHD flow problems concerned with non-linear stretching sheet, use of Homotopy Perturbation Method (HPM) adds new dimension to the study of hydrodynamic flow. HPM helps in solving ordinary or partial differential equations of both linear and nonlinear nature. Most importantly, HPM provides a way to solve nonlinear differential equations by conducting a linearization process thereby reducing huge computational work. Dr. Ji Huan He was the first to advocate and apply HPM method in 1998. Applications of HPM prominently appear in a good number of scholarly works, of which the worth–mentioning ones include J.H.He [5,6,7], Jhankal[10], Cuce and Cuce[2], Kharrat and Toma[12], Jameel et al.[9], He and El-Dib[8] etc. Present study is carried out in the line of the work done by Motsa and Sibanda [13]. They solved the problem using spectral- homotopy analysis method (SHAM). But the present authors have applied the homotopy perturbation method (HPM) here to acquire solution of MHD flow over a non-linear stretching sheet.

2. MATHEMATICAL FORMULATION:

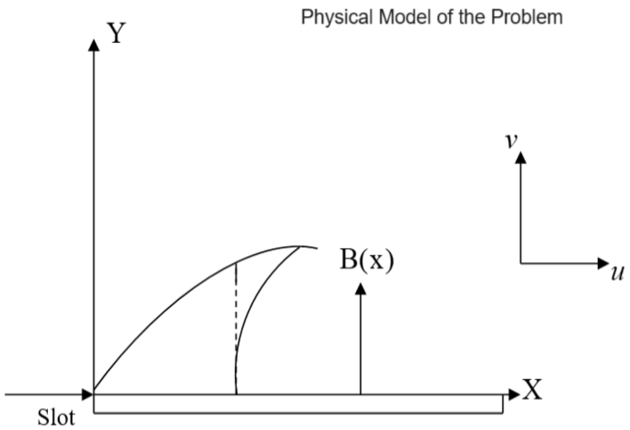
Considered a problem involving two-dimensional steady boundary layer flow from an impervious horizontal non-linear stretching sheet passing through incompressible, viscous electrically conducting fluid. The magnetic field generated by motion of electrically conducting fluid is considered nominal as compared with applied magnetic field. Magnetic Reynolds number is taken to be small and no electric field is applied so that Hall effects become negligible. A constant pressure is also assumed and magnetic pressure term is ignored.

Based on above assumptions, the governing continuity and momentum equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \rightarrow(1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u \qquad \rightarrow(2)$$

Where υ implies kinematic viscosity, ρ and σ imply density and electrical conductivity of fluid respectively. B(x) denotes magnetic field which is set normal to the flow direction.



As presented in Figure1 the non-linear stretching sheet is parallel with X-axis. Let u and v are velocity components along X-axis and Y- axis respectively.

Let $B(x) = B_0\, x^{(n-1)/2}$, B_0 implies strength of applied magnetic force.

It is assumed that the sheet moves with a power-law velocity for which the boundary conditions become

$$u(x,0) = cx^n \quad \rightarrow (3)$$

$$u(x,y) \rightarrow 0 \ \ as \ \ y \rightarrow \infty \quad \rightarrow (4)$$

Where, n>0 and n<0 is for accelerating and decelerating sheet respectively.
With the help of transformations,

$$u = cx^n f'(\eta) \text{ where } \eta = \sqrt{\frac{(n+1)c}{2v}}\, x^{(n-1)/2} y \quad \rightarrow (5)$$

$$v = -\sqrt{\frac{cv(n+1)}{2}}\left[f(\eta) + \frac{n-1}{n+1}\eta f'(\eta)\right] x^{(n-1)/2} \quad \rightarrow (6)$$

Here, $f(\eta)$ indicates non-dimensional stream function. Now the equations (1) and (2) can be converted to following non-linear differential equation:

$$f'''(\eta) + f(\eta) f''(\eta) - \beta (f'(\eta))^2 - Mf'(\eta) = 0 \quad \rightarrow (7)$$

Taking boundary conditions as:

$$f(0) = 0, \qquad f'(0) = 1, \qquad f'(\infty) = 0 \quad \rightarrow (8)$$

Where, β and M indicates stretching parameter and magnetic parameter respectively and their values are:

$$\beta = 2n/_{1+n} \quad \text{and} \quad M = 2\sigma B_0^2/_{\rho c(1+n)}$$

3. SOLUTION WITH HOMOTOPY PERTURBATION METHOD(HPM):

Applying HPM, the equation (1) takes the following form:

$$(1-p)(f''' - M.f') + p\left(f''' + f.f'' - \beta f'^2 - M.f'\right) = 0 \quad \rightarrow (9)$$

Where, p is perturbation parameter.
Considering "*f*" as under:

$$f = f_0 + pf_1 + p^2 f_2 + ... \quad \rightarrow (10)$$

Using (3) in (2) equating the terms free from 'p', the following ordinary differential equation is obtained:

$$f_0''' - M.f_0' = 0 \quad \rightarrow (11)$$

Solving the equation (11) with boundary conditions for zeroth order,

i.e. $f_0(0) = 0,\ f_0'(0) = 1,\ f_0'(6) = 0$, obtained the following solutions:

$$f_0(\eta) = A_1 - A_2(e^{\sqrt{M}\eta} + e^{(12-\eta)\sqrt{M}}) \quad \rightarrow (12)$$

Now,

$$f_0'(\eta) = -A_2\sqrt{M}\,(e^{\sqrt{M}\eta} - e^{(12-\eta)\sqrt{M}}) \quad \rightarrow (13)$$

$$f_0''(\eta) = -A_2 M\,(e^{\sqrt{M}\eta} + e^{(12-\eta)\sqrt{M}}) \quad \rightarrow (14)$$

Again, equating the terms involving coefficients of 'p' from (2), the following ordinary differential equations are obtained:

$$f_1''' - M.f_1' + f_0.f_0'' - \beta(f_0')^2 = 0$$

$$i.e.\ f_1''' - M.f_1' = -A_3(e^{\sqrt{M}\eta} + e^{\sqrt{M}(12-\eta)}) + A_4(e^{2\sqrt{M}\eta} + e^{2(12-\eta)\sqrt{M}}) + A_5 \quad \rightarrow (15)$$

Solving equation (15) with boundary conditions for 1st order

i.e. $f_1(o)=0,\ f_1'(0)=0,\ f_1'(6)=0;$

Solutions are obtained as under:

$$f_1(\eta) = \frac{C_1 e^{\sqrt{M}\eta}}{\sqrt{M}} + \frac{C_2 e^{-\sqrt{M}\eta}}{\sqrt{M}} + C_3 + \frac{A_3\eta}{2M}\left[e^{(12-\eta)\sqrt{M}} - e^{\sqrt{M}\eta}\right] + \frac{A_4}{6M\sqrt{M}}\left[e^{2\sqrt{M}\eta} - e^{2(12-\eta)\sqrt{M}}\right] - \frac{A_5\eta}{M} \quad \rightarrow (16)$$

$$f_1'(\eta) = C_1 e^{\sqrt{M}\eta} - C_2 e^{-\sqrt{M}\eta} + \frac{A_3}{2M}\left[e^{(12-\eta)\sqrt{M}} - e^{\sqrt{M}\eta}\right] + \frac{A_3\eta}{2\sqrt{M}}\left[-e^{(12-\eta)\sqrt{M}} - e^{\sqrt{M}\eta}\right] + \frac{A_4}{3M}\left[e^{2\sqrt{M}\eta} + e^{2(12-\eta)\sqrt{M}}\right] - \frac{A_5}{M} \quad \rightarrow (17)$$

Neglecting higher order perturbed terms, it is finally obtained that

$$f'(\eta) = -A_2\sqrt{M}\,(e^{\sqrt{M}\eta} - e^{(12-\eta)\sqrt{M}}) + p[C_1 e^{\sqrt{M}\eta} - C_2 e^{-\sqrt{M}\eta} + \frac{A_3}{2M}\left(e^{(12-\eta)\sqrt{M}} - e^{\sqrt{M}\eta}\right) + \frac{A_3\eta}{2\sqrt{M}}\left(-e^{(12-\eta)\sqrt{M}} - e^{\sqrt{M}\eta}\right) + \frac{A_4}{3M}\left(e^{2\sqrt{M}\eta} + e^{2(12-\eta)\sqrt{M}}\right) - \frac{A_5}{M}] \quad \rightarrow (18)$$

$$f(\eta) = f_o + pf_1$$
$$= A_1 - A_2(e^{\sqrt{M}\eta} + e^{(12-\eta)\sqrt{M}}) + p[\frac{C_1 e^{\sqrt{M}\eta}}{\sqrt{M}} + \frac{C_2 e^{-\sqrt{M}\eta}}{\sqrt{M}} + C_3 + \frac{A_3\eta}{2M}\left(e^{(12-\eta)\sqrt{M}} - e^{\sqrt{M}\eta}\right) + \frac{A_4}{6M\sqrt{M}}\left(e^{2\sqrt{M}\eta} - e^{2(12-\eta)\sqrt{M}}\right) - \frac{A_5\eta}{M}] \quad \rightarrow (19)$$

4. RESULTS AND DISCUSSION:

In this study, the problem of boundary layer for MHD flow past a nonlinear stretching sheet in the presence of a transverse magnetic field is considered by Homotopy Perturbation Method. The obtained results are revealed graphically and are compared with the accurate solutions. The numerical results are obtained for different values of parameters M, β and P implanted in the flow system.

Figure 2 and 3 implements the influence of magnetic parameter and stretching parameter on velocity profile. An obstructing performance of the fluid flow velocity is depicted in figure 2 due to the magnetic field applied and its strength that is the fluid motion is obliged on account of magnetic intensity. It usually happens when a magnetic field is present in fluid conducting electrically which generates a *Lorentz force* that acts opposite to the flow when magnetic fields is applied in normal direction. Furthermore, acceleration of velocity profile on account of stretching parameter is observed in figure 3 which indicates the fact that the velocity distribution is directly proportional to the stretching parameter.

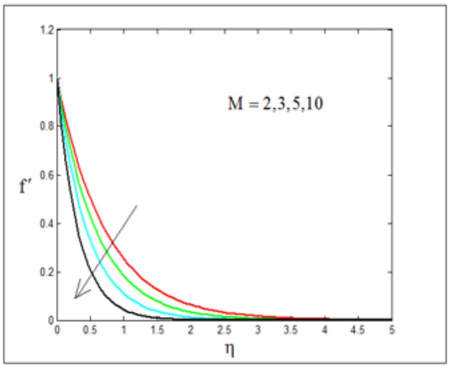


Figure 2: Velocity versus η under β=1.5, P=0.1

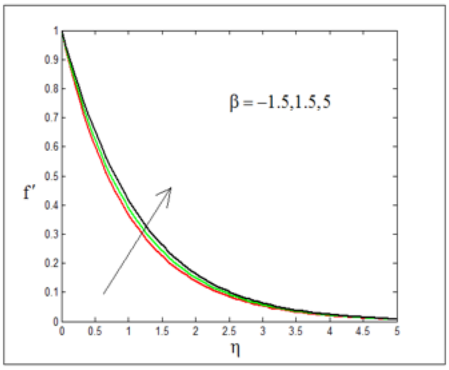


Figure 3: Velocity versus η under M=1, P=0.1

The behaviors of typical velocities against the normal co-ordinate for different values of strength of the applied magnetic field and stretching constraint are depicted in figures 4 and 5. It is empowered from figure 4 that the velocity boundary layer is reduced by virtue of magnetic induction. Moreover, this velocity graph attains its linearity in the infinite direction of the plate. From figure 5, it is revealed that the velocity boundary layer attains its minimum near the plate and greatest value far away from the plate.

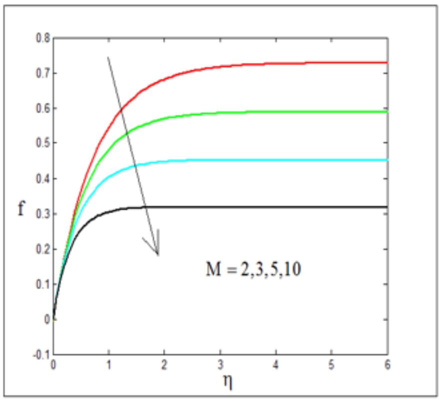


Figure 4: Velocity versus η under β=1.5, P=0.1

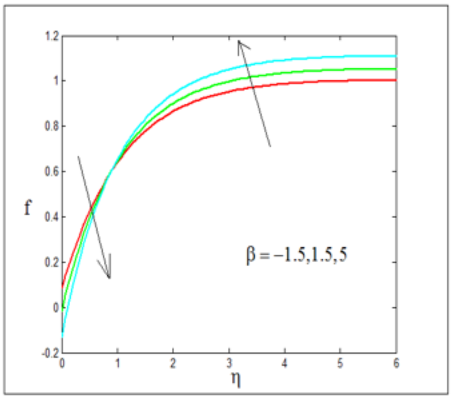


Figure 5: Velocity versus η under M=1, P=0.1

The influence of perturbation parameter on both f' and f are depicted in figure 6 and 7. From these figures, it is observed that the flow pattern of the fluid gets accelerated under the action of perturbation parameter satisfying the boundary restrictions.

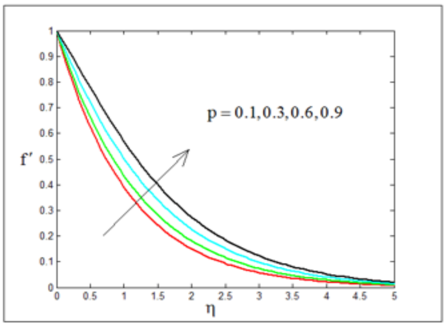


Figure 6: Velocity versus η under β=1.5, M=1

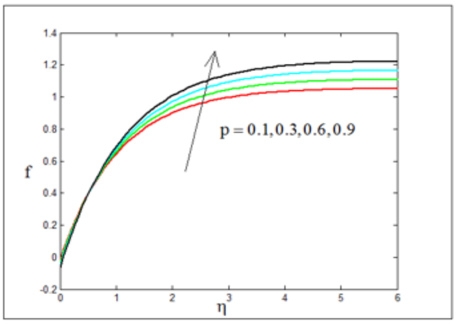


Figure 7: Velocity versus η under β=1.5, M=1

5. COMPARISION OF RESULTS:

For comparing outcomes of present research , the results of Motsa S. S. and Sibanda P. [13] are used. Comparing figures 7 with figure 3 (Motsa S. S. and Sibanda P. [13]), it is observed same kind of behaviour due to the implementation of Hartmann Number, i.e., there is a significant effect of magnetic field on velocity profile. With the imposition of Homotopy Perturbation Technique, the velocity profile is almost similar making an admirable fact with the findings investigated by Motsa S. S. and Sibanda P. [13] and the present authors.

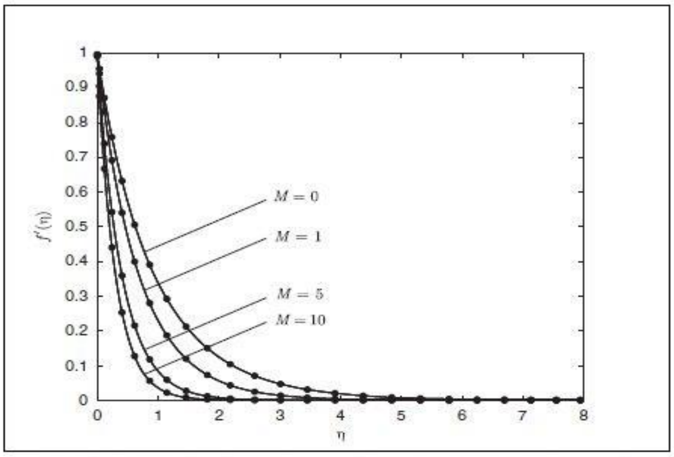


Figure 3. The comparison of the numerical results (solid line) and second-order SHAM solution (filled circles) for $f'(\eta)$ using different values of M when $\beta = 1.5$.

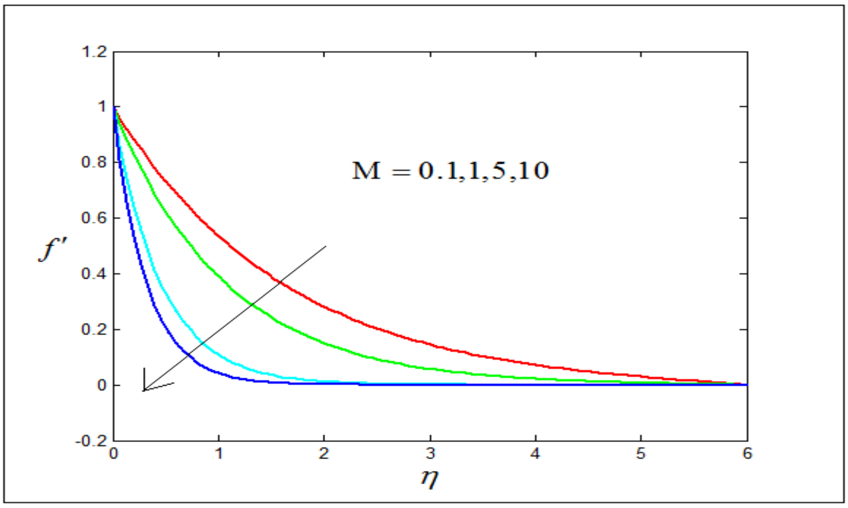


Figure 7: Velocity versus η under β=1.5, P=.1

6. CONCLUSION:

On investigation of the existing problem, it is concluded that these opinions are:

1. Due to the electromagnetic field used and its strength, there has been an obstruction of fluid flow velocity.

2. The acceleration of the speed profile, which indicated that the velocity distribution was directly inverse to the stretch parameter, is established on the basis of the extension parameters.
3. The action of a perturbation parameter shall increase the fluid flow pattern in accordance with the boundary constraints.

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