Section A-Research paper

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REGULAR PERFECT DOMINATION IN GRAPH

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ABSTRACT:

Let G=(V,E) be a graph. A perfect dominating set D⊆ V(G) is called regular perfect dominating set, if the induced subgraph $\langle D\rangle$ is regular. The minimum cardinality of D is called regular perfect domination number in a graph G and is denoted by $\gamma_{rp}(G)$.

In this paper, we study some theoretic properties of $\gamma_{rp}(G)$ and many bounds were attained in terms of vertices, edges and other distinct parameters of G. In addition their relations with different domination parameters were also established.

KEY WORDS: Graph, regular perfect dominating set, regular perfect domination number.

SUBJECT CLASSIFICATION NUMBER: **AMS05C69, 05C70**.

1.INTRODUCTION:

In this paper, the graphs inferred here are simple and finite. Commonly we follow the notations of Harary [5].

As usual p=$|V|$ and q=$|E|$ denote the number of vertices and edges of a graph G. As usual δ(G)(Δ(G)) is the minimum(maximum) degree of a vertex in a graph G.

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The degree of an edge $e = uv$ of G defined by deg(e)=deg(u)+deg(v)-2 and $\delta^1(G)(\Delta^1(G))$ is the minimum (maximum) degree among the edges of G.

The notations α_0(G) ,($\alpha_1(G)$) is the smallest cardinality of vertices(edges) in a vertex(edge) cover of G and β_0(G), (β_1(G)) is the minimum number of vertices(edges) in a maximal independent set of a vertices(edges) of a graph G,

A perfect dominating set $D \subseteq V(G)$ is called perfect dominating set, if for every vertex of $V - D$ is adjacent to exactly one vertex of D. The smallest cardinality of a perfect dominating set of G is a perfect domination number and is denoted by γ_p(G), see [4].

A dominating set D of G is a regular perfect dominating set, if the induced subgraph $\langle D \rangle$ is regular. The regular perfect domination number γ_{rp}(G) is the minimum cardinality of a regular perfect dominating set.

2. RESULTS:

The following theorem gives the regular perfect domination number for some standard graphs.

Theorem1: a. For any path $G = P_{3n}$ with ($n = 1,2,3 \dots \dots \dots \dots . .$), $\gamma_{rp}(P_{3n}) = n.$

b. For any cycle $G = C_{4n}$ with ($n = 1,2,3 \dots \dots \dots$), $\gamma_{rp}(C_{4n}) = 2n$ (1 regular).

c. For any cycle $G = C_{3n}$ with ($n = 1,2,3 \dots . .$), $\gamma_{rp}(C_{3n}) = n$ (0 regular).

d. For any star $G = K_{1,n}$ with $n \geq 2$ vertices, $\gamma_{rp}(K_{1,n}) = 1$.

e. For any complete graph $G = K_n$ with $n \geq 3$ vertices, $\gamma_{rp}(K_n) = 1$.

f. For any bipartite graph $G = K_{m,n}$with $m, n \geq 2$ vertices, $\gamma_{rp}(K_{m,n}) = 2$.

g. For any wheel $G = W_n$ with $n \geq 4$ vertices, $\gamma_{rp}(W_n) = 1$.

The proof of the above theorem is simple. Hence we omit the proof.

Theorem2: For any connected (p, q) graph G, $\gamma_{rp} \geq \left\lceil \frac{diam(G)+1}{3} \right\rceil$.

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Proof: Let $I = \{e_1, e_2, e_3, \dots \dots \dots . e_k\} \subseteq E(G)$ be the minimal set of edges in a graph G, which constitutes the shortest distance between any two distinct vertices $u, v \in V(G)$, such that $dist(u, v) = diam(G)$.

Let $D = \{v_1, v_2, v_3, \dots \dots \dots \dots v_n\} \subseteq V(G)$, such that $\forall\ v_j \in V(G) - D$ is adjacent to exactly one vertex of D and if $|N(u) \cap D|$=1for each $u \in V - D$. Then D is the minimal perfect dominating set of G. If induced subgraph $\langle D \rangle$ is regular, then D is a regular perfect dominating set of G. Since the shortest distance between any two distinct vertices of G includes at most $3\,\gamma_{rp} - 1$ edges joining the neighbourhood of the vertices of D. Hence $|\ 3\gamma_{rp} - 1| \geq |diam(G)|$ which gives

$$\gamma_{rp} \geq \left\lceil \frac{diam(G)+1}{3} \right\rceil$$.One can easily verify for the equality.

In the following theorem, we establish the relationship between vertices of G, $\Delta(G)$, with $\gamma_{rp}(G)$.

Theorem3: For any connected (p, q) graph G, $\gamma_{rp} \geq \left\lceil \frac{p}{\Delta(G)+1} \right\rceil$.

Proof: Let the vertex set of G be $V(G) = \{v_1, v_2, v_3, \dots \dots \dots \dots \dots v_n\}$ with $|V|$=p. Now assume there exists at least one vertex $v \in V(G)$ of maximum degree with $\deg(v) = \Delta(G)$. Let $D = \{v_1, v_2, v_3, \dots \dots \dots \dots \dots v_m\} \subseteq V(G)$ and $\forall v_k \in D, 1 \leq k \leq m$. If each vertex not in D is adjacent to exactly one vertex of D and $N[D] = V(G)$. Then D is minimal perfect dominating set of G. If the induced subgraph $\langle D \rangle$ is regular, then D is a regular perfect dominating set of G. Hence $|D| \geq \left\lceil \frac{|v|}{\deg(v)+1} \right\rceil$ Which gives $\gamma_{rp} \geq \left\lceil \frac{p}{\Delta(G)+1} \right\rceil$.

Theorem4: For any connected (p, q) graph G, $\gamma_{rp}(G) \leq q - \Delta^{'}(G) + 2$.

Proof: Let $E(G) = \{e_1, e_2, e_3 \dots \dots \dots \dots . e_n\}$ *be* the edge set of G. Assume there exists atleast one edge $e \in E(G)$ of maximum degree, then $\deg(e) = \Delta^{'}(G)$. Further let $A = \{v_1, v_2, v_3, \dots \dots \dots \dots \dots v_n\}$=$V(G)$. Select a set $F \subseteq A$ such that $N(v_i) \cap N(v_j) = \emptyset$, $\forall\ i, j \in F$ then F is a minimal perfect dominating set of G. For regularity, if the induced subgraph $\langle F \rangle$ is regular. Then F is a regular perfect dominating set of G. Thus $|F| \leq q - |\deg(e)| + 2$ which gives $\gamma_{rp}(G) \leq q - \Delta^{'}(G) + 2$.

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A dominating set $D \subseteq V(G)$ is an independent dominating set if the induced subgraph $\langle D \rangle$ has no edges. The independent domination number $i(G)$ of G is the smallest cardinality of independent dominating set, see[1].

In the following theorem, we develop a relationship between β_0(G), $i(G)$ with our concept.

Theorem5: For any connected (p, q) graph G, $\gamma_{rp}(G) + p \leq 4\beta_0(G) + i(G)$.

Proof: Suppose $D = \{v_1, v_2, v_3, \dots \dots \dots \dots v_n\} \subseteq V(G)$ be the minimal dominating set of G. If $\forall\ v_i \in D, \deg(v_i) = 0$, then D is an independent dominating set of G. Further let $K = \{v_1, v_2, v_3, \dots \dots \dots v_l\} \subseteq V(G)$ be the maximum set of vertices such that $dist(u, v) \geq 2$ and $N(u) \cap N(v) = x,\ \forall\ u, v \in K$ and $x \in V(G) - K$.

Clearly $|K|$=β_0(G). Further there exists $M \subseteq V(G)$ and $\forall\ v_j \in V(G) - M$ is adjacent to exactly one vertex of M and $|N(u) \cap M| = 1$ for each $u \in V - M$. Then M is perfect dominating set of G. If the induced subgraph $\langle M \rangle$ is regular, then M is regular perfect dominating set of G. Since K $\subset M$, $D \subset M$ and $|V| = p$, then $|M| + p \leq 4|K| + |D|$ which gives $\gamma_{rp}(G) + p \leq 4\beta_0(G) + i(G)$.

In [3], defined the total domination number such as a dominating set $D \subseteq V(G)$ is called a total dominating set, if the induced subgraph $\langle D \rangle$ has no isolated vertices. The total domination number $\gamma_t(G)$ is the minimum cardinality of a total dominating set.

The next theorem gives the relationship between $\gamma_s(G)$, $\gamma_t(G)$ with $\gamma_{rp}(G)$.

Theorem6: For any connected (p, q) graph G, γ_{rp}(G) + $\gamma_s(G) \geq \gamma_t(G)$.

Proof: Let $K = \{v_1, v_2, v_3, \dots \dots \dots \dots \dots v_n\} \subseteq V(G)$ and $\forall\ v_i \in K, 1 \leq i \leq n$, is adjacent to the at least one vertex of $V(G) - K$ and if the induced subgraph $\langle V(G) - K \rangle$ has more than one component, then K forms a γ_s-set of G. Further let $S = \{v_1, v_2, v_3, \dots \dots v_m\} \subseteq V(G)$ be minimum set of vertices which covers all the vertices in G. In the induced subgraph $\langle S \rangle$, if $\deg(v_i) \geq 1, \forall v_i \in S$, $1 \leq i \leq m$, then S forms a minimal total dominating set of G. Otherwise if $\deg(v_i) < 1$ for some $v_i \in S$, then attach the minimum number of vertices $\{v_j\} \in N(v_i)$ to the vertices of S. Then $S \cup \{v_j\}$ forms a minimal total dominating set of

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G. Suppose $M \subset S$ and $H \subseteq V(G) - S$. Assume $\forall v_k \in [\{M\} \cup \{H\}]$ is adjacent to exactly one vertex of $V(G) - (\{M\} \cup \{H\})$ such that $N[M \cup H] = V(G)$. Then $\{M\} \cup \{H\}$ is a minimal perfect dominating set of G. If the induced subgraph $\langle \{M\} \cup \{H\} \rangle$ is regular, then $\{M\} \cup \{H\}$ is a regular perfect dominating set of G. Hence $|\{M\} \cup \{H\}| + |K| \geq |S|$, which gives γ_{rp}(G) + $\gamma_s(G) \geq \gamma_t(G)$.

Next we establish the upper bound for our concept in terms of vertices and domination number of G.

Theorem7: For any Connected (p, q) graph G, $\gamma_{rp}(G) \leq 2p - 3\gamma(G) + 2$.

Proof: Suppose $D \subseteq V(G)$ be the minimal dominating set of G. Further let $H \subseteq V(G)$ and $\forall\ v_i \in V - H$ is adjacent to exactly one vertex of H and $N(v_i) \cap N(v_j) = \emptyset$, $\forall\ i, j \in H$. Then H is minimal perfect dominating set of G. If induced subgraph $\langle H \rangle$ is regular also $N[H] = V(G)$, then H is a regular perfect dominating set of G. If not add the set of vertices $\{v_k\} \in \{V - H\}$ such that $\langle H \cup \{v_k\} \rangle$ is regular. Since $D \subseteq H$ for any connected graph, we have the following result.

$|H \cup \{v_k\}| \leq 2p - 3|D|$+2 gives $\gamma_{rp}(G) \leq 2p - 3\gamma(G) + 2$.

Further we obtained an upper bound for $\gamma_{rp}(G)$ in terms of edges of G.

Theorem8: For any connected (p, q) graph G, $\gamma_{rp}(G) \leq \left\lfloor \frac{2q+1}{2} \right\rfloor$.

Proof: Let $F = \{v_1, v_2, v_3, \ldots \ldots \ldots \ldots \ldots v_n\} \subseteq V(G)$ such that $\forall v_j \in V(G) - F$ is adjacent to exactly one vertex of F and if $|N(u) \cap F| = 1$ for each u $\in V - F$. Since $N[F] = V(G)$, then F is the minimal perfect dominating set of G. If the induced subgraph $\langle F \rangle$ is regular, then F is a γ_{rp}-set of G. Hence $|F| \leq \left\lfloor \frac{2q+1}{2} \right\rfloor$ which gives $\gamma_{rp}(G) \leq \left\lfloor \frac{2q+1}{2} \right\rfloor$.

A dominating set S⊆V(G) is called strong dominating set, if for every vertex $u \in V - S$ and there exists a vertex $v \in S$ with $\deg(v) \geq \deg(u)$ and u is adjacent to v. The strong domination number $\gamma_{st}(G)$ is the minimum cardinality of a minimal strong dominating set, see [6].

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In [2] , defined $\gamma_c^{/}(G)$ such as an edge dominating set $F \subseteq E(G)$ is called a connected edge dominating set, if the edge induced subgraph $\langle F \rangle$ is connected. $\gamma_c^{/}(G)$ is the smallest cardinality of a connected edge dominating set.

Theorem9: For any connected (p, q) graph G, $\gamma_{rp}(G) + p \geq \gamma_c^{/}(G) + \gamma_{st}(G)$+2.

Proof: Suppose D be a minimal dominating set of G. If for every $v_i \in V - D$ is adjacent to atleast one vertex $v_j \in D$ with $\deg(v_j) \geq \deg(v_i)$ and $v_j$is adjacent to v_i. Then D is a minimal strong dominating set of G. Let $E_1 = \{e_1, e_2, e_3, \ldots\ldots, e_n\} \subseteq E(G)$ be the set of all nonend edges in G. Suppose there exists a minimal set of edges such that $N[e_i]$=E(G), $\forall\ e_i \in E_1,\ 1 \leq i \leq n$. Then E_1forms a minimal edge dominating set of G. Further if the induced

subgraph $\langle E_1 \rangle$ has exactly one component, then E_1 itself is a connected edge dominating set of G. Suppose there exists $F \subset V(G)$ such that $\forall v_i \in V - F$ is adjacent to exactly one vertex of F and $N[F] = V(G)$. Thus F be the minimal perfect dominating set of $V(G)$. If the induced subgraph $\langle F \rangle$ is regular, then F is a γ_{rp}- set of G. If not select the set of vertices $\{v_j\}$ from {V-F} which makes $(F \cup \{v_j\})$ is regular. Let $A = \{v_1, v_2, v_3, \ldots\ldots\ldots\ldots\ldots v_n\}$ be the set of vertices incident to the edges of a set E_1. Suppose A_1⊆A and also $A_1 \subset F$. Then $|A_1|$+$|V| \geq |E_1| + |D|$+2 gives $\gamma_{rp}(G) + p \geq \gamma_c^{/}(G) + \gamma_{st}(G)$+2.

A function $f: V \rightarrow \{0,1,2\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex w for which $f(w) = 2$ in G. The weight of Roman dominating function is the value $f(w) = \sum_{u \in w} f(u)$. The minimum weight of Roman dominating function on a graph G is called Roman dominating number of G and is denoted by $\gamma_R(G)$.

Theorem10: For any connected (p, q) graph G, $\gamma_{rp}(G) + \gamma_R(G) \leq p + q - 1$.

Proof: Suppose the function $f: V \rightarrow \{0,1,2\}$ which partition the vertex set V(G) in to (V_0, V_1, V_2) induced by f with $|V_i| = n_i$ for i=0,1,2. Suppose the set V_2dominates V_0. Then $N = V_1 \cup\ V_2$ forms a minimal Roman dominating set of G. Suppose $D \subset N$ and $F \subset V(G) - N$. Then assume $\forall\ v_i \in (D \cup F)$ is adjacent to exactly one vertex of $V(G) - (D \cup F)$ such that $N[(D \cup F)] = V(G)$. Then $\{D \cup F\}$ is γ_p-set of G. If the induced subgraph $\langle (D \cup F) \rangle$ is regular, then

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$(D \cup F)$ is a γ_{rp} −set of G. Hence $|(D \cup F)\ | + |N| \leq p + q - 1$ which gives

$$\gamma_{rp}(G) + \gamma_R(G) \leq p + q - 1.$$

A dominating set F of G is called co total dominating set, if the induced subgraph $\langle V - F \rangle$ has no isolated vertices. The co total domination number $\gamma_{ct}(G)$ is the minimum cardinality of minimal co total dominating set of G, see [7].

The following theorem establish a relationship between the vertices of G, $\gamma_{ct}(G)$, $\beta_1(G)$ with $\gamma_{rp}(G)$.

Theorem11: For any connected (p, q) graph G, $\gamma_{rp}(G) + \gamma_{ct}(G) \leq 2\beta_1(G) + p - 1$.

Proof: Let D be a minimal dominating set of G. Suppose the induced Subgraph $\langle V - D \rangle$ has no isolates. The D is a co total dominating set of G. Further $E(G) = \{e_1, e_2, e_3, \dots \dots \dots \dots, e_n\}$ be the edge set of G and $E^1 = \{e_1^1, e_2^1, e_3^1, \dots \dots \dots \dots, e_n^1\} \subset E(G)$ is a set of maximal edges. Suppose any $e_i^1, e_j^1 \in E^1$, $N(e_i^1) \cap N(e_j^1) = e$ and e$\in E(G) - E^1$. Clearly E^1 is an edge independent set of G with $|E^1|$=$\beta_1(G)$. Since for any connected graph G, $M \subset D$ and $H \subset V(G) - D$. Further assume that $\{M \cup H\}$ is a minimal dominating set of G and every vertex $v_i \in \{M \cup H\}$ is adjacent to exactly one vertex of $V(G) - \{M \cup H\}$. Then $\{M \cup H\}$ forms a minimal perfect dominating set of G. If the induced subgraph $\langle \{M \cup H\} \rangle$ is regular, then $\{M \cup H\}$ is a minimal regular perfect dominating set of G. It is also true that if $C = \{v_1, v_2, v_3, \dots \dots \dots \dots \dots, v_m\}$ be the set of vertices which are incident to the edges of E^1. Hence $M \subset C$, then $|M \cup H| + |D| \leq 2|E^1\ | + p - 1$which gives , $\gamma_{rp}(G) + \gamma_{ct}(G) \leq 2\beta_1(G) + p - 1$.

The next theorem gives relationship between γ_r^1(T(G)), q with $\gamma_{rp}(G)$.

Theorem12: For any connected (p, q) graph G, $\gamma_{rp}(G) + q \geq \gamma_r^1$(T(G))+$\delta(G)$and $G \neq W_n$.

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Proof: Suppose $G = W_n$ .Then $\gamma_{rp}(G) + q \ngeq \gamma_r^{1}$(T(G))+ $\delta(G)$.Hence $G \neq W_n$. Let $E(G) = \{e_1, e_2, e_3, \dots \dots \dots \dots, e_n\}$ be the edge set of G with $|E| = q$ and let $V(G) = \{v_1, v_2, v_3, \dots \dots \dots \dots \dots v_n\}$ be the vertex of G. Suppose there exists at least one vertex $v_k \in V(G)$ of minimum degree $\delta(G)$. In T(G), $V(T(G) = V(G) \cup E(G)$ and suppose $E(T(G)) = \{e_1, e_2, e_3, \dots \dots \dots \dots, e_m\}$ be the edge set of T(G).

If $J \subset E(T(G))$, and $\forall e_i \in J$, $1 \leq i \leq m$ is adjacent to at least one edge of $E(T(G)) - J$ with $N[J] = E(G)$. Then J is a minimal edge dominating set of T(G).

Assume the set J is regular, then J is a regular edge dominating set of T(G). Suppose there exists a dominating set $D \subset V(G)$, such that $\forall v_i \in V - D$ is adjacent to exactly one vertex of D and $N[D] = V(G)$. Clearly D forms a minimal perfect dominating set of G. For regularity, if the induced subgraph $\langle D \rangle$ is regular, then D is a γ_{rp} −set of G. Since $J \subset D$, then $|D| + q \geq |J| + \delta(G)$ gives $\gamma_{rp}(G) + q \geq \gamma_r^{1}$(T(G))+$\delta(G)$.

In [8], the end edge domination number has been defined. An edge dominating set S⊆ $E(G)$ is said to be an end edge dominating set, if S contains all end edges of E(G). The minimum cardinality of edges in such a set is called the end edge domination number of G and is denoted by $\gamma_e^{1}(G)$.

Theorem13: For any connected (p, q) graph G, $\gamma_{rp}(G) + \gamma_e^{1}(G) \leq 2p - 3$.

Proof: We consider two cases.

Case1: If G has end edges. Let $E_1 = \{e_1, e_2, e_3, \dots \dots \dots \dots \dots, e_n\}$ be the set of all end edges in G. Suppose $I = E(G) - E_1$ and $H \subseteq I$ is a minimal edge dominating set of G and $N[H] = E(G)$. Then $E_1 \cup H$ is an end edge dominating set of G. Let $E_2 = \{e_1, e_2, e_3, \dots \dots \dots \dots, e_k\}$ be the set of edges adjacent to the edges of E_1. Then $D_1 = \{v_1, v_2, v_3, \dots \dots \dots \dots \dots v_k\}$ denote the minimal dominating set of subgraph of $\{E_1 \cup E_2\}$ which is incident to the edges of $E_1 \cup E_2$ and let $D_2 = \{v_1, v_2, v_3, \dots \dots \dots \dots \dots v_m\}$ denote the minimal dominating set of subgraph of $E(G) - (E_1 \cup E_2)$=K which is incident

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to the edges of K. Now assume every vertex $v_i \in V(G) - (D_1 \cup D_2)$ is adjacent to exactly one vertex of $D_1 \cup D_2$, then $(D_1 \cup D_2)$ is a minimal perfect dominating set of G. If the induced subgraph $\langle D_1 \cup D_2 \rangle$ is regular, then $(D_1 \cup D_2)$ is regular perfect dominating set of G.Hence $|D_1 \cup D_2| + |E_1 \cup H| \leq 2p - 3$ gives $\gamma_{rp}(G) + \gamma_e^1(G) \leq 2p - 3$.

Case2: If G has no end edges, then $E_1 = \{\emptyset\}$. Clearly from the above case1, we have $|D_1 \cup D_2| + |H| \leq 2p - 3$. Hence $\gamma_{rp}(G) + \gamma_e^1(G) \leq 2p - 3$.

CONCLUSION: In this paper we surveyed selected results on Regular Perfect domination in graph. These results establish key relationship between the Regular perfect domination number and other parameters, including the domination number, the edge domination number, split domination number and entire domination number of a simple, and undirected graph.

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