CUBE PERFECT FUZZY MATCHING FOR FUZZY GRAPH BASED ON VERTICES

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Abstract.

Many real-world problems are represented using Graph theory. Graphs are the models of relations. A graph is an appropriate way of representing information involving relationship between objects where the objects are represented by vertices and the relations are represented by edges. We need to design a fuzzy graph model, when there is vagueness in the description of the objects or in its relationships or in both. In this paper, we introduced **Cube** Fuzzy matching and **Cube** perfect Fuzzy matching for Fuzzy graph based on vertices. We proved the necessary condition under which they are equivalent and also proved that, for a particular condition, a perfect cube Fuzzy matching is not a $(\mathbf{3},\mathbf{k})$ regular Fuzzy graph. We also discussed perfect **Cube** Fuzzy matching and **Cube** Fuzzy matching number for cycle graph.

Keywords: Fuzzy graph, **Cube** Fuzzy matching, **Cube** Perfect Fuzzy matching, **Cube** Fuzzy matching number

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1. Introduction

The concept of Fuzzy sets and Fuzzy relations was introduced by L. A.Zadeh [8] in the year 1965. The concept of Fuzzy graph was introduced by Rosenfeld [4] in the year 1975. The concept of regular Fuzzy graphs and regular property ofFuzzy graphs was introduced by Nagoor Gani and Radha[2][3]. New approach on vertex regular Fuzzy graph was introduced by Kailash Kumar Kakkad and Sanjay Sharma [1]. Shakila Banu and Akilandeswari[7] introduced the concept of square perfect Fuzzy matching. Seethalakshmi and Gnanajothi[5] derived the necessary condition for a Fuzzy graph on a cycle. The concept of d_2-*degree* and total d_2-*degree* of a vertex in a Fuzzy graph was defined by Sekar and Santhimaheswari [6].

2. Preliminaries

Definition2.1:

A fuzzy graph denoted by $G:(\sigma,\mu)$ on $G^*:(V,E)$ is a pair of functions (σ,μ) where $\sigma: V \to [0,1]$ is a fuzzy subset of a non-empty set V and $\mu: V\ X\ V\ \to [0,1]$is a symmetric fuzzy relation on σ such that for all u,v in V the relation $\mu(u,v) = \mu(uv) \le \sigma(u) \wedge \sigma(v)$ is satisfies, where σ and μ are called membership functions. A fuzzy graph G is complete if $\mu(u,v) = \mu(uv) = \sigma(u) \wedge \sigma(v)$ where uv

denotes the edge between u and v. $G^*:(V,E)$ Is called the underlying crisp graph of the fuzzy graph $G:(\sigma,\mu)$.

Definition2.2:

Let $G:(\sigma,\ \mu)$ be a fuzzy graph. The degree of a vertex u is

$$\mathrm{d(u)} = \sum_{u\neq v} \mu(uv)$$

since $\mu(uv) > 0$ for $uv \in E$ and $\mu(uv) = 0$ for $uv \notin E$.

The minimum degree of G is $\delta(G) = \wedge \{d(u)/u \in V\}$.

The maximum degree of G is $\Delta(G) = \vee \{d(u)/u \in V\}$.

Definition2.3:

For a given graph G, the d_3 degree of a vertex u in G denoted by $d_3(u)$ means the number of vertices at a distance 3 away from u.

Definition2.4:

For a given fuzzy graph G, the d_3 degree of a Vertex u is

$$d_3(\mathrm{u}) = \sum_{\substack{u\neq v \\ u,v\in V}} \mu^3(uv)$$

Where,

$\mu^3(uv) = \{\mu(uv_1) \wedge \mu(v_{1,}v_2) \wedge \mu(v_{2,}v)\}$.

Also $\mu(uv) = 0$ for $uv \notin E$.

The minimum d_3 degree of G is $\delta_3(G) = \wedge \{d_3(u)/u \in V\}$.

The maximum d_3 degree of G is $\Delta_3(G) = \vee \{d_3(u)/u \in V\}$.

Definition2.5:

A fuzzy graph G is said to be $(3,k)$ regular or d_3 regular if $d_3(\mathrm{u}) = k$ for all u in G.

3. ***Cube*** Fuzzy Matching

Definition 3.1:

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. a subset S of V is called a ***Cube*** fuzzy matching if for each vertex u we have,

$$\sum_{\substack{u\neq v \\ u,v\in V}} \mu^3(uv) \leq \sigma(u)$$

Example 3.1

Let $\mathrm{G}:(\sigma,\mu)$ Be a fuzzy graph on the cycle $G^*:(V,E)$ where $V = \{v_1,v_2,v_3,v_4,v_5,v_6\}$ and $E = \{e_1,e_2,e_3,e_4,e_5,e_6\}$ with $e_1 = v_1v_2$, $e_2 = v_2v_3$, $e_3 = v_3v_4$, $e_4 = v_4v_5$, $e_5 = v_5v_6$, $e_6 = v_6v_1$

$\sigma(v_1)$ = 0.4, $\sigma(v_2)$ = 0.3, $\sigma(v_3)$ = 0.5, $\sigma(v_4)$ = 0.8, $\sigma(v_5)$ = 0.2, $\sigma(v_6)$ = 0.5

$\mu(e_1)$ =0.2, $\mu(e_2)$ =0.3, $\mu(e_3)$=0.4, $\mu(e_4)$ = 0.2, $\mu(e_5)$=0.1, $\mu(e_6)$=0.35.

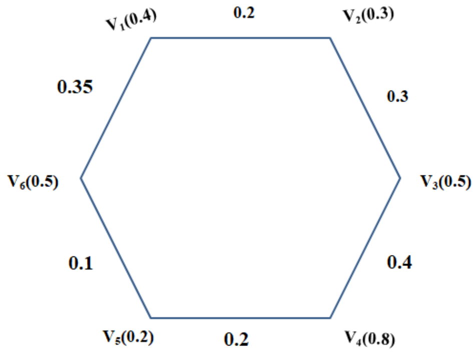

Figure 1: ***Cube*** fuzzy matching

$$\sum_{\substack{v_1 \neq v_4 \\ v_1, v_4 \in V}} \mu^3(v_1 v_4) = \mu^3(v_1 v_4) + \mu^3(v_1 v_4)$$

$= \{\mu(v_1\, v_2) \wedge \mu(v_2 v_3) \wedge \mu(v_3 v_4)\} + \{\mu(v_1\, v_6) \wedge \mu(v_6 v_5) \wedge \mu(v_5 v_4)\}$

$= \{0.2 \wedge 0.3 \wedge 0.4\} + \{0.35 \wedge 0.1 \wedge 0.2\}$

$= 0.2 + 0.1$

$= 0.3 \leq \sigma(v_1)$

$$\sum_{\substack{v_2 \neq v_5 \\ v_2, v_5 \in V}} \mu^3(v_2 v_5) = 0.2 + 0.1 = 0.3 \leq \sigma(v_2)$$

$$\sum_{\substack{v_3 \neq v_6 \\ v_3, v_6 \in V}} \mu^3(v_3 v_6) = 0.1 + 0.2 = 0.3 \leq \sigma(v_3)$$

$$\sum_{\substack{v_4 \neq v_1 \\ v_4, v_1 \in V}} \mu^3(v_4 v_1) = 0.1 + 0.2 = 0.3 \leq \sigma(v_4)$$

$$\sum_{\substack{v_5 \neq v_2 \\ v_5, v_2 \in V}} \mu^3(v_5 v_2) = 0.1 + 0.2 = 0.3 \nleq \sigma(v_5)$$

$$\sum_{\substack{v_6 \neq v_3 \\ v_6, v_3 \in V}} \mu^3(v_6 v_3) = 0.2 + 0.1 = 0.3 \leq \sigma(v_6)$$

Thus $S = \{v_1, v_2, v_3, v_4, v_6\}$ is a ***Cube*** Fuzzy matching in$G$.

Definition 3.2:

A ***Cube*** Fuzzy matching S is called a ***Cube*** Perfect fuzzy matching if,

$$\sum_{\substack{u \neq v \\ u, v \in V}} \mu^3(uv) = \sigma(u)$$

Definition 3.3:

Let G: (σ, μ) be a Fuzzy graph and S be a ***Cube*** fuzzy matching. Then ***Cube*** fuzzy matching number $\Gamma(G)$ is defined to be

$$\Gamma(G) = \sum_{u \epsilon S} \mu^3(uv)$$

Example 3.2

In this examplewe have considered Figure 1 and Example 3.1.

$\Gamma(G) = \sum_{u \epsilon S} \mu^3(uv)$

Where $S = \{v_1, v_2, v_3, v_4, v_6\}$ $\Gamma(G) = 1.5$

Theorem3.1:

Let $G: (\sigma, \mu)$ be a Fuzzy graph on the cycle$G^*: (V, E)$. Then edges of the Fuzzy graph of G is half of their vertices iff, all the vertices of G are ***Cube*** perfect Fuzzy matching and is equivalent to $(3, k)$ regular fuzzy graph.

Proof:

Suppose that σ is a constant function.

Let $\sigma(u) = k$ is a constant for all $u \epsilon V$ and$\mu(uv) = \frac{k}{2}$, for all $(uv) \epsilon E$

Assume that G is a $(3, k)$ regular Fuzzy graph on the cycle$G^*: (V, E)$.

Then $d_3(u) = k$

By definition of d_3 degree of a vertex in Fuzzy graph

$$d_3(u) = \sum_{\substack{u \neq v \\ u,v \in V}} \mu^3(uv)$$

That is,

$$\sum_{\substack{u \neq v \\ u,v \in V}} \mu^3(uv) = k$$

Since G is a $(3, k)$ regular Fuzzy graph

$$\Rightarrow \sum_{\substack{u \neq v \\ u,v \in V}} \mu^3(uv) = \sigma(\mu)$$

Therefore Each vertex of u satisfies the ***Cube*** Perfect Fuzzy matching in$G$.

Now suppose that V is a perfect Fuzzy matching in$G$.

Since G is a Fuzzy graph on the cycle and only two edges are incident with each vertex for cycles.

For any vertex $u \epsilon V$

$$\Rightarrow \sum_{\substack{u \neq v \\ u,v \in V}} \mu^3(uv) = \sigma(u)$$

By definition,

$$d_3(u) = \sum_{\substack{u \neq v \\ u,v \in V}} \mu^3(uv)$$

$$\Rightarrow d_3(u) = \sigma\,(u)$$

$$\Rightarrow d_3(u) = k$$

Hence G is $(3, k)$ regular Fuzzy graph on the cycle.

The converse of the theorem is trivially true.

Example 3.3

Let G: (σ, μ) be a Fuzzy graph on the cycle $G^* : (V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ with$e_1 = v_1v_2$, $e_2 = v_2v_3$, $e_3 = v_3v_4$, $e_4 = v_4v_5$,

$e_5 = v_5v_6$, $e_6 = v_6v_1$

$\sigma(v_1)$ = 0.6, $\sigma(v_2)$ = 0.6, $\sigma(v_3)$ = 0.6, $\sigma(v_4)$ = 0.6, $\sigma(v_5)$ = 0.6, $\sigma(v_6)$ = 0.6

$\mu(e_1)$ =0.3, $\mu(e_2)$ =0.3, $\mu(e_3)$=0.3, $\mu(e_4)$=0.3, $\mu(e_5)$=0.3, $\mu(e_6)$=0.3

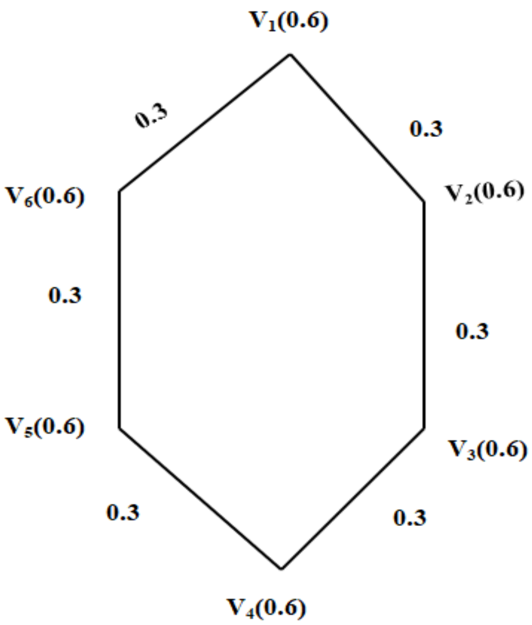

Figure 2: (3, 0.6) - regular fuzzy graph

$$\sum_{\substack{v_1 \neq v_4 \\ v_1, v_4 \in V}} \mu^3(v_1v_4) = \mu^3(v_1v_4) + \mu^3(v_1v_4)$$

$= \{\mu(v_1\,v_2) \wedge \mu(v_2v_3) \wedge \mu(v_3v_4)\} + \{\mu(v_1\,v_6) \wedge \mu(v_6v_5) \wedge \mu(v_5v_4)\}$

$= \{0.3 \wedge 0.3 \wedge 0.3\} + \{0.3 \wedge 0.13 \wedge 0.3\}$

$=0.3 + 0.3$

$=0.6 = \sigma(v_1)$

$$\sum_{\substack{v_2 \neq v_5 \\ v_2, v_5 \in V}} \mu^3(v_2v_5) = 0.3 + 0.3 = 0.6 = \sigma(v_2)$$

$$\sum_{\substack{v_3 \neq v_6 \\ v_3, v_6 \in V}} \mu^3(v_3v_6) = 0.3 + 0.3 = 0.6 = \sigma(v_3)$$

$$\sum_{\substack{v_4 \neq v_1 \\ v_4, v_1 \in V}} \mu^3(v_4v_1) = 0.3 + 0.3 = 0.6 = \sigma(v_4)$$

$$\sum_{\substack{v_5 \neq v_2 \\ v_5, v_2 \in V}} \mu^3(v_5 v_2) = 0.3 + 0.3 = 0.6 = \sigma(v_5)$$

$$\sum_{\substack{v_6 \neq v_3 \\ v_6, v_3 \in V}} \mu^3(v_6 v_3) = 0.3 + 0.3 = 0.6 = \sigma(v_6)$$

Hence G is a ***Cube*** perfect Fuzzy matching and also $(3, 0.6)$ regular Fuzzy graph.

Theorem3.2:

Let G: (σ, μ) be a $(2, k)$ regular Fuzzy graph on the cycle$G^*: (V, E)$. $\sigma(u)$ And $\mu(uv)$are constant functions where $\mu(uv) \leq \sigma(u)$ and $\mu(uv) \neq \frac{1}{2}\sigma(u)$ for all $(uv) \epsilon E$ then V is not a Cube perfect Fuzzy matching of$G$.

Proof:

Let $\sigma(u)$ = k and $\mu(uv) = c$ are constant functions for all $u, v \epsilon V$ where $c \leq k$ and $c \neq k/2$

To prove V is not a ***Cube*** perfect Fuzzy matching of$G$.

Suppose that G is a Fuzzy graph on the cycle and only two edges are incident with each vertex for the cycles.

For any vertex$u \epsilon V$,

$$\Rightarrow \sum_{\substack{u \neq w \\ u, w \in V}} \mu^3(uw) = \mu^3(uw) + \mu^3(uv)$$

$$= c + c$$

$$= 2c \leq k$$

$$= c \leq k/2 \text{ But } c \neq k/2$$

$$= c < k/2$$

$$\Rightarrow \sum_{\substack{u \neq w \\ u, w \in V}} \mu^3(uw) \neq k$$

$$\Rightarrow \sum_{\substack{u \neq w \\ u, w \in V}} \mu^3(uw) \neq \sigma(u)$$

Hence V is not a ***Cube*** perfect Fuzzy matching of$G$.

Example 3.4

Let $G: (\sigma, \mu)$ be a Fuzzy graph on the cycle $G^*: (V, E)$where $V = \{v_1, v_2, v_3, v_4\}$and $E = \{e_1, e_2, e_3, e_4\}$ with$e_1 = v_1 v_2$, $e_2 = v_2 v_3$, $e_3 = v_3 v_4$, $e_4 = v_4 v_1$

$\sigma(v_1)$ = 0.6, $\sigma(v_2)$ = 0.6, $\sigma(v_3)$ = 0.6, $\sigma(v_4)$ = 0.6

$\mu(e_1)$ =0.2, $\mu(e_2)$ =0.2, $\mu(e_3)$=0.2, $\mu(e_4)$=0.2

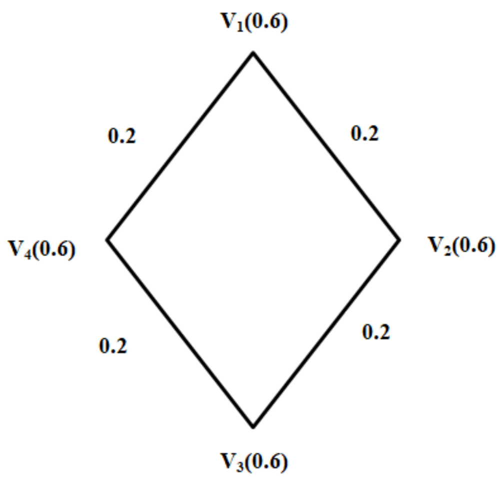

Figure 3: Not a ***Cube*** Perfect fuzzy matching

$$\sum_{\substack{v_1 \neq v_4 \\ v_1, v_4 \in V}} \mu^3(v_1 v_4) = 0.2 + 0.2 = 0.4 \neq \sigma(v_1)$$

$$\sum_{\substack{v_2 \neq v_1 \\ v_2, v_6 \in V}} \mu^3(v_2 v_1) = 0.2 + 0.2 = 0.4 \neq \sigma(v_2)$$

$$\sum_{\substack{v_3 \neq v_2 \\ v_3, v_2 \in V}} \mu^3(v_3 v_2) = 0.2 + 0.2 = 0.4 \neq \sigma(v_3)$$

$$\sum_{\substack{v_4 \neq v_3 \\ v_4, v_3 \in V}} \mu^3(v_4 v_3) = 0.2 + 0.2 = 0.4 \neq \sigma(v_4)$$

Hence G is not a ***Cube*** Perfect Fuzzy matching.

Theorem3.3:

Let G be a ***Cube*** Perfect Fuzzy matching on the cycle $G^*:(V,E)$ of length$n \geq 5$. If

$$\sigma(u_i) = \begin{cases} k/_4 \ , i = 1,2, n-1, n \\ \quad k/_3 \,, i = 3,4, \dots n-2 \\ \quad for\ all\ u \in V \end{cases}$$

And

$$\mu(e_i) = \begin{cases} k/_6 \ , i = 1,2,3,4, \dots n-1 \\ \quad k/_{12} \,, i = n \\ \quad for\ all (uv) \in E \end{cases}$$

Then G is not $(3,k)$ regular Fuzzy graph.

Proof:

Let $G:(\sigma,u)$ be a ***Cube*** Perfect Fuzzy matching on the Cycle $G^*:(V,E)$is any length≥ 5.

To prove that G is not a $(3,k)$ regular Fuzzy graph.

Let $v_1, v_2, v_3, \dots \dots . v_n$ *be the vertices and* $e_1, e_2, \dots \dots\ e_n$ be edges of a cycle onG^* in that order.

By definition,

$$d_3(u) = \sum_{\substack{u \neq v \\ u,v \in V}} \mu^3(uv)$$

$$d_3(v_1) = \sum_{\substack{v_1 \neq v_4 \\ v_1,v_4 \in V}} \mu^3(v_1v_4)$$

$$= \mu^3(v_1v_4) + \mu^3(v_1v_{n-2})$$

$$= \{\mu(v_1\, v_2) \wedge \mu(v_2v_3) \wedge \mu(v_3v_4)\} + \{\mu(v_1\, v_n) \wedge \mu(v_nv_{n-1}) \wedge \mu(v_{n-1}v_{n-2})\}$$

$$= \{\mu(e_1) \wedge \mu(e_2) \wedge \mu(e_3)\} + \{\mu(e_n) \wedge \mu(e_{n-1}) \wedge \mu(e_{n-2})\}$$

$$= \left\{{}^{k}\!/_{6} \wedge {}^{k}\!/_{6} \wedge {}^{k}\!/_{6}\right\} + \left\{{}^{k}\!/_{12} \wedge {}^{k}\!/_{6} \wedge {}^{k}\!/_{6}\right\}$$

$$= {}^{k}\!/_{6} + {}^{k}\!/_{12} = {}^{k}\!/_{4} = \sigma(v_1)$$

$$d_3(v_2) = \{\mu(e_2) \wedge \mu(e_3) \wedge \mu(e_4)\} + \{\mu(e_1) \wedge \mu(e_n) \wedge \mu(e_{n-1})\}$$

$$= \left\{{}^{k}\!/_{6} \wedge {}^{k}\!/_{6} \wedge {}^{k}\!/_{6}\right\} + \left\{{}^{k}\!/_{6} \wedge {}^{k}\!/_{12} \wedge {}^{k}\!/_{6}\right\}$$

$$= {}^{k}\!/_{6} + {}^{k}\!/_{12} = {}^{k}\!/_{4} = \sigma(v_2)$$

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$$d_3(v_i) = \{\mu(e_i) \wedge \mu(e_{i+1}) \wedge \mu(e_{i+2})\} + \{\mu(e_{i-1}) \wedge \mu(e_{i-2}) \wedge \mu(e_{i-3})\}$$

$$= \left\{{}^{k}\!/_{6} \wedge {}^{k}\!/_{6} \wedge {}^{k}\!/_{6}\right\} + \left\{{}^{k}\!/_{6} \wedge {}^{k}\!/_{6} \wedge {}^{k}\!/_{6}\right\}$$

$$= {}^{k}\!/_{6} + {}^{k}\!/_{6} = {}^{k}\!/_{3} = \sigma(v_i)$$

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$$d_3(v_{n-1}) = \{\mu(e_{n-1}) \wedge \mu(e_n) \wedge \mu(e_{n+1})\} + \{\mu(e_{n-2}) \wedge \mu(e_{n-3}) \wedge \mu(e_{n-4})\}$$

$$= \left\{{}^{k}\!/_{6} \wedge {}^{k}\!/_{12} \wedge {}^{k}\!/_{6}\right\} + \left\{{}^{k}\!/_{6} \wedge {}^{k}\!/_{6} \wedge {}^{k}\!/_{6}\right\}$$

$$= {}^{k}\!/_{12} + {}^{k}\!/_{6} = {}^{k}\!/_{4} = \sigma(v_{n-1})$$

$$d_3(v_n) = \{\mu(e_n) \wedge \mu(e_1) \wedge \mu(e_2)\} + \{\mu(e_{n-1}) \wedge \mu(e_{n-2}) \wedge \mu(e_{n-3})\}$$

$$= \left\{{}^{k}\!/_{12} \wedge {}^{k}\!/_{6} \wedge {}^{k}\!/_{6}\right\} + \left\{{}^{k}\!/_{6} \wedge {}^{k}\!/_{6} \wedge {}^{k}\!/_{6}\right\}$$

$$= {}^{k}\!/_{12} + {}^{k}\!/_{6} = {}^{k}\!/_{4} = \sigma(v_n)$$

Hence G is not a$(3, k)$ regular Fuzzy graph

2. Conclusion

In this paper, we have introduced **Cube** perfect Fuzzy matching for Fuzzy graph based on vertices on the cycle. We proved the necessary condition under which they are equivalent and also proved that, for a particular condition, a **Cube** perfect Fuzzy matching is not a $(\mathbf{3}, \mathbf{k})$regular Fuzzy graph. We also discussed **Cube** perfect Fuzzy matching and **Cube** Fuzzy matching number for cycle graph. For the future work we can extend to complete graph regular graph and finally for any connected graph.

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