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HOMOMORPHISM AND ANTI-HOMOMORPHISM OF Q-INTUITIONISTIC L-FUZZY NORMAL ℓ-SUBSEMIRING OF AN ℓ-SEMIRING

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ABSTRACT: In this paper, we introduce the notion of Q-intuitionistic L -fuzzy ℓ -subsemiring of a ℓ -semiring is introduced in this study. We attempted to investigate the algebraic character of ℓ -semiring. We also studied the primary theorem for homomorphism and anti-homomorphism, as well as some aspects of Q-intuitionistic L -fuzzy normal ℓ -subsemiring of an ℓ -semiring.

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KEY WORDS: fuzzy subset, (Q, L)-fuzzy subset, (Q,L)-fuzzy ℓ-subsemiring, Q-intuitionistic L-fuzzy subset, Q-intuitionistic L-fuzzy normal ℓ-subsemiring, Q-intuitionistic L-fuzzy relation, Product of Q-intuitionistic L-fuzzy subsets, Q-intuitionistic L-fuzzy characteristic ℓ-subsemiring.

INTRODUCTION: Following L.A.Zadeh's presentation of fuzzy sets [25], various academics investigated the generalisation of the concept of fuzzy sets. Dedekind first defined the concept of lattice in 1897, and Birkhofft, G.,[7,8] further extended it. Boole created Boolean algebra, which was comparable to a special type of lattice called a Boolean ring with identity. This connection was established between lattice theory and modern algebra as a result of this relationship. K.T.Atanassov [5,6] proposed the concept of intuitionistic fuzzy subset as a hypothesis on the concept of fuzzy set. Abou Zaid.S [1] proposed the concept of fuzzy subnearings and ideals.

Another mathematical design known as Q -fuzzy subgroups was developed and characterised by A.Solairaju and R.Nagarajan [21,22]. We introduce certain features of Q -intuitionistic L-fuzzy normal ℓ -subsemiring of a ℓ -semiring in this study.

1.PRELIMINARIES:

1.1 Definition [25]: Let X be a set that isn't empty. A function μ_A:X→[0, 1] a **fuzzy subset μ_A** of X.

1.2 Definition [21,22]: Let X be a set that isn't empty and L = (L,≤) be a lattice with least element 0 and greatest element 1 and Q be a set that isn't empty. A **(Q, L)-fuzzy subset μ_A** of a function μ_A: X× Q → L.

1.3 Definition [17,18]: Let R be a ℓ-semiring and Q be a set that isn't empty. A (Q, L)-fuzzy subset A of R is referred to as a **(Q, L)-fuzzy ℓ-subsemiring (QLFLSSR)** of R if it meets the following criteria:

(i) μ_A (x+y, q) ≥ μ_A (x, q) ∧ μ_A (y, q),

(ii) μ_A (xy, q) ≥ μ_A (x, q) ∧ μ_A (y, q),

(iii) μ_A (x∨y, q) ≥ μ_A (x, q) ∧ μ_A (y, q),

(iv) μ_A (x∧y, q) ≥ μ_A (x, q) ∧ μ_A (y, q),for every x and y in R and q in Q.

1.1 Example: Let (Z , + , ●,∨,∧) be a ℓ-semiring and Q ={p}, Then the (Q,L)-Fuzzy Set A of Z is defined by

$$A(x, q) = \begin{cases} 1 & if \quad x = 0 \\ 0.33 & if \quad x \epsilon < 2 > -0 \\ 0 & if \quad x \epsilon\, Z - < 2 > \end{cases}$$

A is unmistakably a (Q,L)-Fuzzy ℓ-subsemiring of a ℓ-semiring.

1.4 Definition [5,6]: An **intuitionistic fuzzy subset (IFS)** A in X is defined as an object of the form A ={⟨x, A_μ (x), A_ϑ (x)⟩ / x∈X}, where A_μ : X → [0,1] and A_ϑ : X → [0,1] define the degree of membership and the degree of non-membership of the element x∈ X respectively and for every x ∈ X satisfying 0 ≤ A_μ(x) + A_ϑ(x) ≤ 1.

1.5 Definition [19]: Let (L, ≤) be a complete lattice with an involutive order reversing operation N : L → L and Q be a set that isn't empty. A Q **-intuitionistic L-fuzzy subset (QILFS)** A in X is defined as an object of the form A={<(x, q), A_μ (x, q), A_ϑ (x, q)>/ x in X and q in Q }, where A_μ:X× Q →L and

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A_ϑ:X× Q →L define the degree of membership and the degree of non-membership of the element x∊X respectively and for every x∊X satisfying A_μ (x) ≤ N(A_ϑ (x)).

1.6 Definition [19]: Let Ʀ be a ℓ-semiring. A Q -intuitionistic L-fuzzy subset A of Ʀ is referred to as a Q -i**ntuitionistic L-fuzzy** ℓ**-subsemiring (QILFLSSR)** of Ʀ if it meets the following criteria:

(i) A_μ (x+y, q) ≥ A_μ (x, q) ∧ A_μ (y, q),

(ii) A_μ (xy, q) ≥ A_μ (x, q) ∧ A_μ (y, q),

(iii) A_μ (x∨y, q) ≥ A_μ (x, q) ∧ A_μ (y, q),

(iv) A_μ (x∧y, q) ≥ A_μ (x, q) ∧ A_μ (y, q),

(v) A_ϑ (x+y, q) ≤ A_ϑ (x, q) ∨ A_ϑ (y, q),

(vi) A_ϑ (xy, q) ≤ A_ϑ (x, q) ∨ A_ϑ (y, q),

(vii) A_ϑ (x∨y, q) ≤ A_ϑ (x, q) ∨ A_ϑ (y, q),

(viii) A_ϑ (x∧y, q) ≤ A_ϑ (x, q) ∨ A_ϑ (y, q) , for every x and y ∊ R and q ∊ Q.

1.2 Example: Let (Z , + , ●,∨,∧) be a ℓ-semiring and Q ={p}, Then Q -intuitionistic L-Fuzzy subset

A={< (x, q), A_μ (x, q), A_ϑ (x, q) >/ x in Z and q in Q } of Z is defined by

A_μ (x, q) = $\begin{cases} 0.6 \; if \; x \in <2> \\ 0.3 \; otherwise \end{cases}$

and

A_ϑ (x, q) = $\begin{cases} 0.4 \; if \; x \in <2> \\ 0.7 \; otherwise \end{cases}$

A is unmistakably a Q -intuitionistic L-Fuzzy ℓ-subsemiring of a ℓ-semiring.

1.7 Definition Let say Ʀ be a ℓ-semiring. A Q-intuitionistic L-fuzzy subset A of Ʀ is referred to as a Q**-intuitionistic** L**-fuzzy normal** ℓ**-subsemiring (QILFNLSSR)** of Ʀ if it meets the following criteria:

(i) $A_\mu(x+y,q) = A_\mu(y+x,q)$,

(ii) $A_\mu(xy,q) = A_\mu(yx,q)$,

(iii) $A_\mu(x \vee y,q) = A_\mu(y \vee x,q)$,

(iv) $A_\mu(x \wedge y,q) = A_\mu(y \wedge x,q)$,

(v) $A_\vartheta(x+y,q) = A_\vartheta(y+x,q)$,

(vi) $A_\vartheta(xy,q) = A_\vartheta(yx,q)$,

(vii) $A_\vartheta(x \vee y,q) = A_\vartheta(y \vee x,q)$,

(viii) $A_\vartheta(x \wedge y,q) = A_\vartheta(y \wedge x,q)$, for every x and $y \in \text{Ɽ}$ and $q \in Q$.

1.3 Example Let L say be the complete lattice and $A: Z \rightarrow L$ be an Q-intuitionistic L-fuzzy subset $A = \{< (x,q), A_\mu(x,q), A_\vartheta(x,q) > / \; x \in X, q$ in $Q\}$defined as

$$A_\mu \text{ (x,}q\text{)} = \begin{cases} 0.79 & if \quad x = <4> \\ 0.31 & if \quad x\epsilon <2> - <4> \\ 0 & if \quad otherwise \end{cases}$$

and

$$A_\vartheta \text{ (x,}q\text{)} = \begin{cases} 0.27 & if \quad x = <4> \\ 0.75 & if \quad x\epsilon <2> - <4> \\ 1 & if \quad otherwise \end{cases}$$

A is unmistakably a Q -intuitionistic L-fuzzy normalℓ-subsemiring.

1.8 Definition Let A and B represent any two Q -intuitionistic pairs. Normal ℓ-subsemiring of a ℓ-semiring G and H, respectively, with L-fuzzy normal ℓ-subsemiring. The product of A and B, designated by $A \times B$, is defined as $A \times B = \{< (x,y),q), (A \times B)_\mu\big((x,y),q\big), (A \times B)_\vartheta\big((x,y),q\;\big) >$/for every x in G and y in $H$and q in $Q\}$, where$(A \times B)_\mu\big((x,y),q\big) = A_\mu(x,q) \wedge B_\mu(y,q)$ and $(A \times B)_\vartheta\big((x,y),q\big) = A_\vartheta(x,q) \vee B_\vartheta(y,q)$.

1.9 Definition Let $(\text{Ɽ}, +, \bullet, \vee, \wedge)$ and $\big(\text{Ɽ}^{|}, +, \bullet, \vee, \wedge\big)$ be any two semirings. Let $f: \text{Ɽ} \rightarrow \text{Ɽ}^{|}$ be any function and A be a Q-intuitionistic L-fuzzy normal ℓ-subsemiring in Ɽ, V be an Q-intuitionistic L-fuzzy normal ℓ-subsemiring in $f(\text{Ɽ}) = \text{Ɽ}^{|}$, defined by $V_\mu(y,q) = \sup_{x \in f^{-1}(y)} A_\mu\,(x,q)$ and $V_\vartheta(y,q) = \inf_{x \in f^{-1}(y)} A_\vartheta\,(x,q)$, for every x in Ɽ and y in $\text{Ɽ}^{|}$ and q in the Q. Then A is known as a V preimage under f and is indicated by $f^{-1}(V)$.

1.10 Definition Let $(\text{Ɽ}, +, \bullet, \vee, \wedge)$ be a ℓ-semiring and Q be a non-empty set. An Q-intuitionistic L-fuzzy ℓ-subsemiring A of R is said to be an Q-intuitionistic L-fuzzy characteristic ℓ-subsemiring (QILFCSLSR) of R if $\mu_A(x,q) = \mu_A(f(x),q)$and $\nu_A(x,q) = \nu_A(f(x),q)$,for all x in R and f in Aut-R and q in Q.

2. Some Properties of Q-Intuitionistic L-Fuzzy Normal ℓ-Subsemiring of a ℓ-Semiring:

2.1 Theorem: Let A be an Q-intuitionistic$L$-fuzzy subset in a ℓ-semiring Ɽ and V be the strongest Q-intuitionistic$L$-fuzzy relation on R. Then A is an Q-intuitionistic$L$-fuzzy normal ℓ-subsemiring of Ɽ if and only if V is an Q-intuitionistic$L$-fuzzy normal ℓ-subsemiring of Ɽ × Ɽ.

Proof: Suppose that A is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring Ɽ. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in Ɽ × Ɽ. Clearly V is an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring Ɽ. We have, $V_\mu(x + y, q) = V_\mu[(x_1, x_2) + (y_1, y_2), q]$ $= V_\mu\big((x_1 + y_1, x_2 + y_2), q\big) = A_\mu(x_1 + y_1, q) \wedge A_\mu(x_2 + y_2, q) = A_\mu(y_1 + x_1, q) \wedge A_\mu(y_2 + x_2, q) = V_\mu(y_1 + x_1, q, y_2 + x_2, q) = V_\mu[(y_1, y_2, q) + (x_1, x_2, q)] = V_\mu(y + x, q)$ Therefore, $V_\mu(x + y, q) = V_\mu(y + x, q)$, for all x and y in Ɽ × Ɽ and q in Q. And, $V_\mu(xy, q) = V_\mu[(x_1, x_2)(y_1, y_2), q] = V_\mu((x_1, x_2)(y_1, y_2), q) = A_\mu(x_1y_1, q) \wedge A_\mu(x_2y_2, q) = A_\mu(y_1x_1, q) \wedge A_\mu(y_2x_2, q) = V_\mu\big((y_1x_1,\ y_2\, x_2), q\big) = V_\mu[(y_1, y_2)(x_1, x_2), q] = V_\mu(yx, q)$ Therefore, $V_\mu(xy, q) = V_\mu(yx, q)$, for all x and y in Ɽ × Ɽ and q in Q. Also, $V_\mu(x \vee y, q) = V_\mu[(x_1, x_2) \vee (y_1, y_2), q]$ $= V_\mu\big((x_1 \vee y_1, x_2 \vee y_2), q\big) = A_\mu(x_1 \vee y_1, q) \wedge A_\mu(x_2 \vee y_2, q) = A_\mu(y_1 \vee x_1, q) \wedge A_\mu(y_2 \vee x_2, q)$ $= V_\mu(y_1 \vee x_1, q, y_2 \vee x_2, q) = V_\mu[(y_1, y_2, q) \vee (x_1, x_2, q)] = V_\mu(y \vee x, q)$ Therefore, $V_\mu(x \vee y, q) = V_\mu(y \vee x, q)$, for all x and y in Ɽ × Ɽ and q in Q. And, $V_\mu(x \wedge y, q) = V_\mu[(x_1, x_2) \wedge (y_1, y_2), q] = V_\mu\big((x_1 \wedge y_1, x_2 \wedge y_2), q\big) = A_\mu(x_1 \wedge y_1, q) \wedge A_\mu(x_2 \wedge y_2, q) = A_\mu(y_1 \wedge x_1, q) \wedge A_\mu(y_2 \wedge x_2, q) = V_\mu(y_1 \wedge x_1, q, y_2 \wedge x_2, q) = V_\mu[(y_1, y_2, q) \wedge (x_1, x_2, q)] = V_\mu(y \wedge x, q)$ Therefore, $V_\mu(x \wedge y, q) = V_\mu(y \wedge x, q)$, for all x and y in Ɽ × Ɽ and q in Q. We have, $V_\vartheta(x + y, q) = V_\vartheta[(x_1, x_2) + (y_1, y_2), q] = V_\vartheta\big((x_1 + y_1, x_2 + y_2), q\big) = A_\vartheta(x_1 + y_1, q) \vee A_\vartheta(x_2 + y_2, q) = A_\vartheta(y_1 + x_1, q) \vee A_\vartheta(y_2 + x_2, q) = V_\vartheta\big((y_1 + x_1, y_2 + x_2), q\big) = V_\vartheta[(y_1, y_2) + (x_1, x_2), q] = V_\vartheta(y + x, q)$ Therefore, $V_\vartheta(x + y, q) = V_\vartheta(y + x, q)$, for all x and y in Ɽ × Ɽ and q in Q. And, $V_\vartheta(xy, q) = V_\vartheta[(x_1, x_2)(y_1, y_2), q] = V_\vartheta\big((x_1y_1, x_2y_2), q\big) = A_\vartheta(x_1y_1, q) \vee A_\vartheta(x_2y_2, q) = A_\vartheta(y_1x_1, q) \vee A_\vartheta(y_2x_2, q) = V_\vartheta\big((y_1x_1, y_2x_2), q\big) = V_\vartheta[(y_1, y_2)(x_1, x_2), q] = V_\vartheta(yx, q)$ Therefore, $V_\vartheta(xy, q) = V_\vartheta(yx, q)$, for all x and y in Ɽ × Ɽ and q in Q. And, $V_\vartheta(x \vee y, q) = V_\vartheta[(x_1, x_2) \vee (y_1, y_2), q] = V_\vartheta\big((x_1 \vee y_1, x_2 \vee y_2), q\big) = A_\vartheta(x_1 \vee y_1, q) \vee A_\vartheta(x_2 \vee y_2, q) = A_\vartheta(y_1 \vee x_1, q) \vee A_\vartheta(y_2 \vee x_2, q) = V_\vartheta\big((y_1 \vee x_1, y_2 \vee x_2), q\big) = V_\vartheta[(y_1, y_2) \vee (x_1, x_2), q] = V_\vartheta(y \vee x, q)$ Therefore, $V_\vartheta(x \vee y, q) = V_\vartheta(y \vee x, q)$, for all x and y in Ɽ × Ɽ and q

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in Q. And, $V_\vartheta(x \wedge y, q) = V_\vartheta[(x_1, x_2) \wedge (y_1, y_2), q] = V_\vartheta((x_1 \wedge y_1, x_2 \wedge y_2), q) = A_\vartheta(x_1 \wedge y_1, q) \vee A_\vartheta(x_2 \wedge y_2, q) = A_\vartheta(y_1 \wedge x_1, q) \vee A_\vartheta(y_2 \wedge x_2, q) = V_\vartheta((y_1 \wedge x_1, y_2 \wedge x_2), q) = V_\vartheta[(y_1, y_2) \wedge (x_1, x_2), q] = V_\vartheta(y \wedge x, q)$Therefore, $V_\vartheta(x \wedge y, q) = V_\vartheta(y \wedge x, q)$, for all x and y in $\mathrm{R} \times \mathrm{R}$ and q in Q. This proves that V is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of $\mathrm{R} \times \mathrm{R}$. Conversely assume that V is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of $\mathrm{R} \times \mathrm{R}$, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $\mathrm{R} \times \mathrm{R}$, we know that A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R, if $A_\mu(x_1 + y_1, q) \le A_\mu(x_2 + y_2, q)$, then $A_\mu(x_1 + y_1, q) = A_\mu(x_1 + y_1, q) \wedge A_\mu(x_2 + y_2, q) = V_\mu((x_1 + y_1, x_2 + y_2), q) = V_\mu[(x_1, x_2) + (y_1, y_2), q] = V_\mu(x + y, q) = V_\mu(y + x, q) = V_\mu[(y_1, y_2) + (x_1, x_2), q] = V_\mu((y_1 + x_1, y_2 + x_2), q) = A_\mu(y_1 + x_1, q) \wedge A_\mu(y_2 + x_2, q) = A_\mu(y_1 + x_1, q)$. We get, $A_\mu(x_1 + y_1, q) = A_\mu(y_1 + x_1, q)$, for all $x_1$and y_1 in R and q in Q. If $A_\mu(x_1 y_1, q) \le A_\mu(x_2 y_2, q)$, then $A_\mu(x_1 y_1, q) = A_\mu(x_1 y_1, q) \wedge A_\mu(x_2 y_2, q) = V_\mu((x_1 y_1, x_2 y_2), q) = V_\mu[(x_1, x_2)(y_1, y_2), q] = V_\mu(xy, q) = V_\mu(yx, q) = V_\mu[(y_1, y_2)(x_1, x_2), q] = V_\mu((y_1 x_1, y_2 x_2), q) = A_\mu(y_1 x_1, q) \wedge A_\mu(y_2 x_2, q) = A_\mu(y_1 x_1, q)$. We get, $A_\mu(x_1 y_1, q) = A_\mu(y_1 x_1, q)$, for all $x_1$and y_1 in R and q in Q. If $A_\mu(x_1 \vee y_1, q) \le A_\mu(x_2 \vee y_2, q)$, then $A_\mu(x_1 \vee y_1, q) = A_\mu(x_1 \vee y_1, q) \wedge A_\mu(x_2 \vee y_2, q) = V_\mu((x_1 \vee y_1, x_2 \vee y_2), q) = V_\mu[(x_1, x_2) \vee (y_1, y_2), q] = V_\mu(x \vee y, q) = V_\mu(y \vee x, q) = V_\mu[(y_1, y_2) \vee (x_1, x_2), q] = V_\mu((y_1 \vee x_1, y_2 \vee x_2), q) = A_\mu(y_1 \vee x_1, q) \wedge A_\mu(y_2 \vee x_2, q) = A_\mu(y_1 \vee x_1, q)$. We get, $A_\mu(x_1 \vee y_1, q) = A_\mu(y_1 \vee x_1, q)$, for all $x_1$and y_1 in R and q in Q.
If $A_\mu(x_1 \wedge y_1, q) \le A_\mu(x_2 \wedge y_2, q)$, then $A_\mu(x_1 \wedge y_1, q) = A_\mu(x_1 \wedge y_1, q) \wedge A_\mu(x_2 \wedge y_2, q) = V_\mu((x_1 \wedge y_1, x_2 \wedge y_2), q) = V_\mu[(x_1, x_2) \wedge (y_1, y_2), q] = V_\mu(x \wedge y, q) = V_\mu(y \wedge x, q) = V_\mu[(y_1, y_2) \wedge (x_1, x_2), q] = V_\mu((y_1 \wedge x_1, y_2 \wedge x_2), q) = A_\mu(y_1 \wedge x_1, q) \wedge A_\mu(y_2 \wedge x_2, q) = A_\mu(y_1 \wedge x_1, q)$. We get, $A_\mu(x_1 \wedge y_1, q) = A_\mu(y_1 \wedge x_1, q)$, for all $x_1$and y_1 in R and q in Q.
If $A_\vartheta(x_1 + y_1, q) \ge A_\vartheta(x_2 + y_2, q)$, then $A_\vartheta(x_1 + y_1, q) = A_\vartheta(x_1 + y_1, q) \wedge A_\vartheta(x_2 + y_2, q) = V_\vartheta((x_1 + y_1, x_2 + y_2), q) = V_\vartheta[(x_1, x_2) + (y_1, y_2), q] = V_\vartheta(x + y, q) = V_\vartheta(y + x, q) = V_\vartheta[(y_1, y_2) + (x_1, x_2), q] = V_\vartheta((y_1 + x_1, y_2 + x_2), q) = A_\vartheta(y_1 + x_1, q) \wedge A_\vartheta(y_2 + x_2, q) = A_\vartheta(y_1 + x_1, q)$. We get, $A_\vartheta(x_1 + y_1, q) = A_\vartheta(y_1 + x_1, q)$, for all $x_1$and y_1 in R and q in Q.
If $A_\vartheta(x_1 y_1, q) \ge A_\vartheta(x_2 y_2, q)$, then $A_\vartheta(x_1 y_1, q) = A_\vartheta(x_1 y_1, q) \wedge A_\vartheta(x_2 y_2, q)$

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$= V_{\vartheta}((x_1y_1, x_2y_2), q) = V_{\vartheta}[(x_1, x_2)(y_1, y_2), q] = V_{\vartheta}(xy, q) = V_{\vartheta}(yx, q) =$ $V_{\vartheta}[(y_1, y_2)(x_1, x_2), q] = V_{\vartheta}((y_1x_1, y_2x_2), q) = A_{\vartheta}(y_1x_1, q) \wedge A_{\vartheta}(y_2x_2, q) = A_{\vartheta}(y_1x_1, q)$. We get, $A_{\vartheta}(x_1y_1, q) = A_{\vartheta}(y_1x_1, q)$, for all $x_1$and y_1 in Ɽ and q in Q. If $A_{\vartheta}(x_1 \vee y_1, q) \geq A_{\vartheta}(x_2 \vee y_2, q)$, then $A_{\vartheta}(x_1 \vee y_1, q) = A_{\vartheta}(x_1 \vee y_1, q) \wedge A_{\vartheta}(x_2 \vee y_2, q) = V_{\vartheta}((x_1 \vee y_1, x_2 \vee y_2), q) = V_{\vartheta}[(x_1, x_2) \vee (y_1, y_2), q] = V_{\vartheta}(x \vee y, q) = V_{\vartheta}(y \vee x, q) = V_{\vartheta}[(y_1, y_2) \vee (x_1, x_2), q]$ $= V_{\vartheta}((y_1 \vee x_1, y_2 \vee x_2), q) = A_{\vartheta}(y_1 \vee x_1, q) \wedge A_{\vartheta}(y_2 \vee x_2, q) = A_{\vartheta}(y_1 \vee x_1, q)$. We get, $A_{\vartheta}(x_1 \vee y_1, q) = A_{\vartheta}(y_1 \vee x_1, q)$, for all $x_1$and y_1 in Ɽ and q in Q. If $A_{\vartheta}(x_1 \wedge y_1, q) \geq A_{\vartheta}(x_2 \wedge y_2, q)$, then $A_{\vartheta}(x_1 \wedge y_1, q) = A_{\vartheta}(x_1 \wedge y_1, q) \wedge A_{\vartheta}(x_2 \wedge y_2, q) = V_{\vartheta}((x_1 \wedge y_1, x_2 \wedge y_2), q) = V_{\vartheta}[(x_1, x_2) \wedge (y_1, y_2), q] = V_{\vartheta}(x \wedge y, q) = V_{\vartheta}(y \wedge x, q) = V_{\vartheta}[(y_1, y_2) \wedge (x_1, x_2), q]$

$$= V_{\vartheta}((y_1 \wedge x_1, y_2 \wedge x_2), q) = A_{\vartheta}(y_1 \wedge x_1, q) \wedge A_{\vartheta}(y_2 \wedge x_2, q) = A_{\vartheta}(y_1 \wedge x_1, q).$$

We get, $A_{\vartheta}(x_1 \wedge y_1, q) = A_{\vartheta}(y_1 \wedge x_1, q)$, for all $x_1$and y_1 in Ɽ and q in Q. Therefore A is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of Ɽ.

2.2 Theorem: Let A and B be Q-intuitionistic L-fuzzy ℓ-subsemiring of the ℓ-semirings G and H, respectively. If A and B are Q-intuitionistic L-fuzzy normal ℓ-subsemiring, then $A \times B$ is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of $G \times H$.

Proof: Let A and B be two Q-intuitionistic L-fuzzy normal ℓ-subsemirings of the ℓ-semirings $G$and H respectively. Clearly $A \times B$ is an Q-intuitionistic L-fuzzy ℓ-subsemiring of $G \times H$.

Let x_1 and $x_2 \in G$, y_1 and $y_2 \in H$and $q \in Q$. Then (x_1, y_1) and (x_2, y_2) are in $G \times H$. Now, $(A \times B)_{\mu}[((x_1, y_1) + (x_2, y_2)), q] = (A \times B)_{\mu}((x_1 + x_2, y_1 + y_2), q) = A_{\mu}((x_1 + x_2), q) \wedge B_{\mu}((y_1 + y_2), q) = A_{\mu}((x_2 + x_1), q) \wedge B_{\mu}((y_2 + y_1), q) = (A \times B)_{\mu}((x_2 + x_1, y_2 + y_1), q) = (A \times B)_{\mu}((x_2, y_2) + (x_1, y_1), q)$ Therefore, $(A \times B)_{\mu}[((x_1, y_1) + (x_2, y_2), q)] = (A \times B)_{\mu}((x_2, y_2) + (x_1, y_1), q)$ And, $(A \times B)_{\mu}[((x_1, y_1)(x_2, y_2)), q] = (A \times B)_{\mu}((x_1x_2, y_1y_2), q)$ $= A_{\mu}((x_1x_2), q) \wedge B_{\mu}((y_1y_2), q) = A_{\mu}((x_2x_1), q) \wedge B_{\mu}((y_2y_1), q) =$ $(A \times B)_{\mu}((x_2x_1, y_2y_1), q) = (A \times B)_{\mu}((x_2, y_2)(x_1, y_1), q)$

Therefore, $(A \times B)_{\mu}[((x_1, y_1)(x_2, y_2), q)] = (A \times B)_{\mu}((x_2, y_2)(x_1, y_1), q)$.

Also, $(A \times B)_\mu[((x_1, y_1) \vee (x_2, y_2)), q] = (A \times B)_\mu((x_1 \vee x_2, y_1 \vee y_2), q) = A_\mu((x_1 \vee x_2), q) \wedge B_\mu((y_1 \vee y_2), q) = A_\mu((x_2 \vee x_1), q) \wedge B_\mu((y_2 \vee y_1), q) = (A \times B)_\mu((x_2 \vee x_1, y_2 \vee y_1), q) = (A \times B)_\mu((x_2, y_2) \vee (x_1, y_1), q)$

Therefore, $(A \times B)_\mu[((x_1, y_1) \vee (x_2, y_2), q)] = (A \times B)_\mu((x_2, y_2) \vee (x_1, y_1), q)$.

And, $(A \times B)_\mu[((x_1, y_1) \wedge (x_2, y_2)), q] = (A \times B)_\mu((x_1 \wedge x_2, y_1 \wedge y_2), q) = A_\mu((x_1 \wedge x_2), q) \wedge B_\mu((y_1 \wedge y_2), q) = A_\mu((x_2 \wedge x_1), q) \wedge B_\mu((y_2 \wedge y_1), q) = (A \times B)_\mu((x_2 \wedge x_1, y_2 \wedge y_1), q)$
$= (A \times B)_\mu((x_2, y_2) \wedge (x_1, y_1), q)$

Therefore, $(A \times B)_\mu[((x_1, y_1) \wedge (x_2, y_2), q)] = (A \times B)_\mu((x_2, y_2) \wedge (x_1, y_1), q)$.

Now, $(A \times B)_\vartheta[((x_1, y_1) + (x_2, y_2)), q] = (A \times B)_\vartheta((x_1 + x_2, y_1 + y_2), q) = A_\vartheta((x_1 + x_2), q) \wedge B_\vartheta((y_1 + y_2), q) = A_\vartheta((x_2 + x_1), q) \wedge B_\vartheta((y_2 + y_1), q)$
$= (A \times B)_\vartheta((x_2 + x_1, y_2 + y_1), q) = (A \times B)_\vartheta((x_2, y_2) + (x_1, y_1), q)$

Therefore, $(A \times B)_\vartheta[((x_1, y_1) + (x_2, y_2), q)] = (A \times B)_\vartheta((x_2, y_2) + (x_1, y_1), q)$ And, $(A \times B)_\vartheta[((x_1, y_1)(x_2, y_2)), q] = (A \times B)_\vartheta((x_1x_2, y_1y_2), q) = A_\vartheta((x_1x_2), q) \wedge B_\vartheta((y_1y_2), q)$
$= A_\vartheta((x_2x_1), q) \wedge B_\vartheta((y_2y_1), q) = (A \times B)_\vartheta((x_2x_1, y_2y_1), q) = (A \times B)_\vartheta((x_2, y_2)(x_1, y_1), q)$

Therefore, $(A \times B)_\vartheta[((x_1, y_1)(x_2, y_2), q)] = (A \times B)_\vartheta((x_2, y_2)(x_1, y_1), q)$. Also, $(A \times B)_\vartheta[((x_1, y_1) \vee (x_2, y_2)), q] = (A \times B)_\vartheta((x_1 \vee x_2, y_1 \vee y_2), q) = A_\vartheta((x_1 \vee x_2), q) \wedge B_\vartheta((y_1 \vee y_2), q) = A_\vartheta((x_2 \vee x_1), q) \wedge B_\vartheta((y_2 \vee y_1), q) = (A \times B)_\vartheta((x_2 \vee x_1, y_2 \vee y_1), q)$
$= (A \times B)_\vartheta((x_2, y_2) \vee (x_1, y_1), q)$

Therefore, $(A \times B)_\vartheta[((x_1, y_1) \vee (x_2, y_2), q)] = (A \times B)_\vartheta((x_2, y_2) \vee (x_1, y_1), q)$. And, $(A \times B)_\vartheta[((x_1, y_1) \wedge (x_2, y_2)), q] = (A \times B)_\vartheta((x_1 \wedge x_2, y_1 \wedge y_2), q) = A_\vartheta((x_1 \wedge x_2), q) \wedge B_\vartheta((y_1 \wedge y_2), q) = A_\vartheta((x_2 \wedge x_1), q) \wedge B_\vartheta((y_2 \wedge y_1), q)$
$= (A \times B)_\vartheta((x_2 \wedge x_1, y_2 \wedge y_1), q) = (A \times B)_\vartheta((x_2, y_2) \wedge (x_1, y_1), q)$

Therefore, $(A \times B)_\vartheta[((x_1, y_1) \wedge (x_2, y_2), q)] = (A \times B)_\vartheta((x_2, y_2) \wedge (x_1, y_1), q)$. As a result, $A \times B$ is a Q-intuitionistic L-fuzzy normal ℓ-subsemiring of ℓ-semiring of $G \times H$.

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2.3 Theorem: Let $(\text{R}, +, \cdot, \vee, \wedge)$ and $(\text{R}^{|}, +, \cdot, \vee, \wedge)$ be any two ℓ-semirings. The homomorphic image of an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of R is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of $\text{R}^{|}$.

Proof:

Let $V = f(A)$, where A is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring R. We have to prove that V is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring $\text{R}^{|}$. Now, for $f(x), f(y)$ in $\text{R}^{|}$, clearly V is an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring $\text{R}^{|}$, since A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R. Now, $V_\mu(f(x) + f(y), q) = V_\mu(f(x+y), q) \geq A_\mu(x+y, q) = A_\mu(y+x, q) \leq V_\mu(f(y+x), q) = V_\mu\big(f(y,q) + f(x,q)\big)$Therefore,$V_\mu\big(f(x,q) + f(y,q)\big) = V_\mu(f(y,q) + \big(f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $\text{R}^{|}$ and q in the Q. Again, $V_\mu(f(x)f(y), q) = V_\mu(f(xy), q) \geq A_\mu(xy, q) = A_\mu(yx, q) \leq V_\mu(f(yx), q) = V_\mu\big(f(y,q)f(x,q)\big)$. Therefore,$V_\mu\big(f(x,q)f(y,q)\big) = V_\mu(f(y,q)\big(f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $\text{R}^{|}$ and q in the Q. Also, $V_\mu(f(x) \vee f(y), q) = V_\mu(f(x \vee y), q) \geq A_\mu(x \vee y, q) = A_\mu(y \vee x, q) \leq V_\mu(f(y \vee x), q) = V_\mu\big(f(y,q) \vee f(x,q)\big)$Therefore,$V_\mu\big(f(x,q) \vee f(y,q)\big) = V_\mu(f(y,q) \vee \big(f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $\text{R}^{|}$ and q in the Q. Now,

$V_\mu(f(x) \wedge f(y), q) = V_\mu(f(x \wedge y), q) \geq A_\mu(x \wedge y, q) = A_\mu(y \wedge x, q) \leq V_\mu(f(y \wedge x), q)$
$= V_\mu\big(f(y,q) \wedge f(x,q)\big)$.

Therefore,$V_\mu\big(f(x,q) \wedge f(y,q)\big) = V_\mu(f(y,q) \wedge \big(f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $\text{R}^{|}$ and q in the Q.

And,$V_\vartheta(f(x) + f(y), q) = V_\vartheta(f(x+y), q) \geq A_\vartheta(x+y, q) = A_\vartheta(y+x, q) \leq V_\vartheta(f(y+x), q) = V_\vartheta\big(f(y,q) + f(x,q)\big)$. Therefore,$V_\vartheta\big(f(x,q) + f(y,q)\big) = V_\vartheta(f(y,q) + \big(f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $\text{R}^{|}$ and q in the Q. Also, $V_\vartheta(f(x)f(y), q) = V_\vartheta(f(xy), q) \geq A_\vartheta(xy, q) = A_\vartheta(yx, q) \leq V_\vartheta(f(yx), q) = V_\vartheta\big(f(y,q)f(x,q)\big)$. Therefore,$V_\vartheta\big(f(x,q)f(y,q)\big) = V_\vartheta(f(y,q)\big(f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $\text{R}^{|}$ and q in the Q. And, $V_\vartheta(f(x) \vee f(y), q) = V_\vartheta(f(x \vee y), q) \geq A_\vartheta(x \vee y, q) = A_\vartheta(y \vee x, q) \leq V_\vartheta(f(y \vee x), q) = V_\vartheta\big(f(y,q) \vee f(x,q)\big)$. Therefore,$V_\vartheta\big(f(x,q) \vee f(y,q)\big) = V_\vartheta(f(y,q) \vee \big(f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $\text{R}^{|}$ and q in the Q. Also, $V_\vartheta(f(x) \wedge f(y), q) = V_\vartheta(f(x \wedge y), q) \geq A_\vartheta(x \wedge y, q) = A_\vartheta(y \wedge x, q)$

$\leq V_\vartheta(f(y \wedge x), q) = V_\vartheta\big(f(y,q) \wedge f(x,q)\big)$. Therefore,$V_\vartheta\big(f(x,q) \wedge f(y,q)\big) = V_\vartheta\big(f(y,q) \wedge (f(x,q)\big)$, for every $f(x)$ and $f(y)$ in Ɽ$^{|}$ and q in the Q. As a result, V is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring Ɽ$^{|}$.

2.4 Theorem: Let (Ɽ, +, ●,∨,∧) and (Ɽ$^{|}$, +, ●,∨,∧) be any two ℓ-semirings. The homomorphic preimage of an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of Ɽ| is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of Ɽ.

Proof: Let $V = f(A)$, where V is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring Ɽ$^{|}$. We have to prove that A is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring Ɽ. Let x and y in Ɽ and q in the Q. Then, clearly A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring Ɽ, since V is an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring Ɽ$^{|}$. Now, $A_\mu(x+y,q) = V_\mu\big(f(x+y,q)\big)$ $= V_\mu\big(f(x,q)+f(y,q)\big)$ $= V_\mu\big(f(y,q)+f(x,q)\big)$ $= V_\mu\big(f(y+x,q)\big)$ $= A_\mu(y+x,q)$. Therefore,$A_\mu(x+y,q) = A_\mu(y+x,q)$, for every x and y in Ɽ and q in the Q. $A_\mu(xy,q) = V_\mu\big(f(xy,q)\big)$, since $A_\mu(x,q) = V_\mu\big(f(x,q)\big)$ $= V_\mu\big(f(x,q)f(y,q)\big)$ $= V_\mu\big(f(y,q)f(x,q)\big)$ $= V_\mu\big(f(yx,q)\big)$ $= A_\mu(yx,q)$. Therefore,$A_\mu(xy,q) = A_\mu(yx,q)$, for every x and y in Ɽ and q in the Q. $V_\mu\big(f(x,q)f(y,q)\big) = V_\mu\big(f(x \vee y,q)\big) = A_\mu(x \vee y,q) = A_\mu(y \vee x,q) = V_\mu\big(f(y \vee x,q)\big) = V_\mu\big(f(y,q)f(x,q)\big)$.

Therefore, $V_\mu\big(f(x,q)f(y,q)\big) = V_\mu\big(f(y,q)f(x,q)\big)$, for every $f(x)$ and $f(y)$ in Ɽ$^{|}$ and q in the Q. $V_\mu\big(f(x,q)f(y,q)\big) = V_\mu\big(f(x \wedge y,q)\big) = A_\mu(x \wedge y,q) = A_\mu(y \wedge x,q) = V_\mu\big(f(y \wedge x,q)\big) = V_\mu\big(f(y,q)f(x,q)\big)$. Therefore, $V_\mu\big(f(x,q)f(y,q)\big) = V_\mu\big(f(y,q)f(x,q)\big)$, for every $f(x)$ and $f(y)$ in Ɽ$^{|}$ and q in the Q. $A_\vartheta(x+y,q) = V_\vartheta\big(f(x+y,q)\big)$, since $A_\vartheta(x,q) = V_\vartheta\big(f(x,q)\big)$ $= V_\vartheta\big(f(x,q)+f(y,q)\big)$ $= V_\vartheta\big(f(y,q)+f(x,q)\big)$ $= V_\vartheta\big(f(y+x,q)\big) = A_\vartheta(y+x,q)$, since $A_\vartheta(x,q) = V_\vartheta\big(f(x,q)\big)$. Therefore,$A_\mu(x+y,q) = A_\mu(y+x,q)$, for every x and y in Ɽ and q in the Q. $A_\vartheta(xy,q) = V_\vartheta\big(f(xy,q)\big) = V_\vartheta\big(f(x,q)f(y,q)\big) = V_\vartheta\big(f(y,q)f(x,q)\big) = V_\vartheta\big(f(yx,q)\big)$ $= A_\vartheta(yx,q)$.Therefore,$A_\vartheta(xy,q) = A_\vartheta(yx,q)$, for every x and y in Ɽ and q in the Q.
$V_\vartheta\big(f(x,q)f(y,q)\big) = V_\vartheta\big(f(x \vee y,q)\big) = A_\vartheta(x \vee y,q) = A_\vartheta(y \vee x,q) = V_\vartheta\big(f(y \vee x,q)\big)$ $= V_\vartheta\big(f(y,q)f(x,q)\big)$. Therefore, $V_\vartheta\big(f(x,q)f(y,q)\big) = V_\vartheta\big(f(y,q)f(x,q)\big)$, for every $f(x)$ and $f(y)$ in Ɽ$^{|}$ and q in the Q. $V_\vartheta\big(f(x,q)f(y,q)\big) = V_\vartheta\big(f(x \wedge y,q)\big)$, since $A_\vartheta(x,q) = V_\vartheta\big(f(x,q)\big)$

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$= A_\vartheta(x \wedge y, q) = A_\vartheta(y \wedge x, q) = V_\vartheta\big(f(y \wedge x, q)\big) = V_\vartheta\big(f(y,q)f(x,q)\big)$. Therefore, $V_\vartheta\big(f(x,q)f(y,q)\big) = V_\vartheta\big(f(y,q)f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $Ɽ^{|}$and q in the Q. As a result, A is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring Ɽ.

2.5 Theorem: Let $(Ɽ, +, \bullet, \vee, \wedge)$ and $(Ɽ^{|}, +, \bullet, \vee, \wedge)$ be any two ℓ-semirings. The anti-homomorphic image of an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of Ɽ is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of $Ɽ^{|}$.

Proof: Let $V = f(A)$, where A is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring Ɽ. We have to prove that V is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring $Ɽ^{|}$. Now, for $f(x)$ and $f(y)$ in $Ɽ^{|}$ and q in the Q, clearly V is an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring $Ɽ^{|}$, since A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring Ɽ. Now, $V_\mu\big(f(x,q) + f(y,q)\big) = V_\mu\big(f(y + x, q)\big) \geq A_\mu(y + x, q) = A_\mu(x + y, q) \leq V_\mu\big(f(x + y, q)\big) = V_\mu\big(f(y,q) + f(x,q)\big)$. Therefore, $V_\mu\big(f(x,q) + f(y,q)\big) = V_\mu\big(f(y,q) + f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $Ɽ^{|}$ and q in the Q. And, $V_\mu\big(f(x,q)f(y,q)\big) = V_\mu\big(f(yx,q)\big) \geq A_\mu(yx, q) = A_\mu(xy, q) \leq V_\mu\big(f(xy,q)\big) = V_\mu\big(f(y,q)f(x,q)\big)$.

Therefore, $V_\mu\big(f(x,q)f(y,q)\big) = V_\mu\big(f(y,q)f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $Ɽ^{|}$ and q in the Q. Also, $V_\mu\big(f(x,q)f(y,q)\big) = V_\mu\big(f(x \vee y, q)\big) \geq A_\mu(x \vee y, q) = A_\mu(y \vee x, q) \leq V_\mu\big(f(y \vee x, q)\big) = V_\mu\big(f(y,q)f(x,q)\big)$. Therefore, $V_\mu\big(f(x,q)f(y,q)\big) = V_\mu\big(f(y,q)f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $Ɽ^{|}$ and q in the Q. And, $V_\mu\big(f(x,q)f(y,q)\big) = V_\mu\big(f(x \wedge y, q)\big) \geq A_\mu(x \wedge y, q) = A_\mu(y \wedge x, q) \leq V_\mu\big(f(y \wedge x, q)\big) = V_\mu\big(f(y,q)f(x,q)\big)$. Therefore, $V_\mu\big(f(x,q)f(y,q)\big) = V_\mu\big(f(y,q)f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $Ɽ^{|}$ and q in the Q. Again, $V_\vartheta\big(f(x,q) + f(y,q)\big) = V_\vartheta\big(f(y + x, q)\big) \leq A_\vartheta(y + x, q) = A_\vartheta(x + y, q) \geq V_\vartheta\big(f(x + y, q)\big) = V_\vartheta\big(f(y,q) + f(x,q)\big)$. Therefore, $V_\vartheta\big(f(x,q) + f(y,q)\big) = V_\vartheta\big(f(y,q) + f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $Ɽ^{|}$ and q in the Q. And $V_\vartheta\big(f(x,q)f(y,q)\big) = V_\vartheta\big(f(yx,q)\big) \leq A_\vartheta(yx, q) = A_\vartheta(xy, q) \geq V_\vartheta\big(f(xy,q)\big) = V_\vartheta\big(f(y,q)f(x,q)\big)$. Therefore, $V_\vartheta\big(f(x,q)f(y,q)\big) = V_\vartheta\big(f(y,q)\, f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $Ɽ^{|}$ and q in the Q. Also, $V_\vartheta\big(f(x,q)f(y,q)\big) = V_\vartheta\big(f(x \vee y, q)\big) \leq A_\vartheta(x \vee y, q) = A_\vartheta(y \vee x, q) \geq V_\vartheta\big(f(y \vee x, q)\big) = V_\vartheta\big(f(y,q)f(x,q)\big)$

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Therefore,$V_{\vartheta}\big(f(x,q)f(y,q)\big) = V_{\vartheta}\big(f(y,q)f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $R^{|}$ and q in the Q. And, $V_{\vartheta}\big(f(x,q)f(y,q)\big) = V_{\vartheta}\big(f(x\wedge y,q)\big) \leq A_{\vartheta}(x\wedge y,q) = A_{\vartheta}(y\wedge x,q) \geq V_{\vartheta}\big(f(y\wedge x,q)\big) = V_{\vartheta}\big(f(y,q)f(x,q)\big)$. Therefore,$V_{\vartheta}\big(f(x,q)f(y,q)\big) = V_{\vartheta}\big(f(y,q)f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $R^{|}$ and q in the Q. As a result, V is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring $R^{|}$.

2.6 Theorem: Let $(R,+,\bullet,\vee,\wedge)$ and $\big(R^{|},+,\bullet,\vee,\wedge\big)$ be any two ℓ-semirings. The anti-homomorphic preimage of an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of $R^{|}$ is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of R.

Proof Let $V = f(A)$, where V is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring $R^{|}$. We have to prove that A is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring R. Let x and y in R and q in the Q, then, clearly A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R, since V is an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring $R^{|}$. Now, $A_{\mu}(x+y,q) = V_{\mu}\big(f(x+y,q)\big) = V_{\mu}\big(f(y,q)+f(x,q)\big) = V_{\mu}\big(f(x,q)+f(y,q)\big) = V_{\mu}\big(f(y+x,q)\big) = A_{\mu}(y+x,q)$. Therefore,$A_{\mu}(x+y,q) = A_{\mu}(y+x,q)$, for every x and y in R and q in the Q. And, $A_{\mu}(xy,q) = V_{\mu}\big(f(xy,q)\big) = V_{\mu}\big(f(y,q)f(x,q)\big) = V_{\mu}\big(f(x,q)f(y,q)\big) = V_{\mu}\big(f(yx,q)\big) = A_{\mu}(yx,q)$. Therefore,$A_{\mu}(xy,q) = A_{\mu}(yx,q)$, for every x and y in R and q in the Q. Also, $V_{\mu}\big(f(x,q)f(y,q)\big) = V_{\mu}\big(f(x\vee y,q)\big)$, $= A_{\mu}(x\vee y,q) = A_{\mu}(y\vee x,q) = V_{\mu}\big(f(y\vee x,q)\big) = V_{\mu}\big(f(y,q)f(x,q)\big)$. Therefore, $V_{\mu}\big(f(x,q)f(y,q)\big) = V_{\mu}\big(f(y,q)\,f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $R^{|}$ and q in the Q. And, $V_{\mu}\big(f(x,q)f(y,q)\big) = V_{\mu}\big(f(x\wedge y,q)\big) = A_{\mu}(x\wedge y,q) = A_{\mu}(y\wedge x,q) = V_{\mu}\big(f(y\wedge x,q)\big) = V_{\mu}\big(f(y,q)f(x,q)\big)$. Therefore, $V_{\mu}\big(f(x,q)f(y,q)\big) = V_{\mu}\big(f(y,q)\,f(x,q)\big)$, for every $f(x)$ and $f(y)$ in $R^{|}$ and q in the Q. Again, $A_{\vartheta}(x+y,q) = V_{\vartheta}\big(f(x+y,q)\big)$, since $A_{\vartheta}(x,q) = V_{\vartheta}\big(f(x,q)\big)$ $= V_{\vartheta}\big(f(y,q)+f(x,q)\big) = V_{\vartheta}\big(f(x,q)+f(y,q)\big) = V_{\vartheta}\big(f(y+x,q)\big) = A_{\vartheta}(y+x,q)$. Therefore,$A_{\vartheta}(x+y,q) = A_{\vartheta}(y+x,q)$, for every x and y in R and q in the Q. And, $A_{\vartheta}(xy,q) = V_{\vartheta}\big(f(xy,q)\big) = V_{\vartheta}\big(f(y,q)f(x,q)\big) = V_{\vartheta}\big(f(x,q)f(y,q)\big) = V_{\vartheta}\big(f(yx,q)\big) = A_{\vartheta}(yx,q)$. Therefore,$A_{\vartheta}(xy,q) = A_{\vartheta}(yx,q)$, for every x and y in R and q in the Q. Also, $V_{\vartheta}\big(f(x,q)f(y,q)\big) = V_{\vartheta}\big(f(x\vee y,q)\big) = A_{\vartheta}(x\vee y,q) = A_{\vartheta}(y\vee x,q) = V_{\vartheta}\big(f(y\vee x,q)\big)$

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$= V_{\vartheta}(f(y,q)f(x,q))$. Therefore, $V_{\vartheta}(f(x,q)f(y,q)) = V_{\vartheta}(f(y,q)\, f(x,q))$, for every $f(x)$ and $f(y)$ in Ɽ$^{|}$ and q in the Q.And, $V_{\vartheta}(f(x,q)f(y,q)) = V_{\vartheta}(f(x \wedge y,q)) = A_{\vartheta}(x \wedge y,q)$ $= A_{\vartheta}(y \wedge x,q) \quad = V_{\vartheta}(f(y \wedge x,q)) \ = V_{\vartheta}(f(y,q)f(x,q))$. Therefore, $V_{\vartheta}(f(x,q)f(y,q)) =$ $V_{\vartheta}(f(y,q)\, f(x,q))$, for every $f(x)$ and $f(y)$ in Ɽ$^{|}$ and q in the Q. As a result, A is an Q-intuitionistic L-fuzzy normal ℓ-subsemiring of a ℓ-semiring Ɽ.

Conclusion:

In the study of the structure of an fuzzy algebraic system, we notice that Q-fuzzy with special properties always play an important role. In this paper, we define Q -intuitionistic L-fuzzy normal ℓ -subsemiring of a ℓ -semiring and investigate some important results. We hope that the research along this direction can be continued, and in fact, this work would serve as a foundation for further study of the theory of semiring, it will be important to complete more hypothetical exploration to set up an overall structure for the commonsense application.

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