EFFECT OF THE COUPLE-STRESS ON VISCO-ELASTIC FLUID FLOW SATURATING A POROUS MEDIUM CONTAINING DUST PARTICLES

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Abstract:
In this paper, the effect of couple-stress on a visco-elastic fluid layer that is heated from below within a porous medium with dust particles has been studied. Normal mode method has been used to determine the dispersion relation and MATLAB has been used to numerically analyzed the problem. Dust Particles has a significant effect on the system, and the effects of medium permeability, dust particles and couple-stress have been determined.

Keywords: Visco-elastic Fluid, Couple-stress, Porous Medium.

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1. Introduction

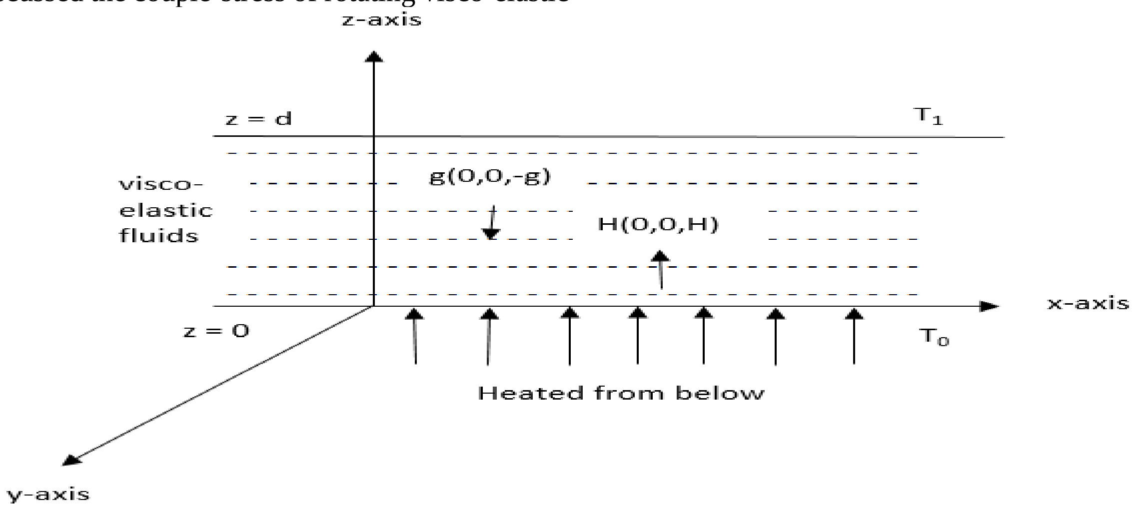
There are some important classes of fluid in technology areas, one of them visco-elastic fluid. The classical theory of viscoelastic fluid and viscoelastic boundary layer flow fluid was developed by Oldroyd [11] and Beard and Walters [2]. Kumar et al. [5] analyzed the viscous fluid and micro polar fluid flow via vertical channel. Sharma [13] investigated the visco-elastic fluid flow on MHD with thermal effect. The analysis of thermal convection on viscoelastic fluid flow with suspended particles was conducted by Vivek and Pardeep [9]. Chamkha [3] studied the problem of hydro magnetic two-phase flow in a channel. The analysis of thermal effect and suspended particles on viscous fluid flow was covered by Singh and Gupta [15]. The effect of parallel plate's on visco-elastic fluid flow with MHD was examined by Chaudhary and Das [4]. Analytical study of Hall Effect on visco-elastic fluid flow was done by Kumar and Mehta [8]. Wooding [17] proposed the notion of the Rayleigh instability via porous media. The analysis of an Oldroydian viscoelastic fluid in a porous media with Hall Effect was studied by Pardeep and Mohan [6].

The classical theory of couple-stress fluid was investigated by Stokes [16]. Kumar [7] et al. discussed the couple-stress of rotating visco-elastic fluid flow with thermal. Banyal and Singh [1] investigated the rotation on the couple-stress fluid flow. Shivakumara et al. [14] analyzed the onset of convection in a couple-stress fluid flow saturating a porous medium by the Galerkin method. Pundir [12] et al. investigated the effect of permeability, couple-stress parameter and magnetization on ferro-magnetic fluid layer heated from below within porous medium with hall current.

In view of the above discussion, application of this work is such as geophysics, medical science, film lubrication, engineering, chemical science and industry. In this research, we attempt to investigate how couple-stress affects the flow of a visco-elastic fluid containing dust particles. To the best of our knowledge, this problem has not yet been investigated using Darcy's generalized model.

2. Mathematical Formulation

The fluid layer is considered to be infinite, incompressible and visco-elastic with thickness d. study adverse temperature gradient $\beta=\left|\frac{dT}{dz}\right|$ has been maintained by maintaining the lower limit at $z=0$ and upper limit at $z=d$ are constant but varying temperatures T_0 and T_1 respectively.



The equations of continuity, motion, energy and basic's state are

$$\nabla.\vec{q}=0 \tag{2.1}$$

$$\frac{\rho_0}{\in}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right]=-\nabla P-\rho g\hat{e}_z+\left(\mu-\frac{\nu}{\rho_0}\nabla^2\right)\nabla^2\vec{q}-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{q}+\frac{KN}{\in}(\vec{q}_d-\vec{q}) \tag{2.2}$$

$$\left[\in\rho_0C_v+(1-\in)\rho_sC_s\right]\frac{\partial T}{\partial t}+\rho_0C_v(\vec{q}.\nabla)T+mNC_{pt}\left(\in\frac{\partial T}{\partial t}+\vec{q}_d.\nabla T\right)=\chi\nabla^2T \tag{2.3}$$

$$\rho=\rho_0\left[1-\alpha(T-T_a)\right] \tag{2.4}$$

where, T – Temperature, P – Pressure, χ – Thermal conductivity, ρ – Fluid density, ρ_s – Density of solid matrix, ρ_0 – Reference density, ν – Couple-stress Viscosity, μ - Viscosity, μ' - Viscoelasticity, $\hat{e}_z$ – Unit vector , $\vec{q}-$ Velocity, t – time, $C_{Pt}-$ Specific heat of dust particles, $\vec{q}_d\left(x,t\right)-$ Velocity of dust particles, C_v- Specific heat at constant volume, C_s- Specific heat of solid, $N\left(x,t\right)-$ Number density of dust particles, $\alpha-$ Coefficient of thermal expansion, T_a- Average temperature is given by $T_a=\frac{\left(T_0+T_1\right)}{2}$.

The equations of continuity and motion for the dust particles are

$$\in\frac{\partial N}{\partial t}+\nabla.\left(N\vec{q}_d\right)=0 \tag{2.5}$$

$$\left[\frac{\partial\vec{q}_d}{\partial t}+\frac{1}{\in}\left(\vec{q}_d.\nabla\right)\vec{q}_d\right]mN=KN\left(\vec{q}-\vec{q}_d\right) \tag{2.6}$$

where $mN-$ Mass of dust particles per unit volume, $KN\left(\vec{q}-\vec{q}_d\right)-$ Extra body force per unit volume.

3. Basic State

The basic's state is

$\vec{q}=\vec{q}_b\left(0,0,0\right),\ \rho=\rho_b\left(z\right),\ \vec{q}_d=\left(\vec{q}_d\right)_b\left(0,0,0\right)$ and $P=P_b\left(z\right)$

using above conditions in equations (2.1) to (2.4), we have

$$\frac{dP_b}{dz}+\rho_b\,g=0 \tag{3.1}$$

$$T=-\beta z+T_a=T_b\left(z\right) \tag{3.2}$$

$$\rho_b=\rho_0+\alpha\beta z\rho_0 \tag{3.3}$$

4. Linearized Perturbation Equations

Now we linearize equations (2.1) to (2.6), we get

$$\nabla.\vec{q}\,'=0 \tag{4.1}$$

$$\frac{\rho_0}{\in}\frac{\partial\vec{q}\,'}{\partial t}=-\nabla P'-\rho' g\hat{e}_z+\left(\mu-\frac{\nu}{\rho_0}\nabla^2\right)\nabla^2\vec{q}\,'-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{q}\,'-\frac{mN_0}{\in}\frac{\partial\vec{q}\,'}{\partial t} \tag{4.2}$$

$$\frac{\partial\theta}{\partial t}\left[E+h_T\in\right]L=\left[\nabla^2\theta k_T+\beta\left(\vec{q}\,'\right)_z\right]L+h_T\beta\left(\vec{q}\,'\right)_z \tag{4.3}$$

$$\rho'=-\rho_0\alpha\theta \tag{4.4}$$

where, $k_T=\frac{\chi}{\rho_0C_v}$, $E=\in+\frac{\left(1-\in\right)\rho_sC_s}{\rho_0C_v}$, $h_T=\frac{mN_0C_{pt}}{\rho_0C_v}-$ Thermal diffusivity and $L=\left[\frac{m}{K}\frac{\partial}{\partial t}+1\right]$.

We converting the equations of (4.1) to (4.3) by the following transformation $\vec{q}\,'=\frac{k_T}{d}\vec{q}*$, $\nabla=\frac{\nabla*}{d}$, $L=\tau\frac{\partial}{\partial t*}+1$, $P'=\frac{\mu k_T}{d^2}P*$, $t=\frac{\rho_0d^2}{\mu}t*$, $\tau=\frac{m\mu}{\rho_0d^2}$ and $\theta=\beta d\,\theta*$, becomes

$$\nabla.\vec{q}=0 \tag{4.5}$$

$$L\left[\frac{1}{\in}\frac{\partial\vec{q}}{\partial t}\right]=L\left[-\nabla P+R\theta\hat{e}_z+\left(1-F\nabla^2\right)\nabla^2\vec{q}-\frac{1}{K_1}\left(1+F_1\frac{\partial}{\partial t}\right)\vec{q}\right]-\frac{f}{\in}\frac{\partial\vec{q}}{\partial t} \tag{4.6}$$

$$LP_rE_r\frac{\partial\theta}{\partial t}=L\left[\nabla^2\theta+\left(\vec{q}\right)_z\right]+h_T\left(\vec{q}\right)_z \tag{4.7}$$

where, $R=\frac{\rho_0\alpha\beta gd^4}{k_T\mu}-$Rayleigh number, $F=\frac{\nu}{\rho_0d^2}$, $F_1=\frac{\mu'}{\rho_0d^2}-$Viscoelastic Parameter, $P_r=\frac{\mu}{k_T\rho_0}-$ Prandtl number, $K_1=\frac{k_1}{d^2}$, $f=\frac{mN_0}{\rho_0}$, $E_r=E+h_T\in$, and $W=\vec{q}.\hat{e}_z$.

5. Boundary conditions

The boundary condition is

$$W = \frac{d^2W}{dz^2} = 0,\ \theta = 0 \text{ at } z = 0 \text{ and } z = d. \tag{5.1}$$

6. Dispersion Relation

Now the taking of curl on both sides in equation (4.6), becomes

$$\left[\left\{\frac{1}{\in}\frac{\partial}{\partial t} + \frac{1}{K_1}\left(1 + F_1\frac{\partial}{\partial t}\right) - \left(1 - F\nabla^2\right)\nabla^2\right\}L + \frac{f}{\in}\frac{\partial}{\partial t}\right]\left(\nabla \times \vec{q}\right) = L\left[R\left(\frac{\partial\theta}{\partial y}\hat{e}_x + \frac{\partial\theta}{\partial x}\hat{e}_y\right)\right] \tag{6.1}$$

Let $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $D = \frac{\partial}{\partial z}$.

Thus, applying the curl and z-component of equation (6.1), we have

$$\left[\left\{\frac{1}{\in}\frac{\partial}{\partial t} + \frac{1}{K_1}\left(1 + F_1\frac{\partial}{\partial t}\right) - \left(1 - F\nabla^2\right)\nabla^2\right\}L + \frac{f}{\in}\frac{\partial}{\partial t}\right]\nabla^2 W = LR\nabla_1^2\theta \tag{6.2}$$

Now applying the z-component of equation (4.7), we have

$$LP_rE_r\frac{\partial\theta}{\partial t} = L\left[\nabla^2\theta + W\right] + h_T W \tag{6.3}$$

The equation of (5.1) becomes

$$W = D^2W = \theta\, at\, z = 0\, to\, z = 1 \tag{6.4}$$

Normal Mode Analysis

Let $\left[\theta, W\right] = \left[\Theta(z), W(z)\right]\exp.\left[i\,k_x x + i\,k_y y + \sigma t\right]$

Using the above normal mode condition of the equations (6.2) and (6.3), becomes

$$\left[\left\{\frac{\sigma}{\in} + \frac{1}{K_1}\left(1 + F_1\sigma\right) + F\left(D^2 - a^2\right)^2 - \left(D^2 - a^2\right)\right\}\left(1 + \tau\sigma\right) + \frac{f}{\in}\sigma\right]W\left(D^2 - a^2\right) = -Ra^2\Theta\left(1 + \tau\sigma\right) \tag{6.5}$$

$$\left[\left\{E_rP_r\sigma - \left(D^2 - a^2\right)\right\}\left(1 + \tau\sigma\right)\right]\Theta = \left(1 + \tau\sigma\right)W + h_T W \tag{6.6}$$

where, $a^2 = k_x^2 + k_y^2 -$ Wave number and $\sigma = \sigma_r + i\,\sigma_r -$ Stability parameter.

Now, the boundary condition becomes

$$W = D^2W = 0, \Theta = 0 \text{ at } z = 0 \text{ to } z = 1 \tag{6.7}$$

$D^{2n}W = 0$ at $z = 0$ to $z = 1,\ \ n > 0$.

The solution of above equation,

$W = W_0 \sin \pi z$, W_0 - Constant.

Eliminating the Θ from equations (6.5) and (6.6), putting the value of W and $b = \pi^2 + a^2$, we have

$$b\left[\left\{\frac{\sigma}{\in} + \frac{1}{K_1}\left(1 + F_1\sigma\right) + Fb^2 + b\right\}\left(1 + \tau\sigma\right) + \frac{f}{\in}\sigma\right]\left[P_rE_r\sigma + b\right] = Ra^2\left(1 + \tau\sigma + h_T\right) \tag{6.8}$$

7. Stationary Convection

Put them $\sigma = 0$ in equation (6.8), becomes

$$R = \frac{b^2}{a^2\left(1 + h_T\right)}\left[\frac{1}{K_1} + Fb^2 + b\right] \tag{7.1}$$

Neglecting the couple-stress and dust particles (i.e. F=0 and h_T), we have

$$R = \frac{b^2}{a^2}\left(\frac{1}{K_1} + b\right) \tag{7.2}$$

Now, we neglecting the porous medium $\left(K_1 \to \infty\right)$ of the equation (7.2), become

$$R = \frac{b^3}{a^2}$$

Above result given by [10].

Result and discussion

Now, we are discussed about the behavior of medium permeability, couple-stress and dust particles, analytically and numerically from equation (7.1).

$$\frac{dR}{dK_1} = \frac{-b^2}{a^2\left(1+h_T\right)K_1^2} \tag{7.3}$$

It is always negative.

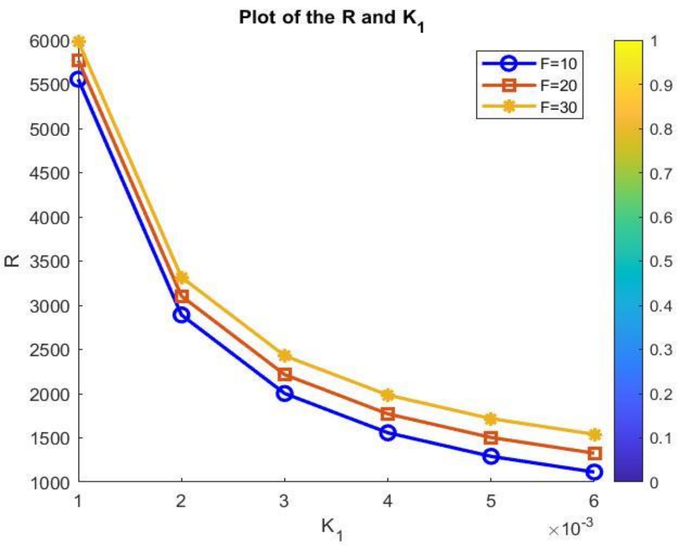


Figure 7.1: Plot between Rayleigh number R and medium K_1.

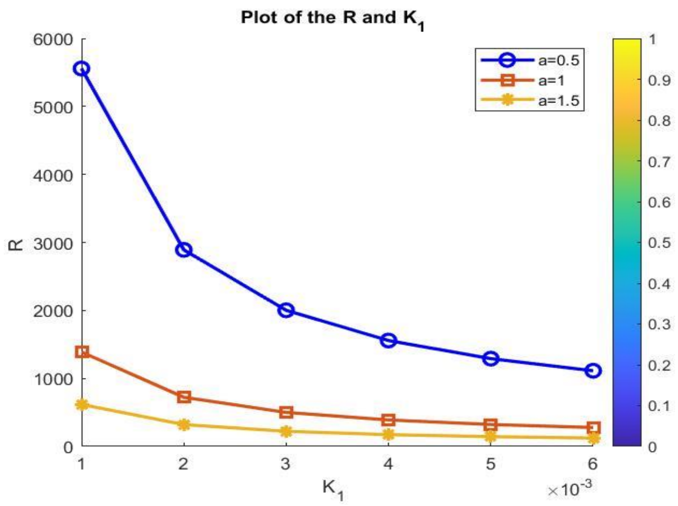


Figure 7.2: Plot of Rayleigh number R and medium permeability K_1.

Figure 7.1 and 7.2 shows that the variation of Rayleigh number R versus medium permeability K_1 i.e. medium permeability increases then Rayleigh number decrease with wave number $a = 0.5, 1.0, 1.5$ and couple stress $F = 10, 20, 30$. Thus, we can say that the effect of permeability is always destabilizing.

$$\frac{dR}{dF} = \frac{b^4}{a^2\left(1+h_T\right)} \tag{7.4}$$

It is always positive.

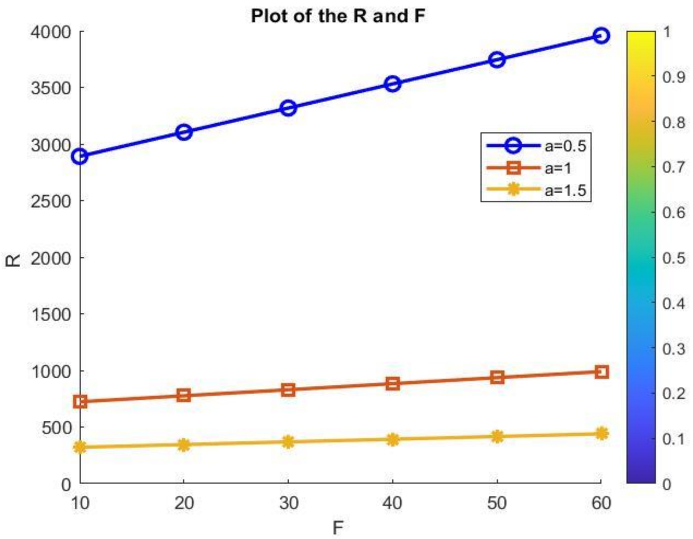


Figure 7.3: Plot between Rayleigh number R and couple stress F.

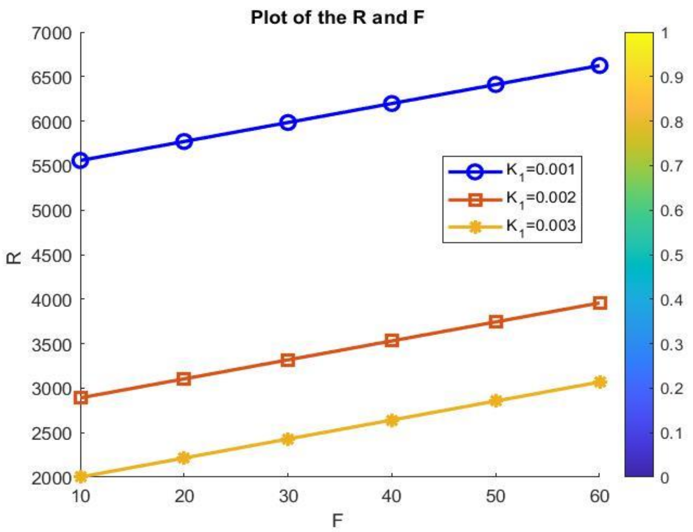


Figure 7.4: Plot of Rayleigh number and couple stress.

Figure 7.3 and 7.4, represent the plot of Rayleigh number R and couple-stress F i.e. couple stress increases then Rayleigh number increases with wave number $a = 0.5, 1.0, 1.5$ and medium permeability $K_1 = 0.001, 0.002, 0.003$. Thus, we can say that the couple stress has stabilizing effect.

$$R = \frac{-b^2}{a^2\left(1+h_T\right)^2}\left[\frac{1}{K_1} + Fb^2 + b\right] \tag{7.5}$$

It is always negative.

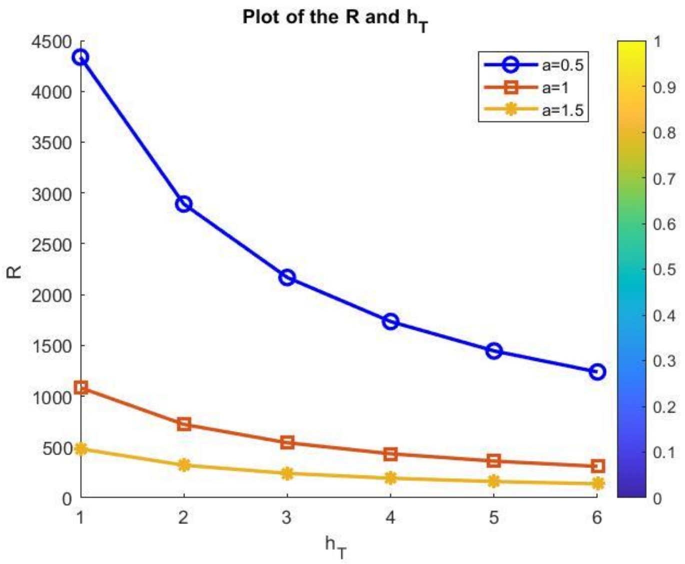


Figure 7.5: Plot of Rayleigh number R and dust particles h_T .

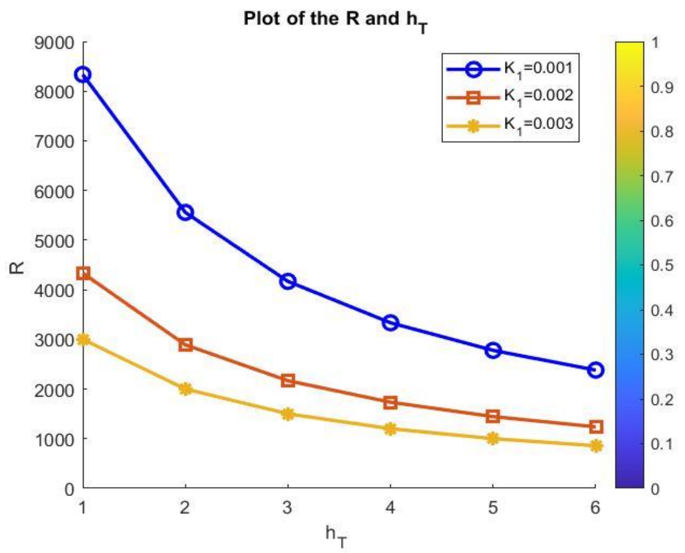


Figure 7.6: Plot of Rayleigh number R and dust particles h_T .

Figure 7.5 and 7.6, represent the plot of Rayleigh number R and dust particles h_T i.e. dust particles increases then Rayleigh number decreases with wave number $a = 0.5, 1.0, 1.5$ and medium permeability $K_1 = 0.001, 0.002, 0.003$. Thus, we can say that the dust particles have destabilizing effect.

8. Conclusions

According to numerically and analytical discussion, we found that the effect of dust particles and permeability are destabilizing. The impact of permeability is always destabilizing when couple stress and dust particles are absent. The effect of couple stress is stabilizing. The most significant finding among these is that dust particles effect the system to become unstable. These results compare with previous work and agree with each other.

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