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Secure domination on silicate chains and silicate network

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ABSTRACT

A set $S \subseteq V(G)$ is said to be secure if for every $X \subseteq S$, $|N[X] \cap S| \geqslant |N[X] - S|$ holds. If set S is secure dominating, then the set must hold both secure and dominating conditions. The secure domination number of G is the minimum cardinality of a secure dominating set in G. Silicate are the inorganic compounds and rock-forming minerals. The basic unit of silicates are tetrahedron (SiO_4). In this paper we have obtained results on secure domination number of silicate chains, silicate network and line graph of silicate chains.

Keywords: Secure domination, Silicate chain and Silicate network.

1.0 Introduction

Chemical graph theory is an interesting field of graph theory. It is important for chemical and pharmacological research in various ways. A graph known as a chemical graph, often referred to as a molecular graph, has atoms as its vertices and molecules as its edges. Let G be the graph with p vertices and q edges, degree of the vertex d(u) is the number of edges that are incident to it. Neighbourhood of the vertex $v \in V(G)$ is the collection of all vertices that are adjacent to vertex v and when the vertex v is included to that vertex set is referred to as closed neighbourhood, denoted by N[v]. The graph whose vertex set corresponds to the edges of a graph G is said to be the line graph L(G) and two vertices of L(G) are said to be adjacent if the corresponding edges of G are also adjacent [6].

The study of domination is one of the fastest growing area in graph theory. This theory arose from the dominating queen's problem studied by European chess enthusiast in 1850's. The set $D \in V(G)$ is said to be dominating set of G if every vertex in V - D is adjacent to at least one vertex in D. The minimum cardinality of a dominating set in G is the domination number $\gamma(G)$ for G [7].

Brigham et al. [2] introduced the security in graphs. While studying the security of graphs, we consider vertices of $|N[X] - S|$ as attackers of X and vertices in $N[X] \cap S$ as defenders of X. A set $S \subseteq V(G)$ is secure if for any subset $X \subseteq S$, any attack on vertices of X can be prevented. Thus, a set S is secure if and only if every attack on S is defendable. A set S is secure if and only if for every $X \subseteq S$, $|N[X] \cap S| \geqslant |N[X] - S|$, Since the whole vertex set is a secure set it observed by Brigham et al.[2]. A set $S \subseteq V(G)$ is a secure dominating set if it is both dominating and secure. The secure domination number of G is the smallest cardinality of a secure dominating set in G and is denoted as $\gamma_s(S(G))$. More works on secure sets and secure domination can be found in [3, 4, 5, 9].

Silicates are incredibly complex and valuable minerals. Silicates are formed by combining metal carbonates with sand or by fusing metal oxides. Silicates act as the fundamental constituents of the typical rock-forming minerals. The silicon atom is joined with equally spaced oxygen atoms when oxygen atoms are placed at the four corners of a tetrahedron, it produces a silicate tetrahedron and when it connects with other tetrahedra linearly, a single-row silicate chain is created. The double silicate chain is formed by two single silicate chains joined together by shared oxygen atom. Now, construction of silicate network, six units of SiO_4 tetrahedra make up the cyclic silicate network SL(1). Six units of the silicate network SL(1) are added to SL(1) to create the silicate network SL(2), where each outer SL(1) shares two consecutive tetrahedra with the inner SL(1). By putting a layer of SL (1) around the edge of SL(n-1), silicate network SL(n) is inductively created from SL(n-1). domination on silicate network can be seen in [8]. There are several uses for dominating sets in graphs for network security, which includes putting in a set of guards at specific network nodes. In the defending technique, some vertices are designated as guards. Secure domination is a defence strategy that can be used as strategy to assess or identify the minimum number of guards required to defend a network or graph network. As per security condition [2] number of the defender ($|N[X] \cap S|$) is greater than or equal to number of attacker($|N[X] - S|$) i.e, $|N[X] \cap S| \geqslant |N[X] - S|$. Such that, in this article we have obtained minimum number of guards required to protect the silicate network i.e, secure domination for silicate network. Secure domination on silicate chains and line graph of the silicate chains are determined.

The following are of immediate use in theorem

Theorem 1.1. [11] The number of nodes in SL(n) is $15n^2+3n$, and the number of edges of SL(n) is $36n^2$.

Theorem 1.2. [11] The number of nodes in Single Silicate chain SL(1, n) is 3n + 1 and edges is 6n, where n is the number of edges in a row line.

Definition 1.1. [8] A double chain silicates network of length n it consists of two condensed identical silicates chains. The number of vertices are $[11n+7/2+1/2(-1)^n]/2$ and the number of edges are 12n, n > 1.

Lemma 1. [3] Security Condition: A set $S \subseteq V(G)$ is secure in G if and only if for any $X \subseteq S$, $|N[X] \cap S| \geqslant |N[X]-S|$.

Lemma 2. [2] $s(G) \geqslant \lceil V(G)/2 \rceil$.

2 **2.0 MAIN RESULTS**

2.1 SILICATE CHAINS:

Theorem 2.1. *Let $SL(1,n)$ be single silicate chain, $n \in N$ then $\gamma_s(SL(1,n))= \lceil\frac{3n+1}{2}\rceil$.*

Proof. By theorem 1.2, single silicate chain $SL(1,n)$ consists of $3n+1$ vertices, by lemma 2, $\gamma_s(G) \geqslant \lceil\frac{|V(G)|}{2}\rceil$. Thus, $\gamma_s(SL(1,n)) \geqslant \lceil\frac{3n+1}{2}\rceil = \lceil\frac{3n+1}{2}\rceil$.

We consider the following cases

Case 1. n is even

Consider $S = \{v_1, v_2, \dots v_{\lceil\frac{3n+1}{2}\rceil}\}$, clearly S is dominating set, since $v_i's$ dominates $v_i''s$. Now, we prove that S is secure in G for this we consider the following subcases.

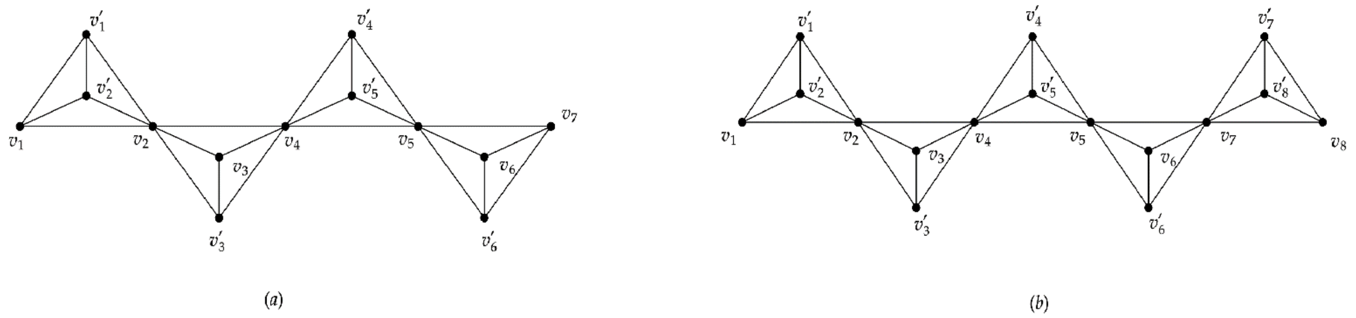

Figure 1: (a) Silicate chain SL(1,4) and (b) Silicate chain SL(1,5).

Subcase 1.1. let X=S then $|N[X] \cap S| = \lceil\frac{3n+1}{2}\rceil$ and $|N[X] - S| = \lceil\frac{3n+1}{2}\rceil - 1$, security condition holds.

Subcase 1.2. let X = $\{v_1, v_2, \dots v_r\} \subseteq \{v_1, v_2, \dots v_{\lceil\frac{3n+1}{2}\rceil}\}$ $r \epsilon N$

(i.) X = $\{v_1, v_2, \dots v_r\}$ each $v_i's$ adjacent to $v_j's$ then possible cardinality of X is 2 or 3. If X contains two vertices then, $|N[X] \cap S|$ =4 or 5 or 6 and $|N[X] - S|$ =3 or 5 or 4 respectively, if X contains three vertices then, $|N[X] \cap S| = 5$ and $|N[X] - S|$ =5.

(ii). X= $\{v_1, v_2, \dots v_r\}$ some of $v_i's$ adjacent to $v_j's$, $N[X]$ = $\{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m, v_1', v_2', \dots v_m'\}$ $m \epsilon N$, here $\{v_i, v_j, \dots v_m\} \subseteq \{v_1, v_2, \dots v_{\lceil\frac{3n+1}{2}\rceil}\}$, $N[X] \cap S = \{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m\}$ and $N[X] - S = \{v_1', v_2', \dots v_m'\}$.

or

$N[X]$=$\{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m, v_1', v_2', \dots v_k'\}$,here $\{v_1', v_2', \dots v_k'\} \subseteq \{v_1', v_2', \dots v_m'\}$, $k \in N$, $N[X] \cap S = \{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m\}$and $N[X] - S = \{v_1', v_2', \dots v_k'\}$, $|N[X] \cap S|$ = m and $|N[X] - S| \leqslant m$.

(iii) X = $\{v_1, v_2, \dots v_r\}$ vertices are not adjacent, $N[X]$ = $\{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m, v_1', v_2', \dots v_m'\}$, here, $\{v_i, v_j, \dots v_m\} \subseteq \{v_1, v_2, \dots v_{\lceil\frac{3n+1}{2}\rceil}\}$ and $\{v_1', v_2', \dots v_k'\} \subseteq \{v_1', v_2', \dots v_m'\}$,
$N[X] \cap S = \{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m\}$ and $N[X] - S = \{v_1', v_2', \dots v_k'\}$ or $\{v_1', v_2', \dots v_m'\}$ $k < m$, $|N[X] \cap S| = m$ and $|N[X] - S| \leqslant m$, in all three cases the security condition will hold.

Subcase 3. X = $\{v_i\}$, $|N[X] \cap S|$= 2 or 4 or 3 and $|N[X] - S|$= 2 or 3 or 1 respectively. Security condition holds.

Case 2: n is odd

Consider S = $\{v_1, v_2, \dots v_{\frac{3n+1}{2}}\}$ clearly S is dominating set, since $v_i's$ dominates $v_i''s$. Now we prove that S is secure in G, consider the following subcases.

Subcase 2.1. let $X = S$ then $|N[X] \cap S| = \frac{3n+1}{2}$ $|N[X] - S| = \frac{3n+1}{2}$, security condition holds.

Subcase 2.2. let X = $\{v_1, v_2, \dots v_r\} \subseteq \{v_1, v_2, \dots v_{\frac{3n+1}{2}}\}$.

(i) X = $\{v_1, v_2, \dots v_r\}$ each $v_i's$ adjacent to $v_j's$ then possible cardinality of X is 2 or 3. If X contains two vertices then, $|N[X] \cap S|$ =4 or 5 or 6 and $|N[X] - S|$=3 or 5 or 4 respectively, if X contains three vertices then, $|N[X] \cap S| = 5$ and $|N[X] - S|$ = 5.

(ii) X = $\{v_1, v_2, \dots v_r\}$ some of $v_i's$ adjacent to $v_j's$, $r \in N$.

$N[X]$=$\{v_1, v_2, \ldots v_r, v_i, v_j, \ldots v_m, v_1', v_2', \ldots v_m'\}$,here$\{v_i, v_j, \ldots v_m\} \subseteq \{v_1, v_2, .. v_{\frac{3n+1}{2}}\}$ $m \epsilon N$,

$N[X] \cap S = \{v_1, v_2, \ldots v_r, v_i, v_j, .. v_m\}$ and $N[X] - S = \{v_1', v_2', \ldots v_m'\}$.

or

$N[X]$=$\{v_1, v_2, \ldots v_r, v_i, v_j, .. v_m, v_1', v_2', \ldots v_k'\}$, here $\{v_i, v_j, .. v_k\} \subseteq \{v_1{}', v_2{}', \ldots v_m{}'\}$, $k \in N$,

$N[X] \cap S = \{v_1, v_2, \ldots v_r, v_i, v_j, \ldots v_m\}$ and $N[X] - S$ = $\{v_1', v_2', \ldots v_m'\}$ $k < m$, $|N[X] \cap S|$ = m and $|N[X] - S| \leqslant m$.

(iii) X = $\{v_1, v_2, \ldots v_r\}$ vertices are not adjacent.

$N[X]$=$\{v_1, v_2, \ldots v_r, v_i, v_j, \ldots v_m, v_1', v_2', \ldots v_k'\}$, or $\{v_1, v_2, \ldots v_r, v_i, v_j, \ldots v_m, v_1', v_2', \ldots v_m'\}$,here $\{v_i, v_j, \ldots v_m\} \subseteq \{v_1, v_2, \ldots v_{\frac{3n+1}{2}}\}$ and $\{v_1', v_2', \ldots v_k'\} \subseteq \{v_1', v_2', \ldots v_m'\}$.

$N[X] \cap S = \{v_1, v_2, \ldots v_r, v_i, v_j, \ldots v_m\}$, $N[X] - S = \{v_1', v_2', \ldots v_k'\}$ or $\{v_1', v_2', \ldots v_m'\}$ $k \leqslant m$, $|N[X] \cap S| = m$ and $|N[X] - S| \leqslant m$. In all these three cases the security condition holds.

Subcase 2.3. X = $\{v_i\}$ $|N[X] \cap S|$ = 2 or 4 or 3 and $|N[X] - S|$= 2 or 3 or 1 respectively. Security condition holds. Hence the proof. ∎

Theorem 2.2 *Let $L(SL(1,n))$ be line graph of single silicate chain for $n \in N$ then, $\gamma_s(L(SL(1,n))) = 3n$.*

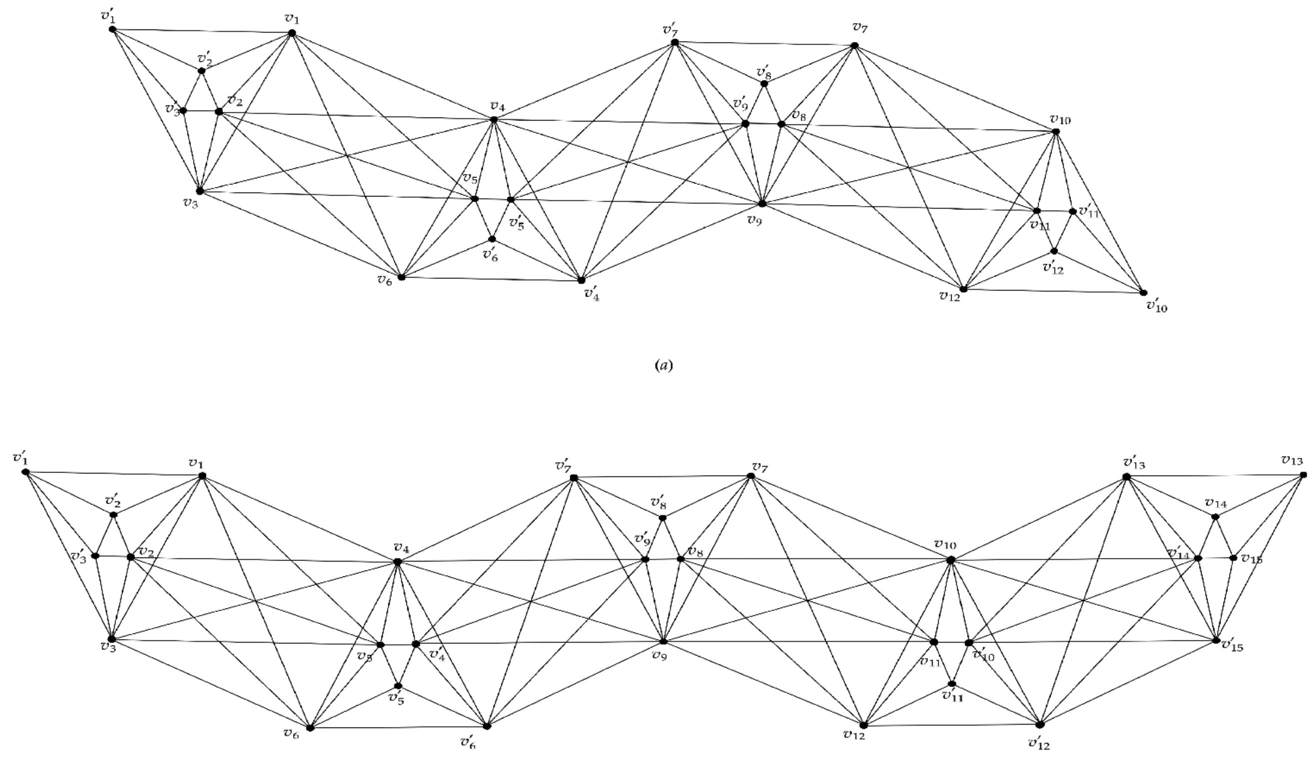

Figure 2: (a) Line graph of SL(1,4) in figure 1.a, and (b) Line graph of SL(1,5) in figure 1.b.

Proof. By theorem 1.2, line graph of single silicate chain $L(SL(1,n))$ consists the $6n$ vertices, by lemma 2, $\gamma_s SL(2,n) \geqslant \frac{6n}{2} = 3n$.

Consider $S = \{v_1, v_2, \ldots v_{3n}\}$, it shows that S is dominating set, since v_i dominates $v_i's$.

To prove S is secure in G.

We consider the following cases.

Case 1. let $X = S$ then, $|N[X] \cap S| = 3n$ and $|N[X] - S| = 3n$, security condition holds.

Case 2. let $X = \{v_1, v_2, \ldots v_r\} \subseteq \{v_1, v_2, \ldots v_{3n}\}$ $r \in N$

(i) X = $\{v_1, v_2, \ldots v_r\}$ each $v_i's$ adjacent to $v_j's$ then the possible cardinality of X is 2 or 3 or 4 or 5 or 6. If X contains 4 or 5 or 6 vertices then, $|N[X] \cap S|$ = 6 and $|N[X] - S| \leqslant 6$ respectively, if X contains 2 vertices then, $|N[X] \cap S|$ = 3 or 6 and $|N[X] - S|$ = 3 or 4 respectively, if X contains 3 vertices then $|N[X] \cap S|$ =3 or 6 and $|N[X] - S|$ = 3 or 5 respectively.

(ii) $X = \{v_1, v_2, \ldots v_r\}$ some of $v_i's$ adjacent to $v_j's$

$N[X]=\{v_1, v_2, \ldots v_r, v_i, v_j, \ldots v_m, v_1', v_2', \ldots v_m'\}$,here$\{v_i, v_j, \ldots v_m\} \subseteq \{v_1, v_2, \ldots v_{3n}\}$, $m \in N$,

$|N[X] \cap S| = \{v_1, v_2, \ldots v_r, v_i, v_j, \ldots v_m\}$ and $N[X] - S = \{v_1', v_2', \ldots v_m'\}$

$|N[X] \cap S| = m$ and $|N[X] - S| = m$

or

$N[X]=\{v_1, v_2, \ldots v_r, v_i, v_j, \ldots v_m, v_1', v_2', \ldots v_k'\}$ $\{v_1', v_2', \ldots v_k'\} \subseteq \{v_1', v_2', \ldots v_m'\}$,$k \in N$,

$N[X] \cap S = \{v_1, v_2, \ldots v_r, v_i, v_j, \ldots v_m\}$ and $N[X] - S = \{v_1', v_2', \ldots v_k'\}$,$k < m$ $|N[X] \cap S| = m$ and $|N[X] - S| < m$

(iii) $X = \{v_1, v_2, \ldots v_r\}$ vertices are not adjacent.

$N[X] = \{v_1, v_2, \ldots v_r, v_i, v_j, \ldots v_m, v_1', v_2', \ldots v_k'\}$, $k < m$, $N[X] \cap S = \{v_1, v_2, \ldots v_r, v_i, v_j, \ldots v_m\}$ and $N[X] - S = \{v_1', v_2', \ldots v_k'\}$ $|N[X] \cap S| = m$ and $|N[X] - S| < m$ in all three cases the security condition will hold.

Case 3. $X = \{v_i\}$ $|N[X] \cap S|$ = 3 or 6 and $|N[X] - S| = 2$. Security condition holds. Hence the proof.
□

Theorem 2.3 *Let $SL(2,n)$ be double silicate chain, $n \geqslant 3$ then $\gamma_s SL(2,n) = 3n + 1$.*

Proof. By defination 1.1, double silicate chain $SL(2,n)$ consists the $\frac{11n+7/2+1/2(-1)^n}{2}$ vertices, by lemma 2, $\gamma_s(G) \geqslant \lceil \frac{|V(G)|}{2} \rceil$ Thus, $\gamma_s L(SL(1,n)) \geqslant \frac{11n+7/2+1/2(-1)^n}{2}$.

Consider $S = \{v_1, v_2, \ldots v_{3n+1}\}$ clearly S is dominating set, since v_i dominates v_i'. To prove S is secure in G.

We consider the following cases.

Case 1. let $X = S$ then $|N[X] \cap S| = 3n + 1$ $|N[X] - S| = \frac{5n+1}{2}$, security condition holds.

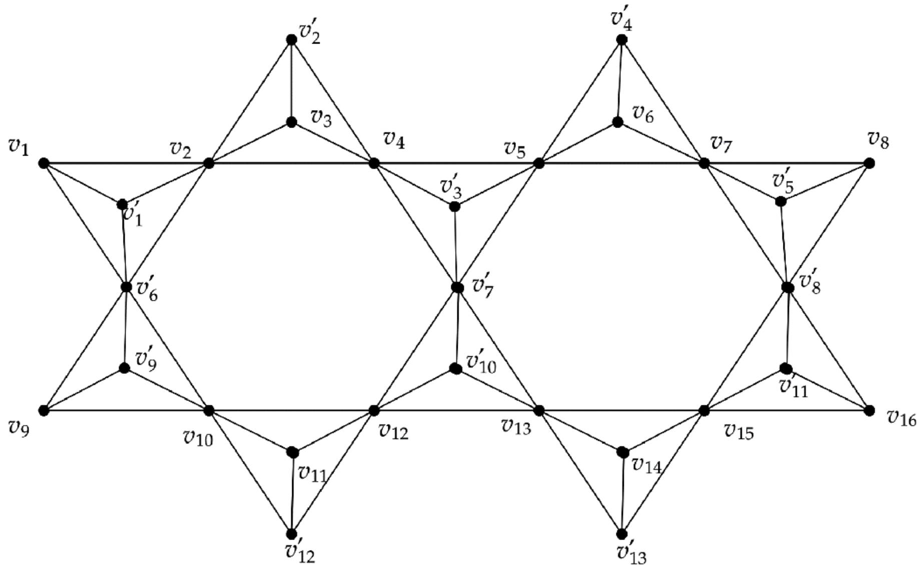

Figure 3: Double silicate chain SL(2,2).

Case 2. let $X = \{v_1, v_2, \dots v_r\} \subseteq \{v_1, v_2, \dots v_{3n+1}\}$ $r \epsilon N$

(i) $X = \{v_1, v_2, \dots v_r\}$ each $v_i's$ adjacent $v_j's$ then, possible cardinality of X is 2 or 3. If X contains two vertices then, $|N[X] \cap S|$ = 4 or 5 or 6 and $|N[X] - S|$= 3 or 5 or 4 respectively, if X contains three vertices then, $|N[X] \cap S| = 5$ and $|N[X] - S|$=5.

(ii) $X = \{v_1, v_2, \dots v_r\}$ some of $v_i's$ adjacent to $v_j's$,

$N[X]$=$\{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m, v_1', v_2', \dots v_m'\}$,here$\{v_i, v_j, \dots v_m\} \subseteq \{v_1, v_2, \dots v_{3n+1}\}$ $m \epsilon N$, $N[X] \cap S = \{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m\}$ and $N[X] - S = \{v_1', v_2', \dots v_m'\}$

or

$N[X]$=$\{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m, v_1', v_2', \dots v_k'\}$, here $\{v_1', v_2', \dots v_k'\} \subseteq \{v_1', v_2', \dots v_m'\}$ $k \in N$, $N[X] \cap S = \{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m\}$ and $N[X] - S = \{v_1', v_2', \dots v_k'\}$ $k < m$, $|N[X] \cap S| = m$ and $|N[X] - S| \leqslant m$

(iii) $X = \{v_1, v_2, \dots v_r\}$ vertices are not adjacent, $N[X] = \{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m, v_1', v_2', \dots v_k'\}$ or $\{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m, v_1', v_2', \dots v_k'\}$, here, $\{v_1', v_2', \dots v_k'\} \subseteq \{v_1', v_2', \dots v_m'\}$ and $k < m$, $N[X] \cap S = \{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m\}$ and $N[X] - S = \{v_1', v_2', \dots v_k'\}$ or $\{v_1', v_2', \dots v_m'\}$ $|N[X] \cap S| = m$ and $|N[X] - S| \leqslant m$. In all these cases the security condition will hold.

Case 3. X = $\{v_i\}$ $|N[X] \cap S|$ = 2 or 4 or 3 and $|N[X] - S|$= 2 or 3 or 1 respectively. Security condition holds. Hence the proof. □

Theorem 2.4 *Let $L(SL(2,n))$ be line graph of double silicate chain, $n \geqslant 3$ then $\gamma_s L(SL(2,n)) = 6n$.*

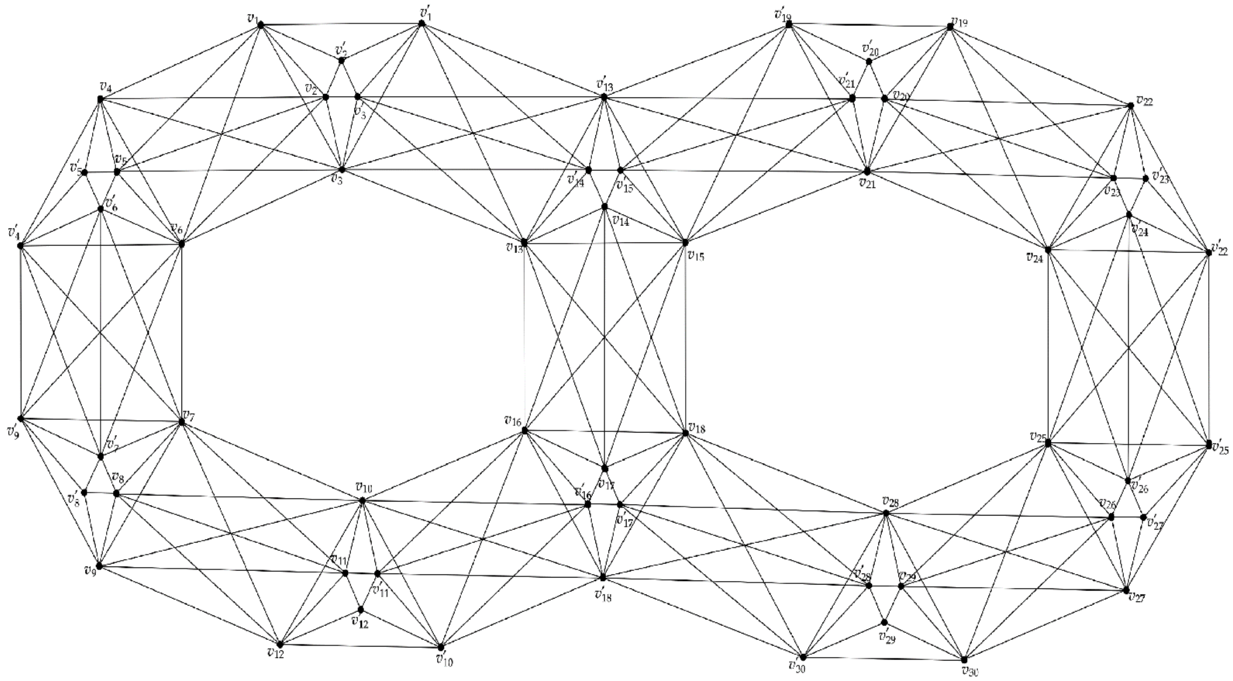

Figure 4: Line graph of SL(2,2) in figure(3).

Proof. By defination 1.1, line graph of double silicate chain $L(SL(2,n))$ consists the $12n$ vertices, and by lemma 2, $\gamma_s L(SL(1,n)) \geqslant \frac{12n}{2} = 6n$.

Consider $S = \{v_1, v_2, \dots v_{6n}\}$ clearly S is dominating set, since $v_i's$ dominates $v_i''s$. To prove S is secure in G.

We consider the following cases.

Case 1. let $X = S$ then $|N[X] \cap S| = 6n$ $|N[X] - S| = 6n$, security condition holds.

Case 2. let $X = \{v_1, v_2, \dots v_r\} \subseteq \{v_1, v_2, \dots v_{6n}\}$ $r \epsilon N$

(i) $X = \{v_1, v_2, \dots v_r\}$ each $v_i's$ adjacent $v_j's$ then, the possible cardinality of X is 2 or 3 or 4 or 5 or 6. If X contains 4 or 5 or 6 vertices then $|N[X] \cap S|$ = 6 and $|N[X] - S| \leqslant 6$ respectively, if X contains 2 vertices then $|N[X] \cap S|$= 3 or 6 and $|N[X] - S|$ = 3 or 4 respectively, if X contains 3 vertices then $|N[X] \cap S|$= 3 or 6 and $|N[X] - S|$ = 3 or 5 respectively.

(ii) X = $\{v_1, v_2, \dots v_r\}$ some of $v_i's$ adjacent $v_j's$

N[X]=$\{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m, v'_1, v'_2, \dots v'_m\}$,here$\{v_i, v_j, \dots v_m\} \subseteq \{v_1, v_2, \dots v_{6n}\}$, $m \epsilon N$, $N[X] \cap S = \{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m\}$ and $N[X] - S = \{v'_1, v'_2, \dots v'_m\}$ $|N[X] \cap S| = m$ and $|N[X] - S| = m$

or

$N[X] = \{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m, v'_1, v'_2, \dots v'_k\}$, $\{v'_1, v'_2, \dots v'_k\} \subseteq \{v'_1, v'_2, \dots v'_m\}$, $k \epsilon N$, $N[X] \cap S = \{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m\}$ and $N[X] - S = \{v'_1, v'_2, \dots v'_k\}$ $k < m$,$|N[X] \cap S| = m$ and $|N[X] - S| < m$.

(iii) X=$\{v_1, v_2, \dots v_r\}$ vertices are not adjacent, $N[X] = \{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m, v'_1, v'_2, \dots v'_k\}$,

$N[X] \cap S = \{v_1, v_2, \dots v_r, v_i, v_j, \dots v_m\}$and $N[X] - S = \{v'_1, v'_2, \dots v'_k\}$, $k < m$ $|N[X] \cap S| = m$ and $|N[X] - S| < m$. In all three cases the security condition hold.

Case 3. $X = \{v_i\}$ $|N[X] \cap S|$ = 6 and $|N[X] - S| = 2$. Security condition holds. Hence the proof.

2.2 SILICATE NETWORK

Theorem 2.5 *Let $SL(n)$ be silicate network, $n \epsilon N$ then $\gamma_s SL(n) = 9n^2 + n$.*

Proof. By theorem 1.1, silicate network $SL(n)$ consists $3n(5n+1)$ vertices, by lemma (2), $\gamma_s(SL(n)) \geqslant \frac{3n(5n+1)}{2}$ So, the set S contains $9n^2 + n$ vertices,

$$i.e, 9n^2 + n = 6\sum_{k=0}^{n-1}(n+k) + 4n$$

We consider the following cases and prove the result by induction on method.

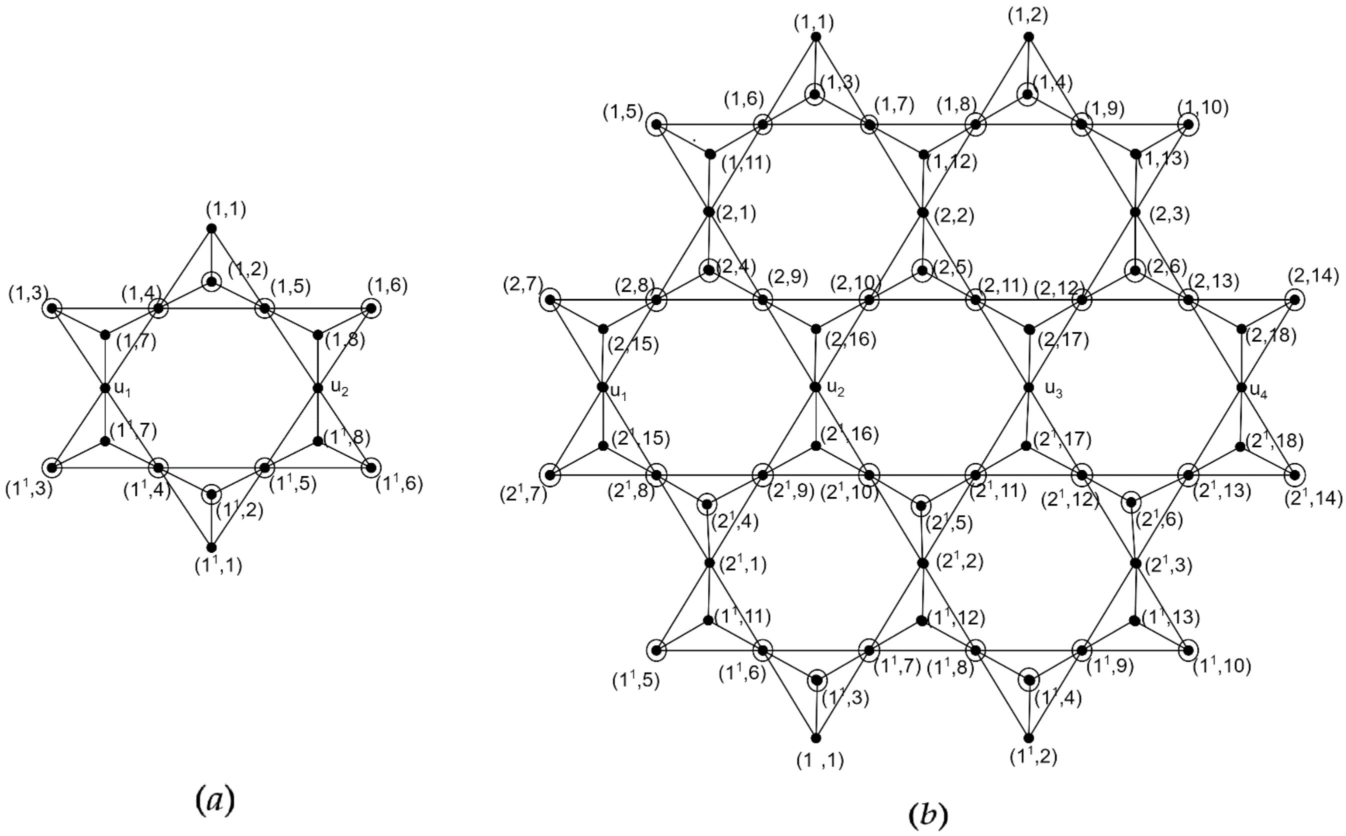

Figure 5: (a) Silicate network of dimension 1 and (b) Silicate network of dimension 2.

Case 1. $n = 1$

From the fig $5(a)$, it is clear that S (circled vertices) is dominating set, where $|S| = 6(1) + 4 = 10$. Now to prove that S is secured, for this we consider the following subcases.

Subcase 1.1. Consider $X = S$ then, $|N[X] \cap S|$=10 and $|N[X] - S| = 8$, security condition holds.

Subcase 1.2. Consider $X = \{(r,s)\}$ and degree $d(r,s) = 3$ then for any X, $|N[X] \cap S|$ =2 or 3 and $|N[X] - S|$= 2 or 1 respectively.

Subcase 1.3. Consider $X = \{(r, s)\}$, $d(r, s) = 6$ then for any X, $|N[X] \cap S|$=4 and $|N[X] - S| = 3$.

Subcase 1.4. Consider X =$\{(r, s), (t, v)\}$ where (r, s) and (t, v) are adjacent in $SL(n)$ and $d(r, s) = 3$ $d(t, v) = 6$ then for any X, $|N[X] \cap S|$=4 and $|N[X] - S| = 3$.

Subcase 1.5. Consider $X = \{(r, s), (t, v)\}$, where (r, s) and (t, v) are not adjacent in $SL(n)$ and $d(r, s) = 3$, $d(t, v) = 6$, then for any X, $|N[X] \cap S|$=7 and $|N[X] - S| = 4$.

Subcase 1.6. Consider $X = \{(r, s), (t, v)\}$ where (r, s) and (t, v) are adjacent in $SL(n)$ and $d(r, s) = 6 = d(t, v)$ then for any X, $|N[X] \cap S|$=5 and $|N[X] - S| = 5$.

Subcase 1.7. Consider $X = \{(r, s), (t, v)\}$ where (r, s) and (t, v) are not adjacent in $SL(n)$ and $d(r, s) = d(t, v) = 6$ then for any X, $|N[X] \cap S|$=8 and $|N[X] - S|$=6.

In any futher subcases, it will be a combination of these seven subcases. Hence for any subset X of S, the security condition will hold. Thus, $\gamma_s(SL(1))$ = 10.

Case 2. $n = 2$

From the fig $5(b)$, it is clear that S (circled vertices) is dominating set, where, $|S|$ = 6(2+3)+4(2) =38. Now we have to prove S is secured, for this we consider the following subcases.

Subcase 2.1. Consider $X = S$ then, $|N[X] \cap S|$=10 and $|N[X] - S|$=8, security condition holds.

Subcase 2.2. Consider $X = \{(r, s)\}$, $d(r, s) = 3$ then for any X, $|N[X] \cap S|$= 2 or 3 and $|N[X] - S|$=2 or 1 respectively.

Subcase 2.3. Consider $X = \{(r, s)\}$, $d(r, s) = 6$ then for any X, $|N[X] \cap S|$=4 and $|N[X] - S|$=3.

Subcase 2.4. Consider $X = \{(r, s), (t, v)\}$, where (r, s) and (t, v) are adjacent in $SL(n)$ and $d(r, s) = 3$, $d(t, v) = 6$ then for any X, $|N[X] \cap S|$=4 and $|N[X] - S|$=3.

Subcase 2.5. Consider $X = \{(r, s), (t, v)\}$, where (r, s) and (t, v) are not adjacent in $SL(n)$ and $d(r, s) = 3, d(t, v) = 6$ then for any, X $|N[X] \cap S|$=7 and $|N[X] - S|$=4.

Subcase 2.6. Consider $X = \{(r, s), (t, v)\}$, where (r, s) and (t, v) are adjacent in $SL(n)$ and $d(r, s) = 6 = d(t, v)$ then for any X, $|N[X] \cap S|$=5 and $|N[X] - S|$=5

Subcase 2.7. Consider $X = \{(r, s), (t, v)\}$, where (r, s) and (t, v) are not adjacent in $SL(n)$ and $d(r, s) = d(t, v) = 6$ then for any X, $|N[X] \cap S|$=8 and $|N[X] - S|$=6.

In any further subcase, these 7 subcases will be combination, hence for any subset X of S, the security condition hold. Thus, $\gamma_s(SL(2))$ = 38 .

Now we have to assume that the result is true for $n - 1$

$i.e, 9n^2 + n = 6\sum_{k=0}^{n-2} ((n - 1) + k) + 4(n - 1) = 9(n - 1)^2 + (n - 1)$ and

security condition holds for any subset X of S.

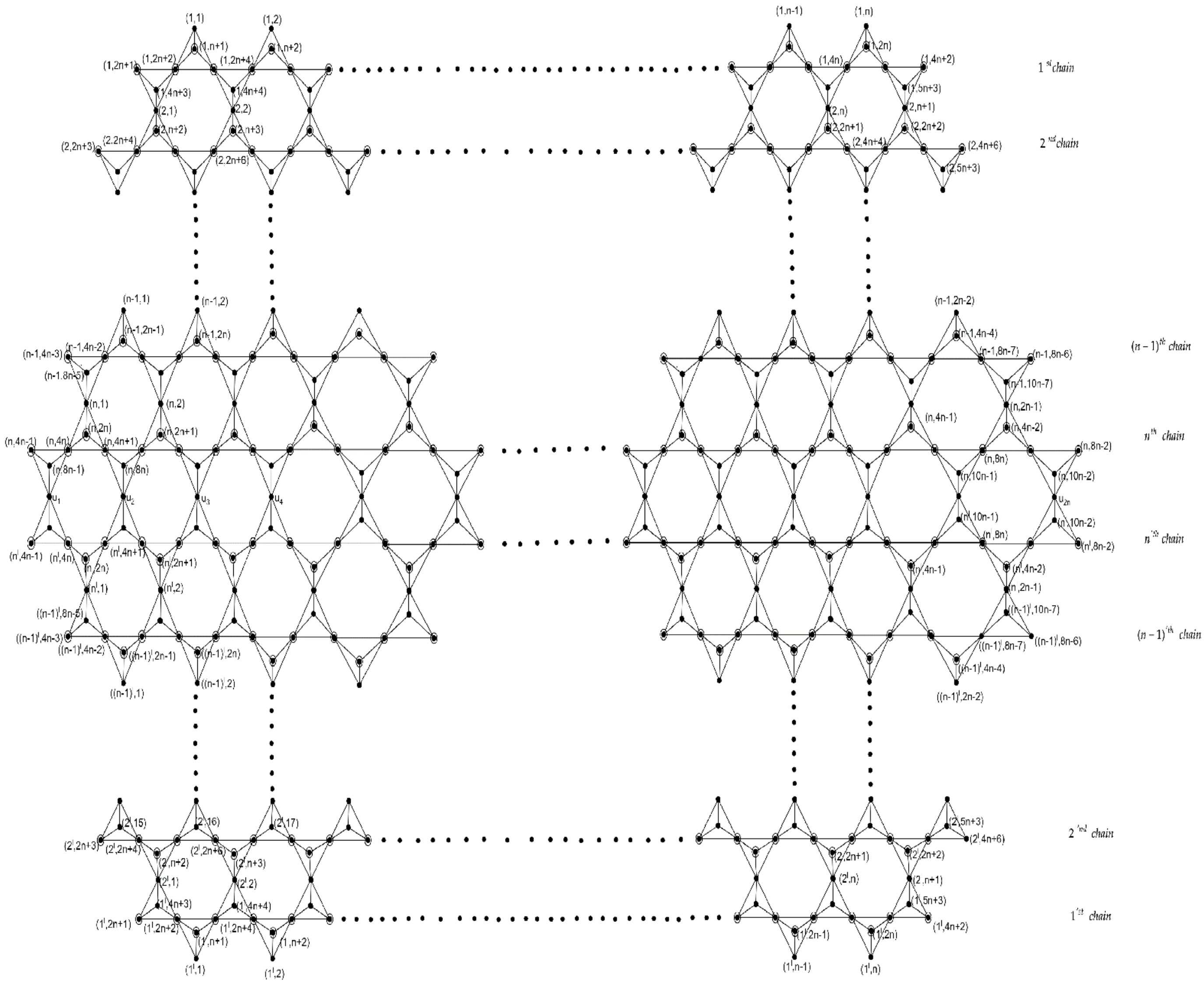

Figure 6: Silicate network of dimension n.

Now we define a set S for $SL(n)$ as follows.

$$S=\{(1,n+1),(1,n+2),(1,n+3),\ldots\ldots\ldots(1,4n+2),$$

$$(2,n+2),(2,n+3),(2,n+4),\ldots\ldots\ldots(2,4n+6)$$

$$(3,n+3),(3,n+4),(3,n+5),\ldots\ldots\ldots(3,4n+10)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$(n,2n),(n,2n+1),(n,2n+2),\ldots\ldots\ldots(n,8n-2)$$

$$(n',2n),(n',2n+1),(n',2n+2),\ldots\ldots\ldots(n',8n-2)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$(3',n+3),(3',n+4),(3',n+5),\ldots\ldots\ldots(3',4n+10)$$

$$(2',n+2),(2',n+3),(2',n+4),\ldots\ldots\ldots(2',4n+6)$$

$$(1',n+1),(1',n+2),(1',n+3),\ldots\ldots\ldots(1',4n+2)\}$$

We have to prove that S is dominating set. For this we choose the vertices from each chain which dominates the entire chain of $SL(n)$.

In 1^{st} chain there are $(n+1)$ vertices, $\{(1,2n+2),(1,2n+4),(1,2n+6)\ldots and(1,4n)\} \in S$,
In 2^{nd} chain there are $(n+2)$ vertices, $\{(2,2n+4),(2,2n+6),(2,2n+8)\ldots and(2,4n+6)\} \in S$

……………………………………………………..

In $(n-1)^{th}$ chain there are $(2n-1)$ vertices, $\{(n-1,4n-2),(n-1,4n-4),(n-1,4n-6)\ldots and(n-1,8n-6)\} \in S$

In n^{th} chain there are $2n$ vertices, $\{(n,4n),(n,4n+2),(n,4n+4)\ldots and(n,8n-2)\} \in S$

In n'^{th} chain there are $2n$ vertices, $\{(n',4n),(n',4n+2),(n',4n+4)\ldots and(n',8n-2)\} \in S$

In $(n-1)'^{th}$ chain there are $(2n-1)$ vertices, $\{((n-1)',4n-2),((n-1)',4n-4),((n-1)',4n-6)\ldots and((n-1)',8n-6)\} \in S$

....

In $2'^{nd}$ chain there are $(n+2)$ vertices, $\{(2',2n+4),(2',2n+6),(2',2n+8)\ldots and(2^1,4n+6)\} \in S$

In $1'^{st}$ chain there are $(n+1)$ vertices, $(1',2n+2),(1',2n+4),(1',2n+6)\ldots and(1',4n) \in S$.

These vertices dominate the entire network $SL(n)$. So, S is dominating set. Now we have to prove that security condition.

Each vertex in silicate network is of degree 3 or 6 and by the same argument as in the cases $n=1$ and $n=2$ it shows that S is secure set.

S satisfy's both secure and dominating condtion, Hence S is secure dominating set. □

3.0 Conclusions

Security is undoubtedly an important aspect for interconnection networks nowadays, therefore this type study has numerous applications. In this paper we have obtained results on secure domination number of silicate chains and line graph of silicate chain and further computed the results on silicate network.

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