Total neighborhood Strongly Multiplicative Labeling of Some Standard Graphs

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Abstract

In this paper, introduce a new type of labeling called total neighborhood strongly multiplicative labeling of some standared graphs. Let $G = (V(G), E(G))$ be a graph with p vertices and q edges. A bijective function $f: V(G) \cup E(G) \rightarrow \{1,2,3, \ldots, p+q\}$ is said to be a total neighborhood strongly multiplicative labeling, if for each vertex v in G with deg $(v) > 1$, the product of the labels of it's neighborhood vertices and edges are all distinct. A graph which admits a total neighborhood strongly multiplicative labeling is called a total neighborhood strongly multiplicative graph. In this manuscript we prove that the Path graph P_n,Comb graph $P_n \odot K_1$, Degree splitting of path $DS(P_n)$, Complete bipartite graph $K_{m,n}$ and the Duplication of complete bipartite graph $D(K_{m,n})$ are all total neighborhood strongly multiplicative graphs.

Keywords: Neighborhood Strongly multiplicative Labeling, Total neighborhood strongly multiplicative, Path graph, Comb graph, Degree splitting of path, Complete bipartite, Duplication of complete bipartite graph.

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Introduction

Graph theory is a new branch of Mathematics with lot of applications. There are different type of parameters that have assumed significant importance. The first paper of graph theory was written by Euler in 1736 when he settled the famous unsolved problems of his day, known as the Konigsberg bridge problem, over the first fifty years, graph theory has become one of the most rapidly growing area of Mathematics. The concept of graph labeling was introduced by Alex Rosa 1967 [1]. A useful survey on graph labeling by J.A Gillian (2014) can be found in [5]. If the vertices of the graph are assigned values subject to certain conditions, then it is known as graph labeling. Vast amount of literature is available on different types of graph labeling. Labeled graph have variety of applications in coding theory, particularly for missile guidance codes with optimal auto correlation properties. Labeled graph play vital role in the communication network. In this paper we deal only finite, simple, connected and undirected graphs. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$. Hence $|V(G)|$ and $|E(G)|$ are the number of vertices and number of edges of G respectively. The concept of strongly multiplicative labeling was first introduced by Beineke and Hedge[2]. S.K Patel and N.Shrimali in[11] introduced the concept of neighborhood prime labeling in 2015.Motivated by the above work, we introduce the concept of total neighborhood strongly multiplicative labeling. "Let $G = (V(G), E(G))$ be a graph with p vertices and q edges. A bijective function $f: V(G) \cup E(G) \rightarrow \{1,2,3, \dots, p+q\}$ is said to be a total neighborhood strongly multiplicative labeling, if for each vertex v in G with $\deg(v) > 1$, the product of the labels of it's neighborhood vertices and edges are all distinct. A graph which admits total neighborhood strongly multiplicative labeling is called a total neighborhood strongly multiplicative graph." In this paper we investigate the total neighborhood strongly multiplicative labeling of some standared graphs.

1 Preliminaries

Definition:1.1 [4] A walk in which no vertex is repeated is called a path P_n. It has n vertices and $n-1$ edges.

Definition:1.2 [12] The comb is the graph obtained from a path P_n by attaching a pendant vertex to each vertex of the path. It is denoted by $P_n \odot K_1$.

Definition:1.33 [10] Let $G = (V, E)$ be a graph with $V = S_1 \cup S_2 \cup S_3 \cup \dots S_i \cup T$ where each S_i is a set of vertices having least two vertices of the same degree and $T = V \backslash \cup S_i$. The Degree Splitting graph of G denoted by $DS(G)$ is obtained from G by adding a vertices w_i and joining to each vertex of S_i for $1 \leq i \leq n$.

Definition:1.4 [7] A complete bipartite graph is a simple bipartite graph with bipartition (X, Y) in which each vertex of X is joined to each vertex of Y; if $|X| = m$ and$|Y| = n$, such a graph is denoted by $K_{m,n}$.

Definition:1.5 [9] Duplication of a vertex v of G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$.In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G'.

2 Main results

Theorem 2.1. The path P_n is a total neighborhood strongly multiplicative graph for all $n \geq 2$.

Proof. Let P_n be the path with vertex set $V(P_n) = \{v_i: 1 \leq i \leq n\}$,where v_1 and v_n are of degree 1. While the remaining vertices having degree 2. Let $E(P_n) = \{v_i v_{i+1}: 1 \leq i \leq n-1\}$ be the edge set of P_n.

We note that $|V(P_n)| = n$ and $|E(P_n)| = n-1$

We define a vertex and edge labelings $f: V(G) \cup E(G) \rightarrow \{1,2,3, \dots, 2n-1\}$ as follows.

The vertex labelings are

$$f(v_i) = 2i - 1, \quad 1 \leq i \leq n$$

The edge labelings are

$$f(v_i v_{i+1}) = 2i, \quad 1 \leq i \leq n-1$$

The labeling pattern defined above satisfies the vertex and edge conditions.
Let v_i be any vertex of P_n, whose degree is greater than 1.
The product of the labels of it's neighborhood vertices and edges of $v_i (2 \leq i \leq n-1)$ are $(2i)(2i+1)(2i-2)(2i-3)$. Thus G admits a total neighborhood strongly multiplicative labeling.
Hence P_n is total neighborhood strongly multiplicative graph.

Illustration 2.1. The Path graph P_n and its total neighborhood strongly multiplicative labeling is shown in figure:1

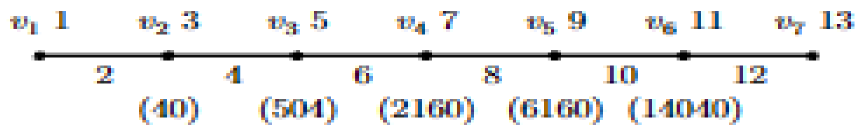

Figure 1:The total neighborhood strongly multiplicative labeling of P_7

Theorem 2.2. The comb graph $P_n \odot K_1$ is a total neighborhood strongly multiplicative graph.

Proof. Let $P_n \odot K_1$ be the comb graph with vertex set $V(P_n \odot K_1) = \{u_i, v_i : 1 \leq i \leq n\}$,where u_1 and u_n having degree 2 and all the internal vertices u_i are of degree 3, While the remaining vertices $v_i's$ are of degree 1. Let $E(P_n \odot K_1) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\}$ be the edge set of $P_n \odot K_1$.
We note that $|V(P_n \odot K_1)| = 2n$ and $|E(P_n \odot K_1)| = 2n-1$
We define a vertex and edge labelings $f: V(P_n \odot K_1) \cup E(P_n \odot K_1) \rightarrow \{1,2,3, \dots, 4n-1\}$ as follows.
The vertex labelings are

$$f(u_i) = 4i-3, \quad 1 \leq i \leq n-1$$
$$f(v_i) = 4i-1, \quad 1 \leq i \leq n-2$$
$$f(u_n) = 4n-2$$
$$f(v_n) = 4n-1$$

The edge labelings are

$$f(u_i u_{i+1}) = 4i\,, \quad 1 \leq i \leq n-2$$
$$f(u_{n-1} u_n) = 4n-5$$
$$f(u_i v_i) = 4i-2\,, \quad 1 \leq i \leq n-1$$
$$f(u_n v_n) = 4n-3$$

The labeling pattern defined above satisfies the vertex and edge conditions.
Here u_i are the vertices of $P_n \odot K_1$, whose degree is greater than 1.
The product of the labels of it's neighborhood vertices and edges of $u_1, u_i (2 \leq i \leq n-2), u_{n-1}$ and u_n are $120, (4i)(4i-7)(4i-4)(4i+1)(4i-2)(4i-1), (4n-2)(4n-8)$

$(4n-6)(4n-4)(4n-5)(4n-11)$ and $(4n-5)(4n-7)(4n-3)(4n-1)$ respectively. Thus the product of the labels of it's neighborhood vertices and edges are all distinct.
Hence $P_n \odot K_1$ is total neighborhood strongly multiplicative graph.

Illustration 2.2. The comb graph $P_n \odot K_1$ and its total neighborhood strongly multiplicative labeling is shown in figure:2

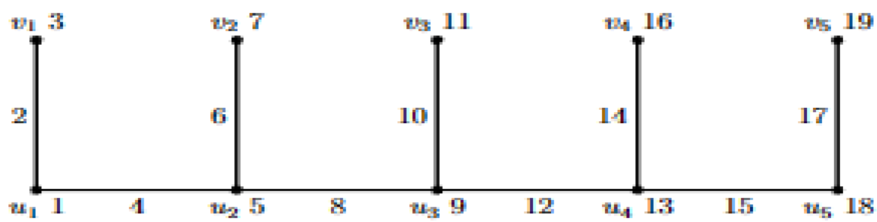

Figure 2:The total neighborhood strongly multiplicative labeling of $P_5 \odot K_1$

Theorem 2.3. The degree splitting graph of path P_n is a total neighborhood strongly multiplicative graph for all $n \geq 4$.

Proof. Let $G = DS(P_n)$ be the degree splitting graph of path graph P_n with vertex set $V(G) = \{u_i : 1 \leq i \leq n\} \cup \{x, y\}$, where u_1, u_n and y having degree 2 and x having degree $n-2$, While the remaining vertices are of degree 3.
Let $E(G) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{x u_i : 2 \leq i \leq n-1\} \cup \{y u_1, y u_n\}$ be the edge set of G.

We note that $|V(G)| = n+2$ and $|E(G)| = 2n-1$
We define a vertex and edge labelings $f : V(G) \cup E(G) \to \{1,2,3, \dots, 3n+1\}$ as follows.
The vertex labelings are

$$f(x) = 2n+3$$
$$f(y) = 1$$
$$f(u_i) = 2i+2, \quad 1 \leq i \leq n-2$$
$$f(u_n) = 2n+2$$
$$f(u_{n-1}) = 2n-1$$

The edge labelings are

$$f(yu_i) = 2$$
$$f(yu_n) = 3$$
$$f(xu_i) = 2n+2+i, \quad 2 \leq i \leq n-1$$
$$f(u_i u_{i+1}) = 2i+3, \quad 1 \leq i \leq n-3$$

The labeling pattern defined above satisfies the vertex and edge conditions.
Let x, y and u_i be the vertices of G, whose degree is greater than 1.
Clearly ,the product of the labels of it's neighborhood vertices and edges of $u_1, u_i \ (2 \leq i \leq n-2), u_{n-1}, u_n, x$ and y are
$60, \prod(2i)(2i+1)(2i+3)(2i+4)(2n+3)\ (2n+2+i)$,
$(2n)(2n-2)(2n+1)(2n+2)(3n+1), 3(2n+1)(2n-1)$,

$\prod_{i-2}^{n-1}(2n+2+i)\prod_{i=2}^{n-2}(2i+2)(2n-1)$ and $24(2n+2)$ respectively.
Thus the product of the labels of it's neighborhood vertices and edges are all distinct.
Hence $G = DS(P_n)$ is total neighborhood strongly multiplicative graph.

Illustration 2.3. The degree splitting of Path graph P_n and its total neighborhood strongly multiplicative labeling is shown in figure:3

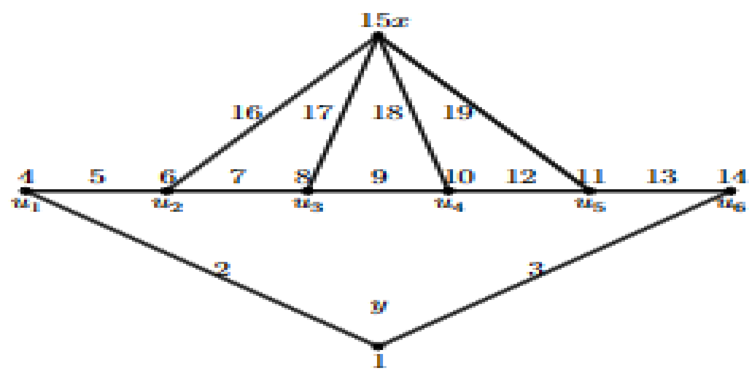

Figure 3:The total neighborhood strongly multiplicative labeling of $DS(P_6)$

Theorem.2.4. The complete bipartite graph $K_{m,n}$ is a total neighborhood strongly multiplicative graph for all $m = n$.

Proof. Let $K_{m,n}$ be the complete bipartite graph with vertex set
$V(K_{m,n}) = \{v_i u_j : 1 \le i \le m, 1 \le j \le n\}$,where each v_i and u_j having degree n and m respectively. Let $E(G) = \{v_i u_j : 1 \le i \le m, 1 \le j \le n\}$ be the edge set of $K_{m,n}$.
We note that $|V(K_{m,n}.)| = m + n$ and $|E(K_{m,n}.)| = mn$
We define a vertex and edge labelings $f: V(K_{m,n}.) \cup E(K_{m,n}.) \to \{1,2,3,\dots,m+n+mn\}$ as follows.
The vertex labelings are

$$f(v_i) = i, \quad 1 \le i \le m$$
$$f(u_j) = m + j, \quad 1 \le j \le n$$

The edge labelings are

$$f(v_i u_j) = m + n + 4i - 4 + j, \quad 1 \le i \le m, \; 1 \le j \le n$$

The labeling pattern defined above satisfies the vertex and edge conditions.
Let v_i, u_j be the vertices of $K_{m,n}$, whose degree is greater than 1.
Clearly ,the product of the labels of it's neighborhood vertices and edges of
$v_i (1 \le i \le m)$ and $u_j (1 \le j \le n)$ are $\prod_{j=1}^{n}(m+j)[m+n+4i-4+j]$ and $\prod_{i=1}^{m}(i)[m+n+4i-4+j]$respectively.
Thus the product of the labels of it's neighborhood vertices and edges are all distinct.
Hence $K_{m,n}$ is total neighborhood strongly multiplicative graph.

Illustration2.4. The complete bipartite graph$K_{m,n}$ and its total neighborhood strongly multiplicative labeling is shown in figure:4

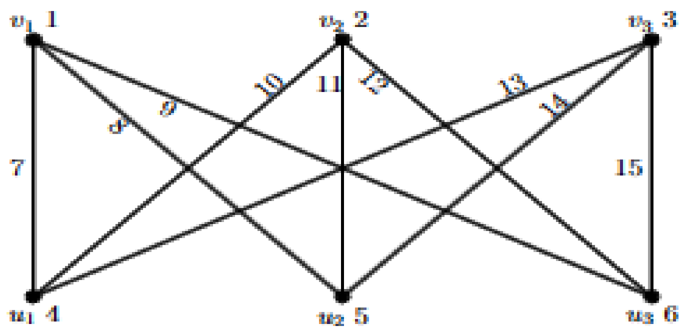

Figure 4:The total neighborhood strongly multiplicative labeling of $K_{3,3}$

Theorem.2.5. Let G be a complete bipartite graph with bipartition (X,Y) such that $|X| = m, |Y| = n$. The new graph G' be obtained by duplicate all the vertices of Y in $K_{m,n}$ is a total neighborhood strongly multiplicative graph.

Proof. Let $V(G') = \{v_i, u_j, u'_j: 1 \leq i \leq m, 1 \leq j \leq n\}$, where each u_j and u'_j having degree m and $v'_i s$ having degree $2n$.
Let $E(G') = \{v_i u_j: 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{v_i u'_j: 1 \leq i \leq m, 1 \leq j \leq n\}$be the edge set of G'.
We note that $|V(G')| = m + 2n$ and $|E(G')| = 2mn$
We define a vertex and edge labelings $f: V(G') \cup E(G') \rightarrow \{1,2,3, \ldots, m + 2n + 2mn\}$ as follows.
The vertex labelings are

$$f(v_i) = i, \quad 1 \leq i \leq m$$
$$f(u_j) = m + j, \quad 1 \leq j \leq n$$
$$f(u'_j) = m + n + j, \quad 1 \leq j \leq n$$

The edge labelings are

$$f(v_i u_j) = m + 2n + i + mj - m, \quad 1 \leq i \leq m, \quad 1 \leq j \leq n$$
$$f(v_i u'_j) = m + 2n + mn + i + mj - m, \quad 1 \leq i \leq m, \quad 1 \leq j \leq n$$

The labeling pattern defined above satisfies the vertex and edge conditions.
Let v_i, u_j, u'_j be the vertices of G',whose degree is greater than 1.
Clearly ,the product of the labels of it's neighborhood vertices and edges of $v_i (1 \leq i \leq m)$, $u_j (1 \leq j \leq n)$ and $u'_j (1 \leq j \leq n)$ are
$\prod_{j=1}^{n} (m + j)(m + 2n + i + mj - m)(m + n + j)(m + 2n + mn + i + mj - m)$,
$\prod_{i=1}^{m} (i)[m + 2n + mj - m + i]$ and $\prod_{i=1}^{m} (i)(m + 2n + mn + i + mj - m)$ respectively.
Thus the product of the labels of it's neighborhood vertices and edges are all distinct.
Hence G' is total neighborhood strongly multiplicative graph.

Illustration.2.5. The duplication of all the vertices of Y in $K_{m,n}$ and its total neighborhood strongly multiplicative labeling is shown in figure:5

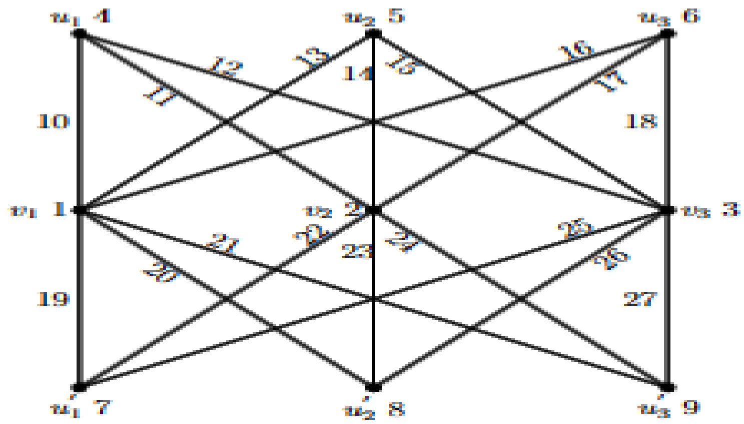

Figure 5:The total neighborhood strongly multiplicative labeling of $D(K_{3,3})$

Conclusion

In this paper, we discussed the total neighborhood strongly multiplicative labeling. The total neighborhood strongly multiplicative labeling conditions are satisfied the some special classes of graphs likely Path, Comb , Complete bipartite graph, Degree splitting of path graph, Duplication of complete bipartite graph. There may be many interesting total neighborhood strongly multiplicative graphs can be constructed in future

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