STUDY OF CHAOS ANTI-SYNCHRONIZATION OF DIFFERENT CHAOTIC SYSTEMS VIA ACTIVE NONLINEAR CONTROL

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Abstract:
This article addresses the Anti-synchronization problem of a class of uncertain chaotic systems with nonlinear inputs in a drive response structure. The high complexity of the unknown parameters of system can provide a new approach for enhancing message security in chaos based encryption systems. With uncertain, advanced three dimensional Chen-Lee, Lorenz-Stenflo and Liu-Chen systems have been considered to demonstrate the anti-synchronization problem. Simulation results are presented to illustrate the effectiveness of the considered chaotic systems.

Keywords: Chen-Lee systems, Lorenz-Stenflo systems, Liu-Chen systems, Active control, Synchronization

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1. Introduction

In dynamical systems chaos is very common and has highly demonstrated phenomena. Around the required fixed point to entitle application of linear control theory the ordinary differential equation of the chaotic dynamical system is linearised. Although this idea is very effective but for the linearised model the approximation region is not exactly defined. Also linearisation may take away the chaotic behaviour to a certain degree. In 1990, Pecora and Carroll [1] developed a method for the synchronization of two identical chaotic systems. After that chaos synchronization has attracted a great attention from different fields and has been encountered by researchers in the last few decades. By synchronization means the trajectory of drive system to govern response system so that trajectory of response system goes along with trajectory of drive system asymptotically.

However, most of the study has based on chaotic systems with certain parameters. But in real application, some or all of the parameters are uncertain. In fact, one can rarely ever attain parameters of chaotic systems in communication theory, so the study on the synchronization of uncertain chaotic systems has some practical application.

In low dimensional chaotic systems synchronization is introduced. Subsequently, in the field of nonlinear dynamics, synchronization of high dimensional chaotic systems has emerged an interesting area. Therefore, the research on synchronization of chaotic systems has obtained reasonable identity. Chaotic systems are naturally connected to systems with memory that convinced for most of biological and engineering systems such as dynamics of optical systems viz. Ikeda system, blood production in patients with leukaemia viz. Mackey-Glass model, population dynamics, laser physics, physiological model, control system, neural networks, etc.

In practical applications of chaos synchronization theory, uncertainties, external disturbances and time delays occur commonly in chaotic synchronization and many more real control systems. There are still many problems whose studied to be worthy of attention in this field. For example, time delay can consider in inaccurate feedback control behaviours, and then result in the instability and non synchronous phenomena of chaotic systems. There is still scope to put some efforts to reduce the conservatism in order to

$y = g\,y(y) + u\,(x, y)$

synchronize chaotic systems efficiently and more accurately.

Motivated by the above discussions, author investigates synchronization of chaotic systems subjected to uncertainties and external disturbances. This paper also addresses the chaotic attractor of two different uncertain, external disturbance chaotic systems with non linearity inputs.

In the last decade, several types of synchronization have been elucidated in interacting chaotic systems, such as complete synchronization, anti-synchronization, phase synchronization, lag synchronization, projective synchronization and transient ladder synchronization etc. Complete synchronization is characterised by the convergence of two chaotic trajectories, $x\,t(t) = y\,t(t)$. In anti-synchronization the state vectors of the two chaotic systems have the same absolute value but are opposite sign i.e. $x\,t(t) = -\,y\,t(t)$. And in the projective synchronization the states vector are in proportional scale i.e. $x\,t(t) = \alpha\,y\,t(t)$ where α is a scaling factor.

The chaos synchronization approaches when two or more chaotic systems are coupled or when a chaotic system derives another chaotic system. These chaotic systems are called master or drive and other chaotic systems are called slave or response systems. And has wide application in several fields such as, biological systems, chemical systems, nonlinear science, and secure communication.

A wide variety of approaches have successfully been applied to the chaos synchronization of chaotic systems, including adaptive control [2], active control [4], back stepping design [3], nonlinear control [5] and observer based control methods [6]. Using these methods, numerous synchronization problems of well known chaotic systems have been worked on by many researchers.

2. System description

Consider the nonlinear chaotic (Hyperchaotic) system as, driver

$$\dot{x} = f\,x(\,)$$

And another chaotic (Hyperchaotic) system as, responser where *x y*, are the states of the systems and *f g R*, : $^n \to R^n$ are nonlinear part of the systems and *u* (*x y*,) are controller.

In order to achieve the chaos anti-synchronization between *x* and *y*, let *e*=*y*+*x* be the synchronization error. Our aim to find a suitable controller *u* (*x y*) such that the trajectory of the responser with initial condition y_0 can asymptotically approach the driver system with initial condition x_0 in converse direction i.e. lim t→∞ *y t*(t) + *x t*(t) ‖ = 0, where . is Euclidean norm.

The chaotic Chen-Lee system [7-8] is given as

$$\dot{x}_1 = -x_2 x_3 + a_1 x_1$$
$$\dot{x}_2 = x_1 x_3 + b_1 x_2$$
$$\dot{x}_3 = \frac{1}{3} x_1 x_2 + c_1 x_3$$

where *x x x R*1 2 3, , ∈ *n* are states vector and *a b c*1 1 1, , are parameters. The system is chaotic when the parameter values $a_1 = 5, b_1 = -10, c_1 = -3.8$.

The chaotic Lorenz-Stenflo system [9] is given as

$$\dot{x}_1 = a(x_2 - x_1)$$
$$\dot{x}_2 = x_1(c - x_3) - x_2$$
$$\dot{x}_3 = x_1 x_2 - d x_3$$

where *x x x R*1 2 3, , ∈ *n* are states vector and *a b c d*, , , are parameters. The system is chaotic when the parameter valuesa $=1, b=1.5, c=26, d=0.7$. The chaotic Liu- Chen system [10] is given as

$$\dot{x}_1 = a x_1 - x_2 x_3$$
$$\dot{x}_2 = -b x_2 + x_1 x_3$$
$$\dot{x}_3 = x_1 x_2 - c x_3$$

where *x x x R*1, 2, 3∈ *n* are states vector and *a b c*, , are parameters. The system is chaotic when the parameter values $a = 0.4, b = 12, c = 5$.

3. Anti-synchronization between chaotic Chen-Lee system and chaotic Lorenz-stenflo systems

Here we are taking chen-Lee system as a master system and Lorenz-Stenflo as a Slave system and design a suitable controller and synchronize the above systems.

$$\begin{aligned} \dot{x}_1 &= -x_2 x_3 + a_1 x_1 \\ \dot{x}_2 &= x_1 x_3 + b_1 x_2 \\ \dot{x}_3 &= \frac{1}{3} x_1 x_2 + c_1 x_3 \end{aligned} \qquad (3.1)$$

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= y_1(c - y_3) - y_2 + u_2 \\ \dot{y}_3 &= y_1 y_2 - d y_3 + u_3 \end{aligned} \qquad (3.2)$$

where *u u u*1 2,, 3 are controller to be design.

Defining an anti-synchronization error between *x* and *y* as

$$e_i = y_i + x_i (i = 1, 2, 3) \qquad (3.3)$$

From (3.1) and (3.2) we get an error dynamics as

$$\begin{aligned}\dot{e}_1 &= a(y_2 - y_1) - ae_1 + a_1x_1 - x_2x_3 + u_1\\ \dot{e}_2 &= y_1(c - y_3) - e_2 + x_2(1 + b_1) + x_1x_3 + u_2\\ \dot{e}_3 &= y_1y_2 - de_3 + \frac{1}{3}x_1x_2 + x_3(d + c_1) + u_3\end{aligned} \qquad (3.4)$$

According to the active control theory defining the controller as

$$\begin{aligned}u_1 &= -a(y_2 + x_1) - a_1x_1 + x_2x_3\\ u_2 &= -y_1(c - y_3) - x_2(1 + b_1) - x_1x_3\\ u_3 &= -y_1y_2 - \frac{1}{3}x_1x_2 - x_3(d + c_1)\end{aligned} \qquad (3.5)$$

Then the error dynamics becomes

$$\begin{aligned}\dot{e}_1 &= -ae_1\\ \dot{e}_2 &= -e_2\\ \dot{e}_3 &= -de_3\end{aligned} \qquad (3.6)$$

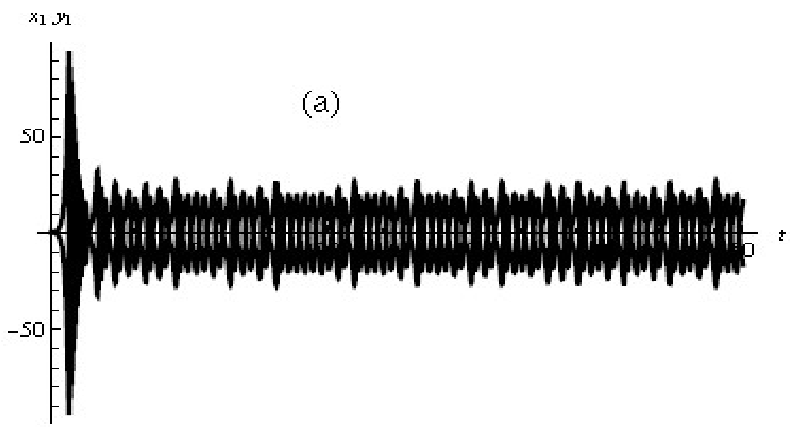
we rewrite the error system as $\dot{e} = Ae$, where

$$A = \begin{bmatrix} -a & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -d \end{bmatrix} \qquad (3.7)$$

The eigenvalue of A are negative, so as t e $\to \infty$ $\to 0$. Hence by Lyapunov stability [11] the error system is asymptotically stable. It implies that the two Chen-Lee and LorenzStenflo chaotic systems are anti-synchronized.

Numerical Simulation:

To validate the feasibility of the control functionsu_i , we simulate the dynamics of the coupled master and slave system. In simulation, we choose the parameters a b c = = − = − 5, 10, 3.8 and a b c d = = = = 1, 1.5, 26, 0.7to ensure two systems are haotic without control. By using the MATHEMATICA to solve the above systems and taking the initial condition of master and slave systems as $x_0 = (0.2,0.2,0.2)$ and $y_0 = (1,2,3)$. Fig. 1. *(a)*, *(b)*, *(c)* Show the time evolution of master and slave. Fig.2 shows the antisynchronization error of master and slave systems.



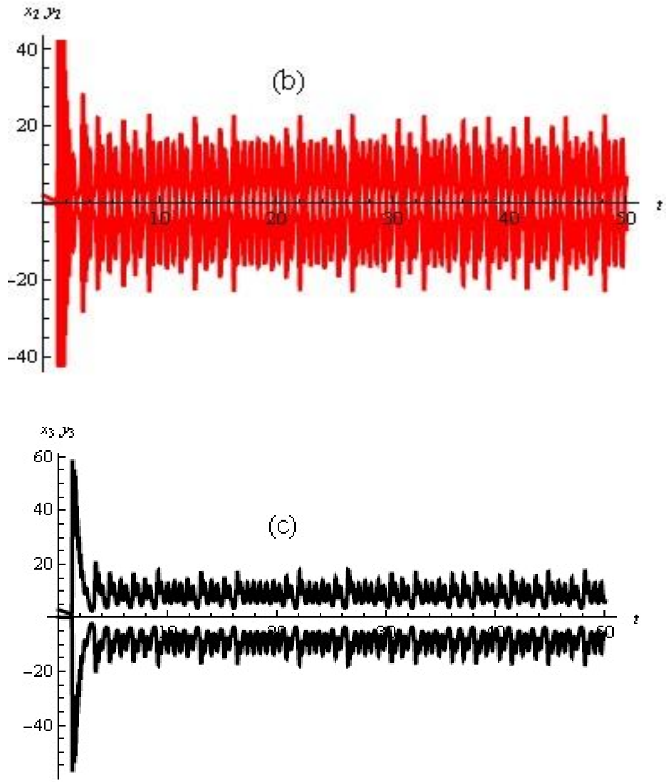
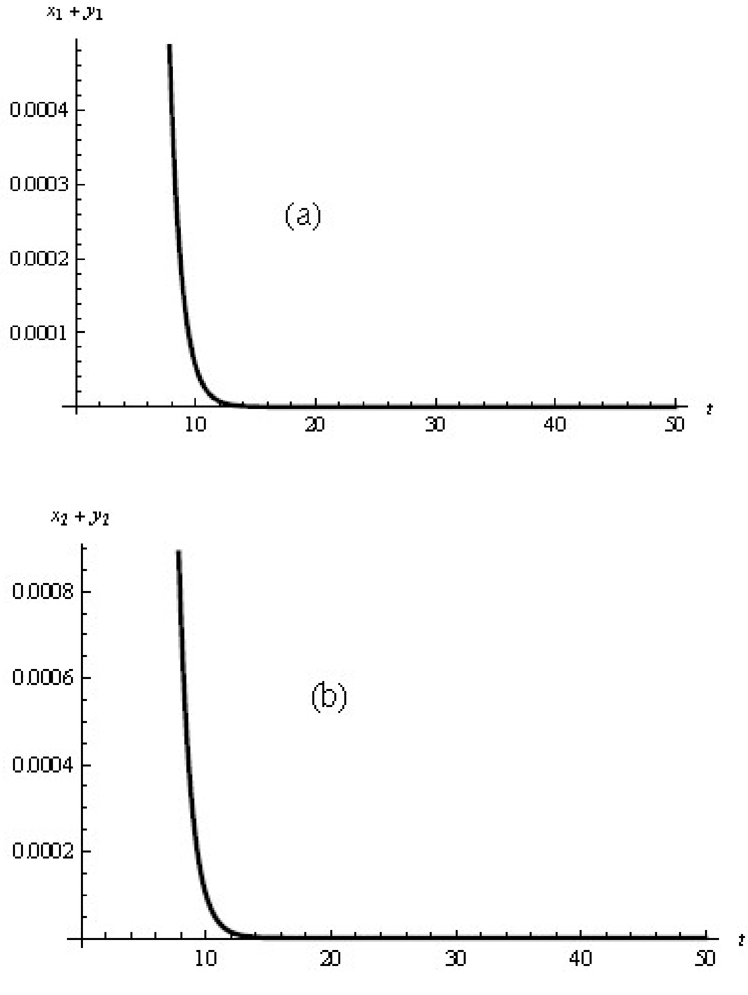


Fig. 1. *(a), (b), (c)* Show the time evolution of the state vector of master and slave systems



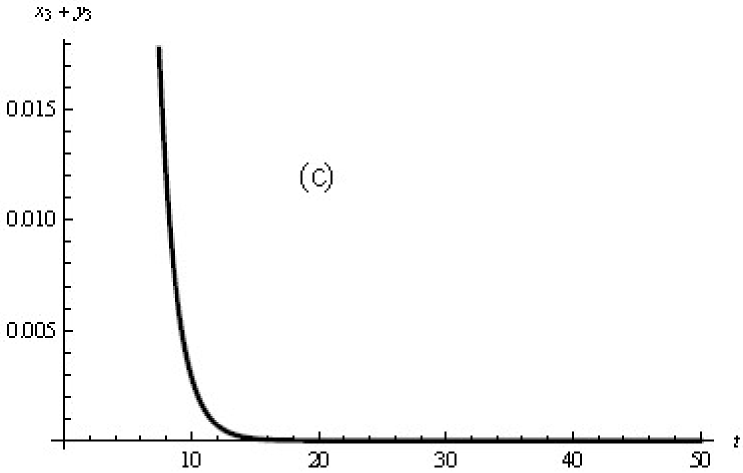


Fig.2 *(a), (b), (c)* shows the anti-synchronization error of master and slave systems.

4.Anti-synchronization of chaotic Chen-Lee and Liu-Chen systems

The master system and Slave systems as

$$\begin{aligned}\dot{x}_1 &= -x_2x_3 + a_1x_1\\ \dot{x}_2 &= x_1x_3 + b_1x_2\\ \dot{x}_3 &= \frac{1}{3}x_1x_2 + c_1x_3\end{aligned} \quad (4.1)$$

$$\begin{aligned}\dot{y}_1 &= ay_1 - y_2y_3 + u_1\\ \dot{y}_2 &= -by_2 + y_1y_3 + u_2\\ \dot{y}_3 &= y_1y_2 - cy_3 + u_3\end{aligned} \quad (4.2)$$

where u_1, u_2, u_3 are controller to be design.

Defining an anti-synchronization error between x and y as

$$e_i = y_i + x_i (i = 1,2,3) \quad (4.3)$$

From (4.1) and (4.2) we get an error dynamics as

$$\begin{aligned}\dot{e}_1 &= ay_1 + a_1x_1 - x_2x_3 - y_2y_3 + u_1\\ \dot{e}_2 &= y_1y_3 + x_1x_3 + x_2(b_1 + b) - be_2 + u_2\\ \dot{e}_3 &= y_1y_2 - ce_3 + \frac{1}{3}x_1x_2 + x_3(c + c_1) + u_3\end{aligned} \quad (4.4)$$

According to the active control theory defining the controller as

$$\begin{aligned}u_1 &= -ay_1 - a_1x_1 + y_2y_3 + x_2x_3 - e_1\\ u_2 &= -y_1y_3 - x_2(b + b_1) - x_1x_3\\ u_3 &= -y_1y_2 - \frac{1}{3}x_1x_2 - x_3(c + c_1)\end{aligned} \quad (4.5)$$

Then the error dynamics becomes

$$\begin{aligned}\dot{e}_1 &= -e_1\\ \dot{e}_2 &= -be_2\\ \dot{e}_3 &= -ce_3\end{aligned} \quad (4.6)$$

Choosing the Lyapunov function as

$$V=\frac{1}{2}(e_1^{\,2}+e_2^{\,2}+e_3^{\,2}) \qquad (4.7)$$

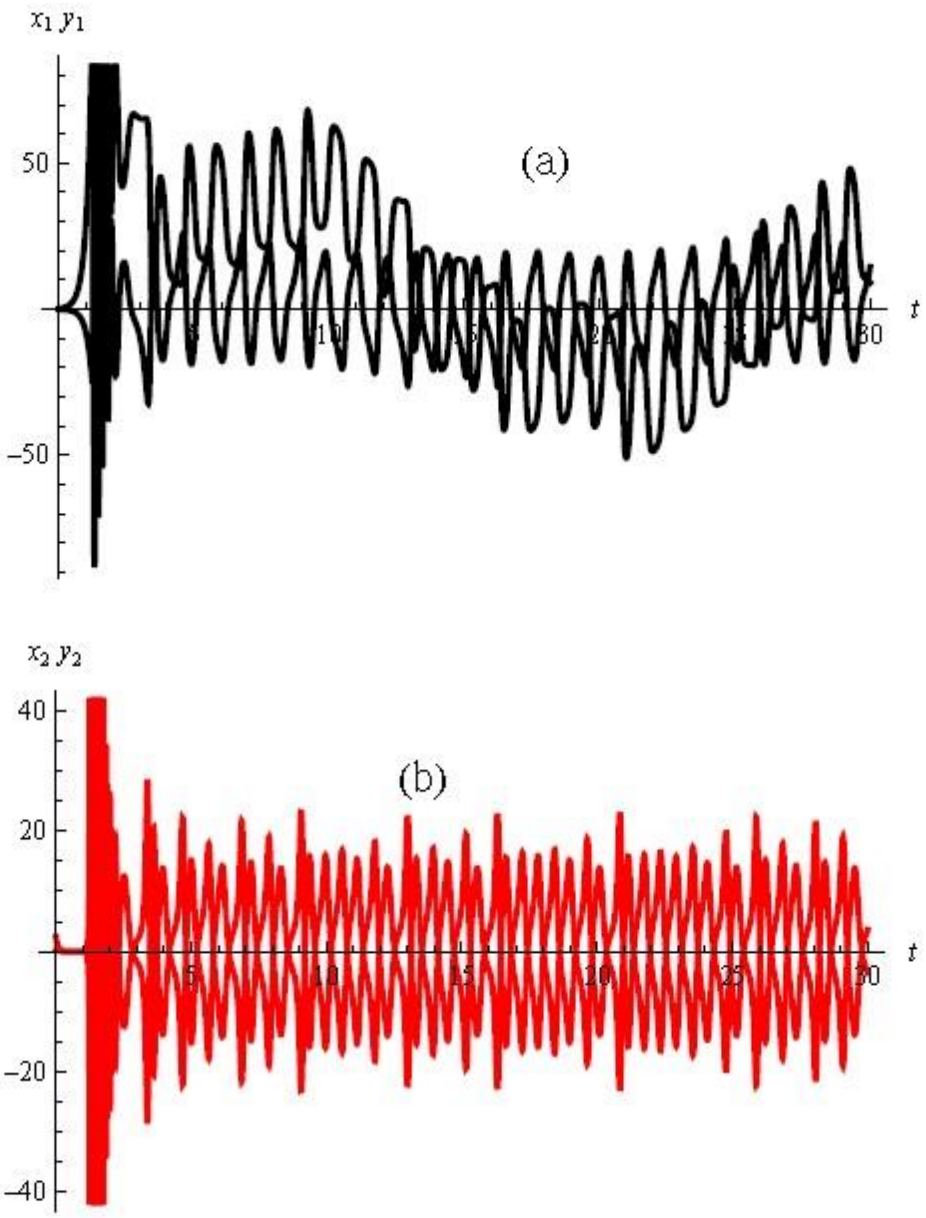
where V is a positive definite function $V\ R{:}\ ^n \rightarrow R$ and taking the derivative of the above function we get

$$\dot{V}=-e_1^{\,2}-be_2^{\,2}-ce_3^{\,2} \qquad (4.8)$$

where $V\ R{:}\ ^n \rightarrow R$is negative definite function. Thus by Lyapunov stability [11] the error system is asymptotically stable. It implies that the two chaotic Chen-Lee and Liu-Chen systems are anti-synchronized.

Numerical Simulation:

To validate the feasibility of the control functionsu_i , we simulate the dynamics of the coupled master and slave system. In simulation, we choose the parameters a b c = = − = − 5, 10, 3.8 and a b c = = = 0.4, 12, 5 to ensure two systems are chaotic without control. By using the MATHEMATICA to solve the above systems and taking the initial condition of master and slave systems as x_0 = (0.2,0.2,0.2) and y_0 = − (0.1,2.4, 3.5) − . Fig.3. *(a), (b), (c)* Show the time evolution of master and slave systems. And Fig.4 shows the anti-synchronization error of the drive and response systems.



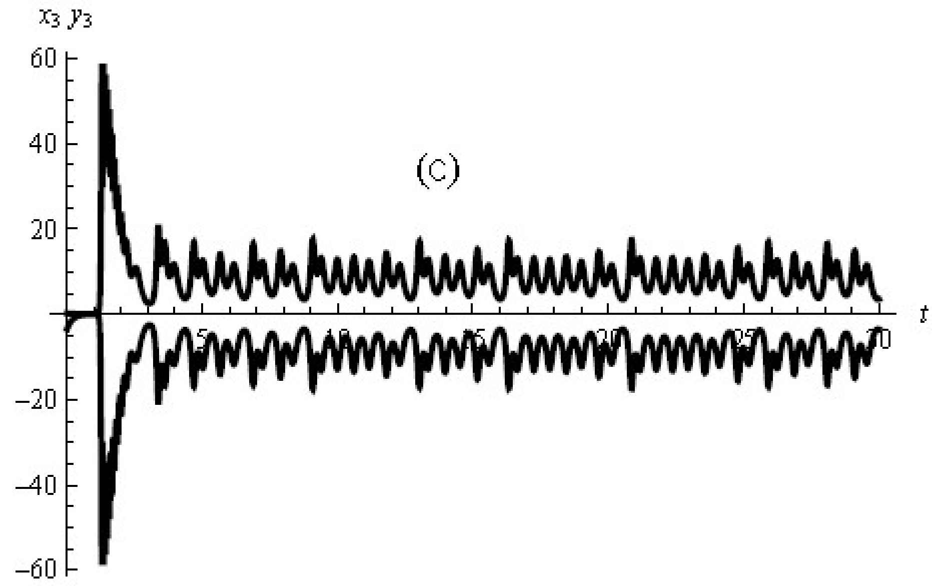
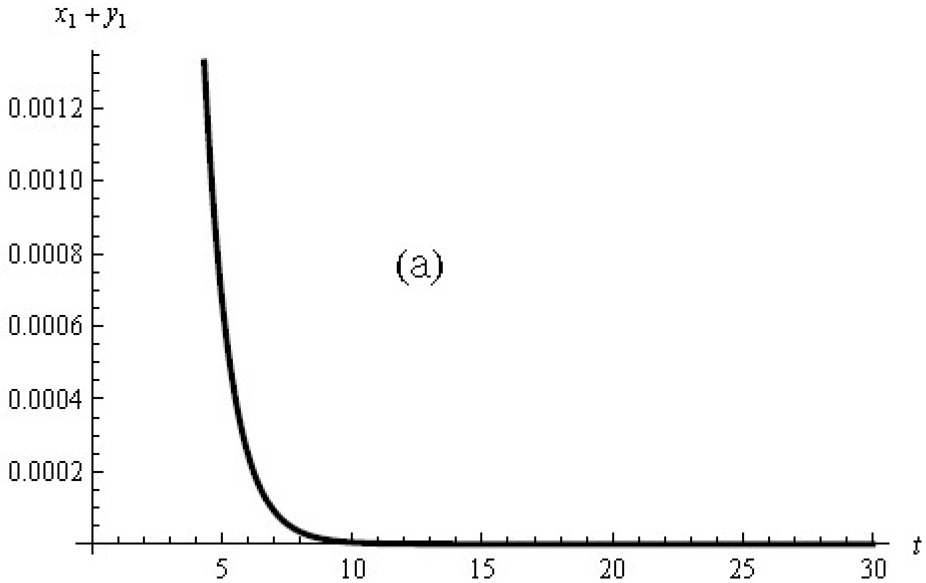
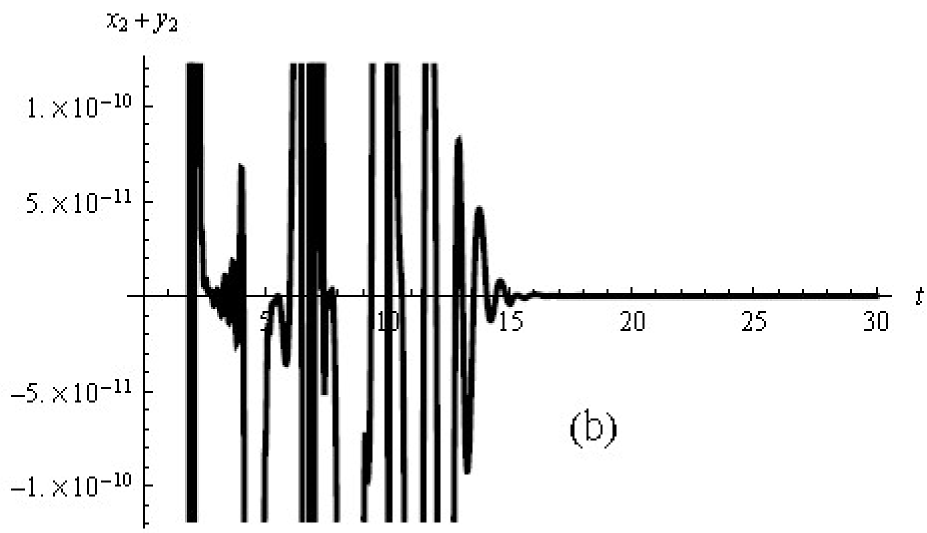


Fig.3. *(a), (b), (c)* Show the time evolution of master and slave systems.





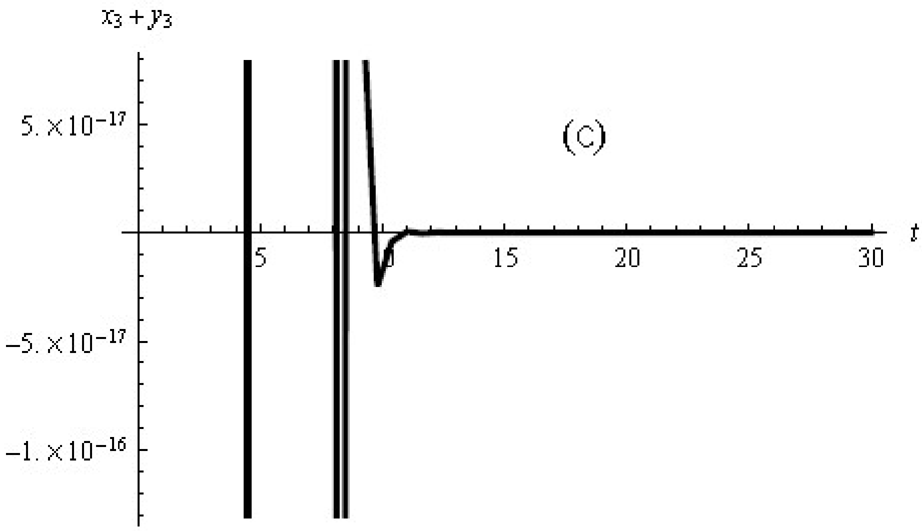


Fig.4 *(a), (b), (c)* shows the anti-synchronization error of the drive and response systems.

Conclusions:

This investigates the anti-synchronization of Chen-Lee, Lrenz-Stenflo and Liu-Chen chaotic systems via active nonlinear control and stability result derived by using Lyapunov stability theory, numerical simulation show the effectiveness of the proposed method.

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