$N_h M_{(m,c)}/N_h M/c$ queuing model with Heterogeneous Arrival, Single Vacation and Breakdown

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Abstract

In this paper, $M_{(m,c)}/M/c$ queuing model with heterogeneous arrival, single vacation and breakdown under Neutrosophic hypersoft environment is analyzed. Customer's arriving in a varied arrival rate is referred to be heterogeneous arrival. Server may become unavailable once for a random period of time is single vacation queuing system. Server may fail at times while serving customers is termed as breakdown. The basic parameters of the model are considered neutrosophic. Here, the steady state equations of the described model are derived. And performance measures and mean system size are calculated.

Keywords: Heterogeneous arrival, Single Vacation, Breakdown, Bi-level threshold, Performance Measures.

1. Introduction

A Mathematical study of queues or waiting in lines was developed by Erlang in the year 1909 is defined to be queuing which plays a significant role in almost all-fields. Finding the number of customers in queue and system and the waiting time of customers in both queue and system are the basic components of queuing theory given by Shortle and Thompson [32]. It was developed to increase the profit of the organizations by providing efficient service without making the customers wait longer. Queuing parameters include arrival and service rate respectively. In addition to this, a server may undergo various conditions and operations. A server may go through vacation in a mean amount of time. That is, a server may leave the system for a random period of time. Shweta U [20] analyzed queuing system with vacations in the year 2016. When the service begins, the service mechanism in subject to breakdown which occur at a constant rate. The service mechanism then goes through repair process of random duration. The performance of the system may be heavily affected by the server breakdowns and limited repair capacity. A vacation queuing model with service breakdowns are given by William Gray, Patrick Wang and Scott [33]. Also, Customers arrive in a varied arrival rate is termed to be Heterogeneous arrival. Heterogeneous batch arrival queue with server startup and breakdown are analyzed by Ke and Wang [8]. Yechiali and Naor [34] analyzed some problems on Heterogeneous arrivals and services.

Here, in this paper, a multi-server queuing model with heterogeneous arrival, single vacation and breakdown are studied. Its considered that the arrival rate takes three quantities $\delta_i, i = 1,2,3$ which signify the probability of arriving customers during idle, busy and breakdown state respectively. This queuing system follows bi-level control (m,c) policy. Chuan Ke [6] explained the Bi-level control for batch arrival queues with early startup and unreliable server. Julia Rose Mary, Afthab Begum and Jemila Parveen [5] gave the Bi-level threshold policy of $M^X/(G_1, G_{2)/1}$ queue with early setup and single vacation. Afthab Begum and Yasotha [1] put forward the Bi-level Control Policy for $M^X_{i(m,N)}/M/1/BD/SV$ Queuing system under Restricted Admissibility. This defines that whenever the system returns from vacation, if it finds m customers, then the server immediately starts the setup operation. And after the setup period, if the queue length is c, the server begins the service to the customers. Or else, the server remains idle for both the cases. Queuing with breakdowns was given by K. Thiruvengadam [31] in the year 1963. S Chakravarthy and Shruti RK [30] studied queuing model with server breakdown, repair, vacations and backup server.

Classical Queuing theory is used in case where the parameters such as arrival and departure rates are precise and clear assumptions. However, when information about the existing system is unreliable, are forced to replace randomness with uncertainty in a possibilistic way. Applying fuzzy logic to queuing theory makes solution for imprecise cases or uncertainty in parameters of queuing system and the Fuzzy logic was firstly introduced by Zadeh in 1965 [35]. Fuzzy queuing models have been considered by Prade [19], Li and Lee [9], Negi [16] and Lee, Kao et al. [7], Pardo and Fuente [18]. Fuzzy approaches to queuing modeling provide a flexible and appropriate way to deal with uncertainty in such problems. This approach in regarding queue performance eliminates problems of estimation under uncertainty. Rather than crisp queues, fuzzy queues are much more sensible in many real circumstances.

Neutrosophic Philosophy in queuing deals with situations in which the parameters of queues are not accurate. Neutrosophic logic which is a generalization of fuzzy logic and intuitionistic fuzzy logic [24, 25, 26, 27] was introduced by Florentin Smarandache in the year 1995. This deals with indeterminacy data realistic and understandable which gives efficient outcomes [21, 22, 23, 28, 29]. The queuing system is said to be a neutrosophic queue if its parameters are neutrosophic numbers i.e. average rate of customers entering the queuing system λ and the average rate of customers being served μ are neutrosophic numbers.An event is said to occur in varied state as (t) truth, (i) indeterminacy and (f) false, where t, i, f are real values ranges from T, I, F $\rightarrow]^{-}0, 1^{+}[$ with no restriction on the sum $t+i+f$. That is, $^{-}0 \leq T+I+F \leq 3^{+}$.

Here, we consider Neutrosophic hypersoft set. Neutrosophic hypersoft set is a real extension of Neutrosophic soft set. This set is observed to enhance the uncertainty in a more precise mode. The arrival and service rates are completely neutrosophic hypersoft denoted as λ_N and μ_N. We find the probabilities of Vacation, Setup, Buildup, Dormant, Busy and in Breakdown state in Neutrosophic hypersoft condition. The parameters applied in certain probabilities are marked neutrosophic. Moreover the probability generating functions of the respective probabilities are defined. This neutrosophic hypersoft set is used as it conceptualize the parameterized family which deals with the sub-attributes to accurately access the deficiencies and uncertainties.

Patro with Smarandache [17] discussed more problems and solutions on Neutrosophic Statistical Distribution. Bisher Zeina [13, 14] studied Erlang Service Queuing Model and Event-Based Queuing Model on Neutrosophic basis in the year 2020. Bisher Zeina [10, 11] studied the M/M/1 Queue's Performance measures on fuzzy environment and Neutrosophic concept of M/M/1, M/M/c, M/M/1/b Queuing system with Interval-valued neutrosophic sets in the year 2020. Also in the year 2021, Mohamed Bisher Zeina [12] analyzed Single Valued Neutrosophic M/M/1 Queue in Linguistic terms. Bhupender Singh Som and Sunny Seth [3, 4] developed M/M/c queuing system with encouraged arrivals with N number of customers and Queuing system with Encouraged arrivals, Impatient customers and Retention of Impatient Customers in the year 2018. Ajay and Joseline C [2] discussed Neutrosophic Hypersoft Topological Spaces in the year 2021. M Saeed, AU Rahman and M Arshad [15] studied some operations and products of Neutrosophic hypersoft graphs.

Using these analyzed study, we define $M_{(m,c)}/M/c$ queuing model in Neutrosophic hyperset environment. Also there occurs vacation and server breakdown. We deduce the steady state equations and find probabilities of the respective states and its performance measures.

2. Model Description

1. Consider a multi-server queuing system with heterogeneous arrival rate λ_{N_i} as the customers arrive in varied arrival rate. A random variable X_N denotes the size with the probability of having k customers, that is, $Pr(X_N = K) = g_k$ where k=1,2,3,.. This represents the probability of k units of heterogeneous arrival in an immeasurable interval $(t, t + \square)$ which is $\lambda g_k h + O(h)$.
2. Not every arriving customers are permitted to join the system at any time. The probability differs based on the system state which contains three classification specifically idle (buildup, setup, dormant and vacation) or busy or breakdown period.
   (a) $\delta_1 (0 \leq \delta_1 \leq 1)$ - probability that a arriving customer is allowed to join the system when the server is idle.
   (b) $\delta_2 (0 \leq \delta_2 \leq 1)$ - probability that a arriving customer joins the system when the server is busy.
   (c) $\delta_3 (0 \leq \delta_3 \leq 1)$ - probability that a arriving customer joins the system during the breakdown period.

3. People entering and joining the system makeup a multiple waiting queue according to their arrivals. They are lined in a First Come First Served (FCFS) manner.
4. Whenever the system is vacant, the cycle starts. The server becomes inactive and quit the system for a vacation of random length V with a parameter ϵ_N.
5. Following the vacation, the server starts the setup operation immediately if it finds **m** or more number of customers in the system. If not, the server goes on in an idle state until the system size attains the minimum of m customers in the system. And then the system start its setup work (i.e.)the single vacation policy is endorsed.
6. The duration which the server remains idle in the system prior to the commencement of setup work is called Buildup period. The setup time has the mean $1/\sigma_N$.
7. At the end of the setup period, the system waits for the queue size to attain at the minimum of c customers to begin the service. If the queue size reaches c or more customers in the system, the server begins the service freely. The service time of each customers is independent and if each random variable has the same probability distribution as the other and are all mutually independent random variable with exponential distribution of $(1 - e^{-\mu_N t})$.
8. The server tends to breakdowns at any moment while operating with Poisson rate α_N. Whenever the system breaks, the server is directed to put right straightaway at a restore space where the restore time is an independent and identically distributed random variable B_N following an exponential distribution $(1 - e^{-\beta_N t})$.
9. During server breakdown, the customer who is being served at that time joins the beginning of the waiting line. Once the server returns form the restore space, the server restarts the service to the impending customers. This process prevails up until the system becomes empty again.
10. This way, a cycle is formed constituting idle period and completion period. Where completion period comprises of the sum of busy period and the breakdown period. And vacation period, buildup period, setup period and dormant period altogether mean an idle period.
11. The initial threshold **m** is used to control the origin of a setup operation, and the

next threshold **c** is used to control the starting condition of service. Also, it is supposed that all the stochastic procedure involved in the model are independent of each other.

12. In this model, Neutrosophic Hypersoft set is utilized. The notion of neutrosophic hypersoft set is a parameterized extraction that deals with the sub attributes of the parameters. Neutrosophic hypersoft set is a proper extension of Neutrosophic set to accurately estimate the insufficiencies and unpredictabilities in decision making.

Let U be the Universal set and P(U) denote the power set of U. Consider $e^1, e^2, e^3, \dots e^n$ for $n \geq 1$ be n well defined attributes, whose corresponding attribute values are respectively the set $E^1, E^2, E^3, \dots E^n$ with $E^i \cap E^j = \Phi$ for $i \neq j$ and $i, j \in \{1,2,3, \dots n\}$, and their relation $E^1 \times E^2 \times E^3 \times \dots E^n = \$$, then the pair ($\mathcal{F}$,\$) is said to be Neutrosophic hypersoft set (NHSS) over U where

$\mathcal{F}: E^1 \times E^2 \times E^3 \times \dots E^n \to P(U)$ and

$\mathcal{F}(E^1 \times E^2 \times E3 \times \dots E^n) = \{< x, T(\square(\$)), I(\square(\$)), F(\square(\$)) >, x \in U\}$

where T - membership value of truthiness

I - membership value of indeterminacy

F - membership value of falsity

such that $T, I, F: U \to [0,1]$

also $0 \leq T(\square(\$)) + I(\square(\$)) + F(\square(\$)) \leq 3$

This model is denoted as $N_h M_{(m,c)}/N_h M/c/BD/SV$.

where BD - Breakdown

SV - Single Vacation.

The below mentioned notations are used to write the steady state equations.

m,c : Bi-level thresholds

λ_{N_i} : Heterogeneous arrival rate

X_N : Random Variable

$Pr(X_N = k)$: $g_k (k \geq 1)$

$X_N(z)$: $\sum_{k=1}^{\infty} g_k z^k$ the probability generating function of X_N.

δ_i; i=1,2,3 : The probability that the arriving customer is allowed to join the system while the server is idle (buildup, setup, dormant and vacation), busy and in breakdown state respectively.

$\mu_N, \epsilon_N, \sigma_N$ and β_N : The parameter of the exponential distributions of the random variables for service time (S_N), vacation time (V_N), setup time (D_N) and repair time (B_δ).

$V_N^*(\theta)$, $D_N^*(\theta)$ and $B_\delta^*(\theta)$: For an exponentially distributed random variable Y with rate parameter λ, its the Laplace-Stieltjes transformation is

$V_N(\theta) = \frac{\epsilon_N}{\epsilon_N + \theta}$, $D_N(\theta) = \frac{\sigma}{\sigma + \theta}$ and $B_\delta(\theta) = \frac{\beta}{\beta + \theta}$

Ω_N(0,1,2,3,4 and 5) : Accordingly as the system is in vacation, buildup, setup, dormant, busy and in breakdown state respectively.

3. System Size Distribution

The steady state equation for $N_h M_{(m,c)}/N_h M/c$ with heterogeneous arrival, Breakdown and Single vacation can be written as

$$(\lambda_{N_1} + \epsilon_N)Q_{N_0} = \mu_N P_{N_1} \tag{1}$$

$$(\lambda_{N_1} + \epsilon_N)Q_{N_n} = \lambda_{N_1} \sum_{k=1}^{n} Q_{N_{n-k}} g_k; n \geq 1 \tag{2}$$

$$\lambda_{N_1} R_{N_0} = \epsilon_N Q_{N_0} \tag{3}$$

$$\lambda_{N_1} R_{N_n} = \epsilon_N Q_{N_n} + \lambda_{N_1} \sum_{k=1}^{n} R_{N_{n-k}} g_k; 1 \leq n \leq m-1 \tag{4}$$

$$(\lambda_{N_1} + \sigma_N)D_{N_m} = \lambda_{N_1} \sum_{k=1}^{m} R_{N_{m-k}} g_k + \epsilon_N Q_{N_m} \tag{5}$$

$$(\lambda_{N_1} + \sigma_N)D_{N_n} = \lambda N_1 \sum_{k=n-m+1}^{n} R_{N_{n-k}} g_k + \lambda_{N_1} \sum_{k=1}^{n-m} D_{N_{n-k}} g_k + \epsilon_N Q_{N_n}; n \geq m+1 \tag{6}$$

$$\lambda_{N_1} U_{N_m} = \sigma_N D_{N_m} \tag{7}$$

$$\lambda_{N_1} U_{N_n} = \sigma_N D_{N_n} + \lambda_{N_1} \sum_{k=1}^{n-m} U_{N_{n-k}} g_k; m+1 \leq n \leq c-1 \tag{8}$$

$$(\lambda_{N_2} + \mu_N + \alpha_N)P_{N_1} = \beta_N B_{N_1} + 2\mu_N P_{N_2} \tag{9}$$

$$(\lambda_{N_2} + n\mu_N + \alpha_N)P_{N_n} = \beta_N B_{N_n} + n\mu_N P_{N_{n+1}} + \lambda_{N_2} \sum_{k=1}^{n-1} P_{N_{n-k}} g_k; 2 \leq n \leq c-1 \tag{10}$$

$$\begin{aligned}(\lambda_{N_2} + c\mu_N + \alpha_N)P_{N_n} &= \beta_N B_{N_n} + c\mu_N P_{N_{n+1}} + \lambda_{N_2} \sum_{k=1}^{n-1} P_{N_{n-k}} g_k \\ &+\sigma_N D_{N_n} + \lambda_{N_1} \sum_{k=n-c+1}^{n-m} U_{N_{n-k}} g_k; n \geq c\end{aligned} \tag{11}$$

$$(\lambda N_3 + \beta_N)B_{N_1} = \alpha_N P_{N_1} \tag{12}$$

$$(\lambda N_3 + \beta_N)B_{N_n} = \alpha_N P_{N_n} + \lambda_{N_3} \sum_{k=1}^{n-1} B_{N_{n-k}} g_k; n \geq 2 \tag{13}$$

The following partial probability generating functions of $R_{N_n}, D_{N_n}, U_{N_n}, P_{N_n}$ and B_{N_n} are defined to obtain the system size distribution of the model.

$$\left.\begin{aligned} R_N(z) &= \sum_{n=0}^{m-1} R_{N_n} z^n D_N(z) = \sum_{n=m}^{\infty} D_{N_n} z^n \\ U_N(z) &= \sum_{n=m}^{c-1} U_{N_n} z^n P_N(z) = \sum_{n=1}^{\infty} nP_{N_n} z^n \\ Q_N(z) &= \sum_{n=0}^{\infty} Q_{N_n} z^n B_N(z) = \sum_{n=1}^{\infty} B_{N_n} z^n \end{aligned}\right\} \tag{14}$$

Now, adding equations (1) and (2), implies

$$(\lambda_{N_1} + \epsilon_N)Q_{N_0} + (\lambda_{N_1} + \epsilon_N)\sum_{n=1}^{\infty} Q_{N_n} z^n = \mu_N P_{N_1} + \lambda_{N_1} \sum_{k=1}^{n} \sum_{n=1}^{\infty} Q_{N_{n-k}} g_k z^n$$

$$Q_N(z) = \frac{\mu_N P_{N_1}}{[\lambda_{N_1}(1 - X_N(z)) + \epsilon_N]}$$

Let us take, $W_{N_X}^i(z) = \lambda_{N_1}(1 - X_N(z)); i = 1,2,3$

which implies,

$$Q_N(z) = \frac{\mu_N P_{N_1}}{W_{N_X}^1(z) + \epsilon_N} \tag{15}$$

Suppose that the α_n denotes the probability the n customers arrive during a single vacation then,

$$\alpha_n = \int_0^{\infty} \sum_{i=0}^{n} e^{\lambda_{N_1} t} \frac{(\lambda_{N_1} t)^i}{i!} g_n(i) dV_N(t)$$

which gives $V_N^*(W_X^1(z)) = \sum_{n=0}^{\infty} \alpha_n z^n$

$V_N^*(W_X^1(z)) = \frac{\epsilon_N}{\epsilon_N + W_X^1(z)}$ [By Laplace-Stieltje's transformation]

$$\epsilon_N Q_{N_n} = \mu_N P_{N_1} \alpha_{N_n} \tag{16}$$

Therefore,

$$Q_N(z) = \mu_N P_{N_1} \left[\frac{1 - V_N^*(W_X^1(z))}{W_X^1(z)}\right] \tag{17}$$

Adding equation (3) and (4),

$$\lambda_{N_1} R_N(z) = \epsilon_N \sum_{n=0}^{m-1} Q_{N_n} z^n + \lambda_{N_1} \sum_{n=1}^{m-1} z^n \sum_{k=1}^{n} R_{N_{n-k}} g_k ---------- (A)$$

From eq(5) and (6),

$$(\lambda_{N_1} + \sigma_N) \sum_{n=m}^{\infty} D_{N_n} z^n =$$
$$\lambda_{N_1} \sum_{n=m}^{\infty} z^n \sum_{k=n-m+1}^{n} R_{N_{m-k}} g_k + \lambda_{N_1} \sum_{k=m+1}^{\infty} z^n \sum_{k=1}^{n-m} D_{N_{n-k}} g_k$$
$$+\epsilon_N \sum_{n=m}^{\infty} Q_{N_n} z^n ---------- (B)$$

Adding (A) and (B),

$$\lambda_{N_1} R_N(z) + (\lambda_{N_1} + \sigma_N) \sum_{n=m}^{\infty} D_{N_n} z^n =$$
$$\epsilon_N \sum_{n=0}^{m-1} Q_{N_n} z^n + \lambda_{N_1} \sum_{n=1}^{m-1} z^n \sum_{k=1}^{n} R_{N_{n-k}} g_k$$
$$+\lambda_{N_1} \sum_{n=m}^{\infty} z^n \sum_{k=n-m+1}^{n} R_{N_{m-k}} g_k + \lambda_{N_1} \sum_{k=m+1}^{\infty} z^n \sum_{k=1}^{n-m} D_{N_{n-k}} g_k +$$
$$\epsilon_N \sum_{n=m}^{\infty} Q_{N_n} z^n$$

$$D_N(z) = \frac{\epsilon_N Q_N(z) - W_X^1(z) R_N(z)}{[W_X^1(z) + \sigma_N]}$$

$$D_N(z) = \frac{\mu_N P_{N_1}}{\sigma_N} D_N^*(W_X^1(z)) \left[V_N^*(W_X^1(z)) - \frac{W_X^1(z) R_N(z)}{\mu_N P_{N_1}}\right] \tag{18}$$

(Or)

$$D_N(z) = \mu_N P_{N_1} \left[\frac{1 - D_N^*(W_X^1(z))}{W_X^1(z)}\right] \left[V_N^*(W_X^1(z)) - \frac{W_X^1(z) R_N(z)}{\mu_N P_{N_1}}\right] \tag{19}$$

Since, $h_k = \int_0^{\infty} \sum_{i=1}^{k} e^{\lambda_{N_1} t} \frac{(\lambda_{N_1} t)^i}{i!} g_k^{(i)} dD_N(t)$ gives the probability that k customers arrive during a setup period,

$$D_N^*(W_X^1(z)) = \frac{\sigma_N}{\sigma_N + W_X^1(z)} = \sum_{k=0}^{\infty} h_k z^k \tag{20}$$

Remark

1. $\pi_{N_0} = 1, \pi_{N_n} = \sum_{k=1}^{n} \pi_{N_{n-k}} g_k$

2. $\psi_{N_0} = \alpha_{N_0}, \psi_{N_n} = \sum_{i=0}^{n} \alpha_{N_i} \pi_{N_{n-i}} (1 \le n \le m-1)$

By using equations (3), (4), (17) and Remark 2, it can be be shown by recursion that $\lambda_{N_1} R_{N_n} = \mu_N P_{N_1} \psi_{N_n};\ 0 \le n \le m-1$

Thus,

$$R_N(z) = \mu_N P_{N_1} (1/\lambda_{N_1}) \sum_{n=0}^{m-1} \psi_{N_n} z^n = \mu_N P_{N_1} \psi_N(z) \tag{21}$$

where $\psi_N(z) = (1/\lambda_{N_1}) \sum_{n=0}^{m-1} \psi_{N_n} z^n$

Remark For $0 \le n \le m-1$

1. $\sum_{i=0}^{n-1} \psi_{N_i} g_{n-i} = \sum_{i=0}^{n-1} \alpha_{N_i} \pi_{N_{n-i}}$
2. $\alpha_{N_n} + \sum_{i=0}^{n-1} \psi_{N_i} g_{n-i} = \psi_{N_n}$

$$\sum_{k=1}^{n} g_k \psi_{N_{n-k}} = \sum_{k=1}^{n} g_k \left[\sum_{i=0}^{n-k} \alpha_{N_i} \pi_{N_{n-k-i}}\right]$$

$$= \sum_{k=0}^{n-1} \alpha_{N_k} \pi_{N_{n-k}}$$

Hence,

$$\alpha_{N_n} + \sum_{i=0}^{n-1} \psi_{N_i} g_{n-i} = \sum_{k=0}^{n} \alpha_{N_k} \pi_{N_{n-k}} = \psi_{N_n}$$

Eq (20) can be written as,

$$D_N(z) = \mu_N P_{N_1} \left[\frac{1-D_N^*(W_X^1(z))}{W_X^1(z)}\right] [V_N^*(W_X^1(z)) - W_X^1(z)\psi_N(z)] \qquad (22)$$

(Or)

$$D_N(z) = \frac{\mu_N P_{N_1}}{\sigma_N} D_N^*(W_X^1(z))[V_N^*(W_X^1(z)) - W_X^1(z)\psi_N(z)] \qquad (23)$$

Adding Equation (12) and (13),

$$(\lambda_{N_3} + \beta_N)[B_{N_1} + B_{N_n}] = \alpha_N [P_{N_1} + P_{N_n}] + \lambda_{N_3} \sum_{k=1}^{n-1} B_{N_{n-k}} g_k$$

$$B_N(z) = \frac{\alpha_N}{\beta_N} B_\delta^*(W_X^3(z)) P_N(z) \qquad (24)$$

From (7), (8), (9), (10) and (11),

$$\lambda_{N_1} U_N(z) + (\lambda_{N_2} + \mu_N(n+c+1) + \alpha_N) P_N(z) = \beta_N B_N(z) + \sigma_N D_N(z) + \lambda_{N_1} U_N(z) X_N(z)$$

$$+\lambda_{N_2} X_N(z) P_N(z) + \frac{\mu_N}{z}\left[P_N(z) - P_{N_1} z\right] \qquad (25)$$

Substituting for $D_N(z)$ and $B_N(z)$ from eq (24) and (25),

$$P_N(z)[z(W_X^2(z) + \alpha_N[1 - B_\delta^*(W_X^3(z))]) + \mu_N(n+c+1)(z-1)]$$

$$= z\mu_N P_{N_1} \left[[D_N^*(W_X^1(z)) - 1] - W_X^1(z)\left[D_N^*(W_X^1(z))\psi_N(z) + \frac{U_N(z)}{\mu_N P_{N_1}}\right]\right] \qquad (26)$$

Remark

1. $[V_N^*(W_X^1(z)) - W_X^1(z)\psi_N(z)] = \sum_{n=m}^{\infty} \zeta_{N_n} z^n$

2. $D_N^*(W_X^1(z))[V_N^*(W_X^1(z)) - W_X^1(z)\psi_N(z)] = \sum_{k=m}^{\infty} z^k \sum_{i=m}^{k} \zeta_i h_{k-i}$

Using eq (7) and (8),

$$\lambda_{N_1} U_{N_m} + \lambda_{N_1} U_{N_n} = \sigma_N D_{N_m} + \sigma_N D_{N_{n+\lambda_{N_1}}} \sum_{k=1}^{n-m} U_{N_{n-k}} g_k$$

$$\lambda_{N_1} U_{N_n} = \sigma_N D_{N_m} + \sigma_N D_{N_n}$$

$$\lambda_{N_1} U_{N_n} = \sigma_N \sum_{k=m}^{n} D_{N_k} \pi_{N_{n-k}} (m \le n \le c-1)$$

where D_{N_k} = co-efficient of z^k in $D_N(z)$

$= \left(\frac{\mu_N P_{N_1}}{\sigma_N}\right)$ co-efficient of z^k in $D_N^*(W_X^1(z))[V_N^*(W_X^1(z)) - W_X^1(z)\psi_N(z)]$

$= \left(\frac{\mu_N P_{N_1}}{\sigma_N}\right) \sum_{i=m}^{k} \zeta_{N_i} h_{k-i}$ [from Remark 2]

$$\lambda_{N_1} U_{N_n} = \mu_N P_{N_1} \phi_{N_n}^s$$

where

$$\phi_{N_n}^s = \sum_{k=m}^{n} \pi_{N_{n-k}} \sum_{i=m}^{k} \zeta_{N_i} h_{k-i} \qquad (27)$$

$$= \sum_{k=m}^{n} \zeta_{N_{n-k}} \sum_{i=0}^{n-k} h_i \pi_{N_{n-ki}}$$

i.e. $U_N(z) = \mu_N P_{N_1} \sum_{n=m}^{c-1} \frac{\phi_{N_n}^s z^n}{\lambda_{N_1}}$

$$U_N(z) = \mu_N P_{N_1} \phi_N^s(z)$$

Substituting $U_N(z)$ in equation (27),

$$P_N(z)[z(W_X^2(z) + \alpha_N[1 - B_\delta^*(W_X^3(z))]) + \mu_N(n + c + 1)(z - 1)]$$
$$= -z\mu_N P_{N_1} W_X^1(z)\left[\left[\frac{1 - D_N^*(W_X^1(z))D_N^*(W_X^1(z))}{W_X^1(z)}\right] + D_N^*(W_X^1(z))\psi_N(z) + \phi_N^s(z)\right] \quad (28)$$

Thus the total probability generating function (PGF) $P_{N_{(m,c)}}^s(z)$ is given by,

$$P_{N_{(m,c)}}^s(z) = P_{N_I}(z) + P_N(z) + B_N(z)$$

where $P_{N_I}(z)$ = PGF of the system size when the server is idle.

$$= Q_N(z) + R_N(z) + D_N(z) + U_N(z)$$

By adding, (18), (22), (23), and (28)

$$\Rightarrow \mu_N P_{N_1}\left[\frac{1 - V_N^*(W_X^1(z))}{W_X^1(z)}\right] + \mu_N P_{N_1}\psi_N(z)$$
$$+\mu_N P_{N_1}\left[\frac{1 - D_N^*(W_X^1(z))}{W_X^1(z)}\right][V_N^*(W_X^1(z)) - W_X^1(z)\psi_N(z)] + \mu_N P_{N_1}\phi_N^s(z)$$

$$\Rightarrow P_{N_I}(z) = \mu_N P_{N_1} I_{N_s}(z)$$

where

$$I_{N_s}(z) = [\left[\frac{1 - V_N^*(W_X^1(z))}{W_X^1(z)}\right] + \left[\frac{1 - D_N^*(W_X^1(z))}{W_X^1(z)}\right][V_N^*(W_X^1(z)) - W_X^1(z)\psi_N(z)]$$
$$+\psi_N(z) + \phi_N^s(z)]$$

$$I_{N_s}(z) = \left[\frac{1 - D_N^*(W_X^1(z))V_N^*(W_X^1(z))}{W_X^1(z)} + D_N^*(W_X^1(z))\psi_N(z) + \phi_N^s(z)\right]$$

Using eq (25)

$$P_N(z) + B_N(z) = P_N(z)\left[1 + \frac{\alpha_N}{\beta_N} B_\delta^*(W_X^3(z))\right]$$

Substituting for $P_N(z)$ from eq (29),

$$P_{N_{(m,c)}}^s(z) = \mu_N P_{N_1} I_{N_s}(z)\left[1 - \frac{zW_X^1(z)\left(1 + \frac{\alpha_N}{\beta_N} B_\delta^*(W_X^3(z))\right)}{z\left(W_X^2(z) + \alpha_N[1 - B_\delta^*(W_X^3(z))]\right) + \mu_N(n + c + 1)(z - 1)}\right] \quad (29)$$

The value of $\mu_N P_{N_1}$ can be calculated using the normalizing condition $P_{N_{(m,c)}}^s(1) = 1$

4. Performance Measures

Let the steady state system size probabilities $P_{N_v}, P_{N_{build}}, P_{N_{set}}, P_{N_{dor}}, P_{N_I}, P_{N_{busy}}$ and $P_{N_{br}}$ denote the probability that the server is in vacation, buildup, setup, dormant, idle, busy and in breakdown state respectively.

Then,

1. $P_{N_v} = lim_{z\to 1} Q_N(z)$

$$Q_{N_1} = \frac{\mu_N P_{N_1}}{\epsilon_N}$$

2. $P_{N_{build}} = lim_{z\to 1} \mu_N P_{N_1} \psi_N(z)$

$$R_N(1) = P_{N_{build}} = \mu_N P_{N_1} \sum_{n=0}^{m-1} \frac{\psi_{N_n}}{\lambda_{N_1}}$$

3. $P_{N_{set}} = D_{N_1}$

$$D_N(1) = \frac{\mu_N P_{N_1}}{\sigma_N}$$

4. $P_{N_{dor}} = lim_{z\to 1}\mu_N P_{N_1} Q_{N_s}(z)$

$$U_N(1) = \mu_N P_{N_1} \sum_{n=m}^{c-1} \frac{\phi_{N_n}^s}{\lambda_{N_1}}$$

5. $P_{N_I} = lim_{z\to 1} P_{N_I}(z)$

$$P_{N_I}(1) = \mu_N P_{N_1} D_{N_s}(m,c)$$

6. $P_{N_{busy}} = lim_{z\to 1} P_N(z)$

$$P_N(1) = \mu_N P_{N_1}\left[\frac{\rho_{N_1}}{1-\rho_{N_{23}}^{br}}\right] D_{N_s}(m,c)$$

7. $P_{N_{br}} = lim_{z\to 1} B_N(z)$

$$B_N(1) = \frac{\alpha_N}{\beta_N} B_\delta^*(W_{N_X}^3(1)) P_N(1)$$

$$= \frac{\alpha_N}{\beta_N} P_N(1) P_{N_{br}} = \frac{\alpha_N}{\beta_N} P_{N_{busy}}$$

Thus the normalizing condition $P_{N_{(m,c)}}^s = 1$ implies

$$1 = \mu_N P_{N_1} I_{N_s}(z)\left[1 - \frac{zW_X^1(z)\left(1+\frac{\alpha_N}{\beta_N}B_\delta^*(W_X^3(z))\right)}{\left(W_X^2(z)+\alpha_N[1-B_\delta^*(W_X^3(z))]\right)+\mu_N(n+c+1)(z-1)}\right]$$

$$\frac{1}{I_{N_s}(1)} = \mu_N P_{N_1}$$

$$\mu_N P_{N_1} = \frac{R_N}{D_{N_s}(m,c)}$$

where, $D_{N_s}(m,c) = I_{N_s}(1)$

$$= E(D_N) + E(V_N) + \sum_{n=0}^{m-1} \frac{\psi_{N_n}}{\lambda_{N_1}} + \sum_{n=m}^{c-1} \frac{\phi_{N_n}^s}{\lambda_{N_1}} \quad (30)$$

$$R = \frac{1-\rho_{N_{23}}}{br}\left((1-\rho_{N_{23}}^{br}) + \rho_{N_1}^{br}\right)$$

$$\rho_{N_i} = \left(\frac{\lambda_{N_i}}{\mu_N}\right) E(X_N); i = 1,2,3$$

$$\rho_{N_1}^{br} = \rho_{N_1}\left[1+\frac{\alpha_N}{\beta_N}\right]$$

$$\rho_{N_{23}}^{br} = \rho_{N_2} + \rho_{N_3}\left(\frac{\alpha_N}{\beta_N}\right)$$

Thus, $P_{N_{busy}} = \frac{\rho_{N_1}}{1-\rho_{N_{23}}^{br}+\rho_{N_1}^{br}}$

$$P^{s}_{N_{(m,c)}}(z) = \mu_N P_{N_1} I_{N_s}(z)\left[1 - \frac{zW^1_X(z)\left(1+\frac{\alpha_N}{\beta_N}B^*_\delta(W^3_X(z))\right)}{z\left(W^2_X(z)+\alpha_N[1-B^*_\delta(W^3_X(z))]\right)+\mu_N(n+c+1)(z-1)}\right]$$

Substitute $\mu_N P_{N_1}$ from eq (37) in eq (36)

$$P^{s}_{N_{(m,c)}}(z) = \frac{R_N}{D_{N_s}(m,c)} I_{N_s}(z)\left[1 - \frac{zW^1_X(z)\left(1+\frac{\alpha_N}{\beta_N}B^*_\delta(W^3_X(z))\right)}{z\left(W^2_X(z)+\alpha_N[1-B^*_\delta(W^3_X(z))]\right)+\mu_N(n+c+1)(z-1)}\right]$$

$$P^{s}_{N_{(m,c)}}(z) = R_N\left[1 - \frac{zW^1_X(z)\left(1+\frac{\alpha_N}{\beta_N}B^*_\delta(W^3_X(z))\right)}{\left[z\left(W^2_X(z)+\alpha_N[1-B^*_\delta(W^3_X(z))]\right)+\mu_N(n+c+1)(z-1)\right]}\right]\frac{I_{N_s}(z)}{I_{N_s}(1)} \qquad (31)$$

5. Expected System Size

Let $L_{N_v}, L_{N_{build}}, L_{N_{set}}, L_{N_{dor}}, L_{N_{busy}}$ and $L_{N_{br}}$ denote the expected system size when the server is in vacation, buildup, setup, dormant, busy and in breakdown state respectively. Then

1. $L_{N_v} = \frac{d}{dz}[Q_N(z)]_{z=1}$

$$= \mu_N P_{N_1}\lambda_N E(X_N) E\left(\frac{(V_N)^2}{2}\right)$$

2. $L_{N_{build}} = \frac{d}{dz}[R_N(z)]_{z=1}$

$$= \frac{\mu_N P_{N_1}}{\lambda_{N_1}}\sum_{n=0}^{m-1} n\psi_{N_n}$$

3. $L_{N_{set}} = \frac{d}{dz}[D_N(z)]_{z=1}$

$$= \mu_N P_{N_1}\lambda_{N_1} E(X_N)\left[\frac{E(D_N^2)}{2} + E(D_N)(E(V_N)+\psi_N(1))\right]$$

4. $L_{N_{dor}} = \frac{d}{dz}[U_N(z)]_{z=1}$

$$= \mu_N P_{N_1}\frac{\sum_{n=m}^{c-1} n\phi^s_{N_n}}{\lambda_{N_1}}$$

5. $L_{N_{br}} = \frac{d}{dz}[B_N(z)]_{z=1}$

$$= \frac{\alpha_N}{\beta_N}\left[L_{N_{busy}} + \lambda_{N_3}\left(\frac{E(X_N)}{\beta_N}\right)P_{N_{busy}}\right]$$

6. $L_{N_{busy}} = \frac{d}{dz}[P_N(z)]_{z=1}$

$$= P_{N_{busy}} + \frac{\mu_N P_{N_1}\rho_{N_1}}{\left(1-\rho^{br}_{N_{23}}L_{N_s}(m,c) + \frac{P_{N_1}D_{N_s}(m,c))}{2\mu_N\left(1-\rho^{br}_{N_{23}}\right)^2}\right)}$$

$$\left[\lambda_{N_1}\mu_N E(X_N(X_N-1)) + 2\lambda_{N_1}E(X_N)\left[\alpha_N\left(\lambda_{N_3}E(X_N)\right)^2\frac{\left(E(B_\delta^2)\right)}{2} + \mu_N\rho_{N_{23}}^{br}\right]\right]$$

where

$$L_{N_s}(m,c) = L_{N_0} + \lambda_{N_1}E(X_N)E(D_N)\psi_N(1) + \sum_{n=0}^{m-1}\frac{n\psi_{N_n}}{\lambda_{N_1}} + \sum_{n=m}^{c-1}\frac{n\phi_{N_n}^s}{\lambda_{N_1}} \quad (32)$$

with

$$L_{N_0} = \lambda_{N_1}E(X_N)\left[\frac{E(D_N^2)}{2} + E(D_N)E(V_N) + \frac{E(V_N^2)}{2}\right] \quad (33)$$

Let $L_{N_{(m,c)}}^s$ denote the expected system size of $NM_{i(m,c)}/NM/c/SV/BD$ queuing system under consideration. Then

$$L_{N_{(m,c)}}^s = L_{N_v} + L_{N_{build}} + L_{N_{set}} + L_{N_{dor}} + L_{N_{busy}} + L_{N_{br}}$$

implies

$$L_{N_{(m,c)}}^s = \frac{L_{N_S}(m,c)}{D_{N_S}(m,c)} + L_{N_1} \quad (34)$$

where

$$L_{N_1} = \frac{1}{1-\rho_{N_{23}}^{br}+\rho_{N_1}^{br}}\left[\rho_{N_1}^{br} + \frac{\lambda_{N_1}E(X_N(X_N-1))E(H_N)+\lambda_{N_1}\lambda_{N_3}(E(X_N))^2E(H_N^2)}{2(1-\rho_{N_{23}}^{br})}\right] \quad (35)$$

with

$$E(H_N) = \frac{1}{\mu_N}\left[1+\frac{\alpha_N}{\beta_N}\right] = E(S_N)\left[1+\frac{\alpha_N}{\beta_N}\right]$$

$$and$$

$$E(H_N^2) = \alpha_N E(B_\delta^2)E(S_N)\left[1+\rho_{N_3}-\rho_{N_2}\right] + E(S_N^2)\left[1+\frac{\alpha_N}{\beta_N}\right]\left[\frac{\lambda_{N_2}}{\lambda_{N_3}}+\frac{\alpha_N}{\beta_N}\right]$$

L_{N_1} give the mean system size of neutrosophic hypersoft $M_i/M/c/BD$ queuing model with unreliable server under restricted admissibility.

6. Conclusion

In this paper, $M_{(m,c)}/M/c$ queuing model with heterogeneous arrival, single vacation and breakdown under neutrosophic hypersoft environment is analyzed. Probability generating functions of the respective states are defined. Also their performance measures are derived. Here, the arrival and service rates depends on servers, truth, indeterminacy and falsity membership values. (m,c) policy is been carried out which defines the system to work reaching m customers after vacation and c customers after setup operation.

Neutrosophic hypersoft set applied here provides detailed approach of uncertain and imprecise quantities. Also it deals with sub-attributes of the parameters. Using these, numerical illustrations can be given. And it can be further developed with other classical queuing models. Also the system can be analyzed with different other situations.

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