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COMMUTATIVE MONOIDS ON ALGEBRAIC SUM AND ALGEBRAIC PRODUCT OVER BIPOLAR FUZZY SET AND BIPOLAR FUZZY MATRICES

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ABSTRACT

An extension of fuzzy set, called Bipolar fuzzy set with positive and negative membership values, whose membership degree range is [-1,1]. Bipolar fuzzy set have potential impacts on many fields, including artificial intelligence, computer science, decision science etc. In this paper, we introduce the algebraic sum $\oplus$ and algebraic product $\odot$ on Bipolar fuzzy set and Bipolar fuzzy matrices and prove that the set of all Bipolar fuzzy set and Bipolar fuzzy matrices form a commutative monoids over the above operations. In addition we discuss some properties like reflexive, symmetric, associative of the operators.

Keywords and Phrases: Bipolar Fuzzy Set (BFS), Bipolar Fuzzy Matrix (BFM).

1. Introduction

Zadeh [15] introduced the concept of a fuzzy set in 1965, as the generalization of a crisp set. Operations of fuzzy sets are union, intersection, complement, algebraic sum and algebraic product. Fuzzy sets and their applications to automata theory, logic, control, game, topology, pattern recognition, taxonomy etc. Algebraic sum and algebraic product are also used in the study of fuzzy events by Zadeh [16]. Fuzzy matrices are introduced for the first time by Thomason [14] in 1977. It plays a vital role in scientific development. Some Results on Fuzzy Matrices are discussed by Gilchrist in 2019. [2]. The theories of fuzzy matrices are developed by Kim and Roush [3] as an extension of Boolean matrices. Meenakshi [7] deals with $\mathcal{F}_{mn}$, the set of all $m\times n$ fuzzy matrices over the fuzzy algebra $\mathcal{F}$=[0,1] under the max-min operations and with the usual ordering of real numbers. Punithavalli [10] discussed the concept of Symmetric and centro Symmetric fuzzy matrices in 2021. Atanossov [1] introduced theory of intuitionistic fuzzy set as generalization of fuzzy set. Muthuraji T and Sriram S [8, 9] deals with commutative monoids on lukasiwicz conjunction and disjunction operators over Intuitionistic fuzzy matrices in 2015. And also discussed commutative Monoid on symmetrical difference operator over Intuitionistic Fuzzy Matrices in 2020. Silambarasan [12] introduced the concept of algebraic sum and algebraic product of picture fuzzy matrices in 2020.

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Bipolar fuzzy set is introduced by Zhang in 1994.[17, 18]. Bipolar fuzzy set theory becomes popular due to its wide applications to model real life situation. Bipolar logic is also used to represent bistable devices in computer and communication systems. Advantages of bipolar fuzzy sets are formalizes a unified approach to polarity and fuzziness. It captures two sided nature of human perception and cognition. In recent years bipolar fuzzy sets seem to have been studied and applied a bit enthusiastically and a bit increasingly. Madhumangal Pal and Sanjib Mondal [5] introduced the concepts of bipolar fuzzy matrices over bipolar fuzzy algebra and bipolar fuzzy relation. Bipolar fuzzy matrix is now essential to model and solve the problem containing bipolar information. Bipolar intuitionistic fuzzy matrices are discussed by Lalitha and Dhivya.[4].

Fuzzy sets and their operations are discussed by Mizumoto [6]. Two binary operators which is algebraic sum and algebraic product on fuzzy matrices are discussed by shyamal AK and Madhumangal pal in 2004[11]. Sriram S and Boobalan J [13] are studied the monoids on intuitionistic fuzzy matrices in 2015. The intuitionistic fuzzy matrices have the condition when add the membership and non membership elements it should be the values between 0 to 1. But the bipolar does not have the condition. In this paper, we study more results of algebraic operators in related to bipolar fuzzy sets are extended to bipolar fuzzy matrices and also its algebraic properties.

2. Preliminaries

In this section, we recollect some basic definitions and properties will be refer later.

Definition 2.1 (Fuzzy Set) [6]

A fuzzy set A in a universe of discourse $\boldsymbol{U}$ is characterized by a membership function μ_A which takes the values in the unit interval [0,1] i.e.,

$$\mu_A : \boldsymbol{U} \rightarrow [0,1]$$

The value of μ_A at u ($\in \boldsymbol{U}$), $\mu_A(u)$ represent the grade of membership (grade for short) of u in A and is a point in [0,1].

Definition 2.2 (Fuzzy Matrix) [2]

Let A be an $n\times m$ matrix defined by

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$

The matrix A is a Fuzzy matrix if and only if $a_{ij} \in [0,1]$ for $1 \le i \le n$ and $1 \le j \le m$. In other words, any $n\times m$ matrix A is a fuzzy matrix if the elements of A are in the interval [0,1].

Definition 2.3 (Bipolar fuzzy Set) [5]

A Bipolar fuzzy set $\boldsymbol{\mathcal{B}}_F$ in X (universe of discourse) is an object having the form

$$\boldsymbol{\mathcal{B}}_F = \{(x, \mu_n(x)\mu^{+}(x))\}$$

where μ_n: X→ $[-1,0]$ and μ^{+}: X→ $[0,1]$ are two mappings.

The positive membership degree $\mu^{+}(x)$ denotes the satisfaction degree of an element x to the property corresponding to a BFS $\boldsymbol{\mathcal{B}}_F$, and the negative membership degree $\mu_n(x)$ denotes the satisfaction degree of x to some implicit counter-property of $\boldsymbol{\mathcal{B}}_F$.

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Definition 2.4 (Different types of Bipolar Fuzzy set) [5]

(i) The Zero element of a BFS is denoted by $o_b = (0,0)$

(ii) The Unit element of a BFS is denoted by $i_b = (-1,1)$.

(iii) The Identity element of a BFS in respect to serial conjunction is denoted by e_b and is defined by $e_b = (0,1)$.

Definition 2.5 (Bipolar Fuzzy Matrix) [5]

Let A= $(a_{ij})_{\times m} \in \mathcal{M}_{lm,}$ then $a_{ij} = (-\, a_{ijn},\, a_{ijp}) \in \mathcal{B}_F$, where a_{ijn}, $a_{ijp} \in [0,1]$ are the negative and positive membership values of the element a_{ij}, respectively.

Definition 2.6 (Different types of Bipolar Fuzzy matrix) [5]

(i) The Zero matrix O_m of order $m \times m$ is the matrix where all the elements are $o_b = (0,0)$

(ii) The Identity matrix I_m of order $m \times m$ is the matrix where all the diagonal entries are $i_b = (-1,1)$ and all other entries are $o_b = (0,0)$.

(iii) The Unit matrix J_m of order $m \times m$ is the matrix where all the elements are $J_m = (-1,1)$.

Definition 2.7 (operations on Bipolar Fuzzy Matrix) [5]

Let A= (a_{ij}), $B = (b_{ij}) \in \mathcal{M}_{lm}$ be two BFMs. Therefore a_{ij}, $b_{ij} \in \mathcal{B}_F$, then,

(i) $A+B = (a_{ij} + b_{ij})_{l\times m} = (-\max\{ a_{ijn}\, b_{ijn}\}, \max \{ a_{ijp}\, b_{ijp}\}_{l\times m}$

(ii) $A\cdot B = (a_{ij} \cdot b_{ij})_{l\times m} = (-\min\{ a_{ijn}\, b_{ijn}\}, \min \{ a_{ijp}\, b_{ijp}\}_{l\times m}$

(iii) $A^c = (-\, a_{ijn}, a_{ijp})^c = (-(1-a_{ijn}),\, 1-a_{ijp})$

(iv) Let $\mathcal{B}_F$ be a BFM_s over X and let $A, B \in \mathcal{B}_F$ where A= $(-a_{ijn},\, a_{ijp})$ and $B = (-\, b_{ijn},\, b_{ijp})$, then $A \leq B$ if and only if $a_{ijn} \leq b_{ijn}$ and $a_{ijp} \leq b_{ijp}$.
That is $A \leq B$ if and only if $A+B = B$.

Definition 2.8 (Algebraic sum and Algebraic product on Fuzzy matrix) [11]

Let A= $[a_{ij}]$, $B = [b_{ij}]$ be two fuzzy matrices of order $m \times n$. Then

(i) $A \oplus B = [a_{ij} + b_{ij} - a_{ij}\, b_{ij}]$

(ii) $A \odot B = [a_{ij}\, b_{ij}]$

3. Some Results

In this section, we introduce two operators namely algebraic sum and algebraic product on Bipolar fuzzy sets and also Bipolar fuzzy matrices.

Definition 3.1

Let A, $B \in$ BFS (Bipolar fuzzy set) where A= $(-x_n,\, x_p)$ and $B = (-y_n,\, y_p)$ then

$A \oplus B = [-(\, x_n + y_n - x_n\, y_n),\, (x_p + y_p - x_p y_p)]$

$A \odot B = [-(\, x_n y_n), (x_p y_p)]$

Definition 3.2

Let A, $B \in$ BFM (Bipolar fuzzy matrix) where A= $(-a_{ijn},\, a_{ijp})$ and $B = (-\, b_{ijn},\, b_{ijp})$ then

$A \oplus B = \{-\,(a_{ijn} + b_{ijn} - a_{ijn}\, b_{ijn}),\, (a_{ijp} + b_{ijp} - a_{ijp} b_{ijp})\}$

$A \odot B = \{-(a_{ijn}\, b_{ijn}), (a_{ijp} b_{ijp})\}$

Theorem 3.1

If A and B are BFS, then
Operations $\oplus$ and $\odot$ are commutative.
(i) $A \oplus B = B \oplus A$
(ii) $A \odot B = B \odot A$

Proof:

A $= (-x_n, x_p)$ and $B = (-y_n, y_p)$
(i) LHS

$$A \oplus B = (-x_n, x_p) \oplus (-y_n, y_p)$$

$$A \oplus B = [-(x_n + y_n - x_n y_n), (x_p + y_p - x_p y_p)] \quad (3.1)$$

RHS

$$B \oplus A = (-y_n, y_p) \oplus (-x_n, x_p)$$

$$B \oplus A = [-(x_n + y_n - x_n y_n), (x_p + y_p - x_p y_p)] \quad (3.2)$$

From (3.1) and (3.2) we get.
Similarly we can prove $A \odot B = B \odot A$

Theorem 3.2 (De morgan's law)

If A and B are BFS, then
(i) $(A \oplus B)^c = A^c \odot B^c$
(ii) $(A \odot B)^c = A^c \oplus B^c$

Proof:

(i) LHS

$$A \oplus B = [-(x_n + y_n - x_n y_n), (x_p + y_p - x_p y_p)]$$

$$(A \oplus B)^c = [-(x_n + y_n - x_n y_n), (x_p + y_p - x_p y_p)]^c$$

$$= [-(1-(x_n + y_n - x_n y_n)), 1-(x_p + y_p - x_p y_p)] \quad (3.3)$$

RHS

$$A = (-x_n, x_p)$$

$$A^c = (-x_n, x_p)^c$$

$$= (-(1-x_n), 1-x_p)$$

$$B = (-y_n, y_p)$$

$$B^c = (-y_n, y_p)^c$$

$$= (-(1-y_n), 1-y_p)$$

$$A^c \odot B^c = [(-(1-x_n)), (1-x_p) \odot (-(1-y_n)), (1-y_p)]$$

$$= [-((1-x_n)(1-y_n)), (1-x_p)(1-y_p)]$$

$$= [-(1 - x_n - y_n + x_n y_n), (1 - x_p - y_p + x_p y_p)]$$

$$= [-(1-(x_n + y_n - x_n y_n)), 1-(x_p + y_p - x_p y_p)] \quad (3.4)$$

From (3.3) and (3.4) we get
Similarly we can prove $(A \odot B)^c = A^c \oplus B^c$

Theorem 3.3

If A be an BFS then O=(0,0) be the identity BFS with respect to $\oplus$ and $\boldsymbol{e}=(-1,1)$ be the identity BFS with respect to $\odot$. Then
(i) $A \oplus O = O \oplus A = A$
(ii) $A \odot \boldsymbol{e} = \boldsymbol{e} \odot A = A$

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Proof:

(i) $A = (-x_n, x_p)$

$A \oplus O = (-x_n, x_p) \oplus (0,0)$

$= \{-(x_n+0-0),(x_p+0-0)\}$

$= (-x_n, x_p)$

$= A$

Similarly we can prove $O \oplus A = A$

(ii) $A \odot \boldsymbol{e} = (-x_n, x_p) \odot (-1,1)$

$= (-x_n, x_p)$

$= A$

Similarly we can prove $\boldsymbol{e} \odot A = A$.

Theorem 3.4 (Associative law)

If A, B and C are BFS, Then

(i) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

(ii) $(A \odot B) \odot C = A \odot (B \odot C)$

Proof:

(i) LHS $=(A \oplus B) \oplus C$

$=\{-(x_n+y_n - x_n y_n), (x_p+y_p-x_p y_p) \oplus (-z_n, z_p)\}$

$=\{-(x_n+y_n - x_n y_n \oplus z_n), (x_p+y_p-x_p y_p \oplus z_p)\}$

$=\{-((x_n+y_n - x_n y_n) + z_n-(x_n+y_n - x_n y_n)(z_n)),$

$(x_p+y_p-x_p y_p) + z_p-(x_p+y_p-x_p y_p)(z_p)\}$

$=\{-(x_n+y_n+z_n- x_n y_n- x_n z_n - y_n z_n+ x_n y_n z_n),$

$(x_p+y_p + z_p-x_p y_p-x_p z_p-y_p z_p+x_p y_p z_p)\},$ (3.5)

RHS $= A \oplus (B \oplus C)$

$=\{(-x_n, x_p) \oplus -(y_n+ z_n- y_n z_n), (y_p+ z_p-y_p z_p)\}$

$=\{-(x_n \oplus y_n+ z_n- y_n z_n),(x_p \oplus y_p+ z_p- y_p z_p)\}$

$=\{-((x_n) + (y_n+ z_n- y_n z_n)- (x_n)(y_n+ z_n- y_n z_n)),$

$(x_p) + (y_p+ z_p-y_p z_p)-(x_p)(y_p+ z_p- y_p z_p)\}$

$=\{-(x_n+y_n+z_n- y_n z_n- x_n y_n- x_n z_n + x_n y_n z_n),$

$(x_p+y_p + z_p -y_p z_p- x_p y_p- x_p z_p+x_p y_p z_p)\}$ (3.6)

From (3.5) and (3.6) we get.

(ii) LHS $=(A \odot B) \odot C$

$=\{-(x_n y_n),(x_p y_p) \odot (-z_n, z_p)\}$

$=\{-(x_n y_n \odot z_n) , (x_p y_p \odot z_p)\}$

$=\{- (x_n y_n z_n),(x_p y_p z_p)\}$ (3.7)

RHS $=A \odot (B \odot C)$

$=\{(-x_n, x_p) \odot (-(y_n z_n)), (y_p z_p)\}$

$=\{-(x_n \odot y_n z_n) (x_p \odot y_p z_p)\}$

$=\{- (x_n y_n z_n), (x_p y_p z_p)\}$ (3.8)

From (3.7) and (3.8) we get.

Theorem 3.5

If A and B are BFMs of same order, then

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(i) $A \oplus B = B \oplus A$
(ii) $A \odot B = B \odot A$

Operations $\oplus$ and $\odot$ are commutative.

Proof:

$A = (-a_{ijn}, a_{ijp})$ and $B = (-b_{ijn}, b_{ijp})$

(i) LHS

$A \oplus B = (-a_{ijn}, a_{ijp}) \oplus (-b_{ijn}, b_{ijp})$

$$A \oplus B = [-(a_{ijn}+b_{ijn}-a_{ijn}\, b_{ijn}), (a_{ijp}+b_{ijp}-a_{ijp}b_{ijp})] \qquad (3.9)$$

RHS

$B \oplus A = (-b_{ijn}, b_{ijp}) \oplus (-a_{ijn}, a_{ijp})$

$$B \oplus A = [-(a_{ijn}+b_{ijn}-a_{ijn}\, b_{ijn}), (a_{ijp}+b_{ijp}-a_{ijp}\, b_{ijp})] \qquad (3.10)$$

From (3.9) and (3.10) we get.

Similarly we can prove $A \odot B = B \odot A$

Theorem 3.6 (De morgan's law)

If A and B are BFMs of same order, then

(i) $(A \oplus B)^c = A^c \odot B^c$
(ii) $(A \odot B)^c = A^c \oplus B^c$

Proof:

(i) LHS

$$A \oplus B = [-(a_{ijn}+b_{ijn}-a_{ijn}\, b_{ijn}), (a_{ijp}+b_{ijp}-a_{ijp}b_{ijp})]$$

$$(A \oplus B)^c = [-(a_{ijn}+b_{ijn}-a_{ijn}\, b_{ijn}), (a_{ijp}+b_{ijp}-a_{ijp}b_{ijp})]^c$$

$$= [-(1-(a_{ijn}+b_{ijn}-a_{ijn}\, b_{ijn})), 1-(a_{ijp}+b_{ijp}-a_{ijp}b_{ijp})] \qquad (3.11)$$

RHS

$A = (-a_{ijn}, a_{ijp})$

$A^c = (-a_{ijn}, a_{ijp})^c$

$= (-(1-a_{ijn}), 1-a_{ijp})$

$B = (-b_{ijn}, b_{ijp})$

$B^c = (-b_{ijn}, b_{ijp})^c$

$= (-(1-b_{ijn}), 1-b_{ijp})$

$A^c \odot B^c = [-(1-a_{ijn}), 1-a_{ijp} \odot -(1-b_{ijn}), 1-b_{ijp}]$

$= [-((1-a_{ijn})\,(1-b_{ijn})), (1-a_{ijp})\,(1-b_{ijp})]$

$= [-(1-a_{ijn}-b_{ijn}+a_{ijn}b_{ijn}), (1-a_{ijp}-b_{ijp}+a_{ijp}b_{ijp})]$

$$= [-(1-(a_{ijn}+b_{ijn}-a_{ijn}\, b_{ijn})), 1-(a_{ijp}+b_{ijp}-a_{ijp}b_{ijp})] \qquad (3.12)$$

From (3.11) and (3.12) we get

Similarly we can prove $(A \odot B)^c = A^c \oplus B^c$

Theorem 3.7

If A be an BFMs then O=(0,0) be the identity BFMs with respect to $\oplus$ and $\boldsymbol{e}$=(–1,1) be the identity BFMs with respect to $\odot$. Then

(i) $A \oplus 0 = 0 \oplus A = A$
(ii) $A \odot \boldsymbol{e} = \boldsymbol{e} \odot A = A$

Proof:

(i) $A = (-a_{ijn}, a_{ijp})$

$A \oplus 0 = (-a_{ijn}, a_{ijp}) \oplus (0,0)$

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$= \{-(a_{ijn}+0-0),(a_{ijp}+0-0)\}$
$= (-a_{ijn}, a_{ijp})$
$= A$

Similarly we can prove $0\oplus A= A$

(ii) $A\odot \boldsymbol{e}=(-a_{ijn}, a_{ijp}) \odot(-1,1)$
$=(-a_{ijn}, a_{ijp})$
$=A$

Similarly we can prove $\boldsymbol{e}\odot A = A$.

Theorem 3.8

If A, B and C are BFMs of same order, then

(i) $(A\oplus B) \oplus C = A\oplus(B\oplus C)$

(ii) $(A\odot B) \odot C = A\odot(B\odot C)$

Proof:

(i) LHS= $(A\oplus B) \oplus C$

$= \{-(a_{ijn}+b_{ijn}-a_{ijn} b_{ijn}), (a_{ijp}+b_{ijp}-a_{ijp}b_{ijp}) \oplus(-c_{ijn},c_{ijp})\}$
$= \{-(a_{ijn}+b_{ijn}-a_{ijn} b_{ijn} \oplus c_{ijn}), (a_{ijp}+b_{ijp}-a_{ijp}b_{ijp} \oplus c_{ijp})\}$
$= \{-((a_{ijn}+b_{ijn}-a_{ijn} b_{ijn}) + c_{ijn}-(a_{ijn}+b_{ijn}-a_{ijn} b_{ijn})(c_{ijn})),$
$(a_{ijp}+b_{ijp}-a_{ijp}b_{ijp}) +c_{ijp}-(a_{ijp}+b_{ijp}-a_{ijp}b_{ijp})(c_{ijp})\}$
$= \{-(a_{ijn}+b_{ijn}+ c_{ijn}-a_{ijn} b_{ijn}-a_{ijn} c_{ijn}-b_{ijn} c_{ijn}+a_{ijn} b_{ijn} c_{ijn}),$
$(a_{ijp}+b_{ijp}+c_{ijp}-a_{ijp}b_{ijp}-a_{ijp}c_{ijp}-b_{ijp}c_{ijp}+a_{ijp}b_{ijp} c_{ijp})\}$ (3.13)

RHS $= A\oplus(B\oplus C)$

$=\{(-a_{ijn}a_{ijp}) \oplus-(b_{ijn}+ c_{ijn}-b_{ijn} c_{ijn}),(b_{ijp}+c_{ijp}-b_{ijp}c_{ijp})\}$
$=\{-(a_{ijn} \oplus b_{ijn}+ c_{ijn}-b_{ijn} c_{ijn}), (a_{ijp} \oplus b_{ijp}+c_{ijp}-b_{ijp}c_{ijp})\}$
$=\{-((a_{ijn}) + (b_{ijn}+ c_{ijn}-b_{ijn} c_{ijn})-(a_{ijn})(b_{ijn}+ c_{ijn}-b_{ijn} c_{ijn})),$
$(a_{ijp})+(b_{ijp}+c_{ijp}-b_{ijp}c_{ijp})-(a_{ijp})(b_{ijp}+c_{ijp}-b_{ijp}c_{ijp})\}$
$=\{-(a_{ijn} +b_{ijn}+ c_{ijn}-b_{ijn} c_{ijn}-a_{ijn} b_{ijn} - a_{ijn} c_{ijn} +a_{ijn} b_{ijn} c_{ijn}),$
$(a_{ijp}+b_{ijp}+c_{ijp}-b_{ijp}c_{ijp}-a_{ijp}b_{ijp}-a_{ijp}c_{ijp}+a_{ijp}b_{ijp}c_{ijp})\}$ (3.14)

From(3.13) and (3.14) we get

(ii) LHS = $(A\odot B) \odot C$

$=\{-(a_{ijn} b_{ijn}),(a_{ijp} b_{ijp}) \odot(-c_{ijn}, c_{ijp})\}$
$=\{-(a_{ijn} b_{ijn}\odot c_{ijn}), (a_{ijp} b_{ijp} \odot c_{ijp})\}$
$=\{-(a_{ijn} b_{ijn}c_{ijn}),(a_{ijp}b_{ijp} c_{ijp})\}$ (3.15)

RHS = $A\odot(B\odot C)$

$=\{(-a_{ijn} a_{ijp}) \odot -(b_{ijn}c_{ijn}), (b_{ijp} c_{ijp})\}$
$=\{-(a_{ijn} \odot b_{ijn}c_{ijn}), (a_{ijp}\odot b_{ijp} c_{ijp})\}$
$=\{-(a_{ijn} b_{ijn}c_{ijn}), (a_{ijp}b_{ijp} c_{ijp})\}$ (3.16)

From (3.15) and (3.16) we get.

Theorem3.9

If A and B are BFMs of Symmetric where A$= (-a_{ijn}, a_{ijp})$ and $B= (-b_{ijn}, b_{ijp})$,then $(-a_{ijn}, a_{ijp}) = (-a_{jin}, a_{jip})$ and $(-b_{ijn}, b_{ijp}) = (-b_{jin}, b_{jip})$ be the ij^{th} elements of $A\oplus B$ and $(A\odot B)$.

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Proof:

$$A \oplus B = \{-(a_{ijn}+b_{ijn}-a_{ijn}\, b_{ijn}), (a_{ijp}+b_{ijp}-a_{ijp} b_{ijp})\}$$
$$= \{-(a_{jin}+b_{jin}-a_{jin} b_{jin}), (a_{jip}+b_{jip}-a_{jip} b_{jip})\}$$
$$= \{-(a_{jin}, a_{jip})\}$$
$$A \odot B = \{-(a_{ijn}, b_{ijn}), (a_{ijp}, b_{ijp})\}$$
$$= \{-(a_{jin}\, b_{jin}), (a_{jip}, b_{jip})\}$$

Proposition 3.1

If A, B and C are BFMs of same order, then

(i) $A \oplus (B \odot C) \neq (A \oplus B) \odot (A \oplus C)$

(ii) $A \odot (B \oplus C) \neq (A \odot B) \oplus (A \odot C)$

Distributive law is not satisfied.

We illustrate the Distributive law by the following example.

Proof:

(i)LHS= A$\oplus(B \odot C)$

$$A = \begin{bmatrix} (-0.2,0.3) & (-0.4,0.6) \\ (-0.1,0.5) & (-0.3,0.6) \end{bmatrix}$$

$$B = \begin{bmatrix} (-0.1,0.6) & (-0.7,0.2) \\ (-0.8,0.3) & (-0.2,0.5) \end{bmatrix}$$

$$C = \begin{bmatrix} (-0.2,0.5) & (-0.6,0.5) \\ (-0.9,0.1) & (-0.1,0.4) \end{bmatrix}$$

$$(B \odot C) = \begin{bmatrix} (-0.02,0.3) & (-0.42,0.1) \\ (-0.72,0.03) & (-0.02,0.2) \end{bmatrix}$$

$$A \oplus (B \odot C) = \begin{bmatrix} (-0.21,0.51) & (-0.65,0.64) \\ (-0.74,0.51) & (-0.31,0.68) \end{bmatrix}$$

RHS=$(A \oplus B) \odot (A \oplus C)$

$$(A \oplus B) = \begin{bmatrix} (-0.28,0.72) & (-0.82,0.68) \\ (-0.82,0.65) & (-0.44,0.8) \end{bmatrix}$$

$$(A \oplus C) = \begin{bmatrix} (-0.36,0.65) & (-0.76,0.8) \\ (-0.91,0.55) & (-0.37,0.76) \end{bmatrix}$$

$$(A \oplus B) \odot (A \oplus C) = \begin{bmatrix} (-0.10,0.46) & (-0.62,0.54) \\ (-0.74,0.35) & (-0.16,0.60) \end{bmatrix}$$

therefore A$\oplus(B \odot C) \neq (A \oplus B) \odot (A \oplus C)$

Again,

(ii) LHS=A$\odot(B \oplus C)$

$$(B\oplus C)=\begin{bmatrix}(-0.28,0.8) & (-0.88,0.6)\\(-0.98,0.37) & (-0.28,0.7)\end{bmatrix}$$

$$A\odot(B\oplus C)=\begin{bmatrix}(-0.05,0.24) & (-0.35,0.36)\\(-0.09,0.18) & (-0.08,0.42)\end{bmatrix}$$

RHS=(A⊙B) ⊕(A⊙C)

$$(A\odot B) = \begin{bmatrix}(-0.02,0.18) & (-0.28,0.12)\\(-0.08,0.15) & (-0.06,0.3)\end{bmatrix}$$

$$(A\odot C) = \begin{bmatrix}(-0.04,0.15) & (-0.24,0.3)\\(-0.09,0.05) & (-0.03,0.24)\end{bmatrix}$$

$$(A\odot B)\oplus(A\odot C) = \begin{bmatrix}(-0.05,0.30) & (-0.45,0.38)\\(-0.16,0.19) & (-0.08,0.46)\end{bmatrix}$$

Therefore $A \odot(B\oplus C)\neq (A\odot B) \oplus(A\odot C)$

Proposition 3.2

If A, B and C are BFMs of same order, then

(i) $A\oplus(A\odot B) \geq A$

(ii) $A\odot(A\oplus B) \leq A$

Proof:

$$A= \begin{bmatrix}(-0.2,0.3) & (-0.4,0.6)\\(-0.1,0.5) & (-0.3,0.6)\end{bmatrix}$$

$$B= \begin{bmatrix}(-0.1,0.6) & (-0.7,0.2)\\(-0.8,0.3) & (-0.2,0.5)\end{bmatrix}$$

(i) $$(A\odot B) = \begin{bmatrix}(-0.02,0.18) & (-0.28,0.12)\\(-0.08,0.15) & (-0.06,0.30)\end{bmatrix}$$

$$A\oplus(A\odot B) = \begin{bmatrix}(-0.21,0.42) & (-0.56,0.64)\\(-0.17,0.57) & (-0.34,0.72)\end{bmatrix}$$

Therefore $A\oplus(A\odot B) \geq A$

(ii) $$(A\oplus B) = \begin{bmatrix}(-0.28,0.72) & (-0.82,0.68)\\(-0.82,0.65) & (-0.44,0.8)\end{bmatrix}$$

$$A\odot(A\oplus B) = \begin{bmatrix}(-0.05,0.21) & (-0.32,0.40)\\(-0.08,0.32) & (-0.13,0.48)\end{bmatrix}$$

Therefore $A\odot(A\oplus B) \leq A$

4. Conclusion:

Bipolar is an important topics in several domains. In this topic we introduce the set of all BFS and BFMs with respect to the algebraic sum and algebraic product form a commutative monoid, and also satisfy the De morgan's law. Both operations does not satisfy distributivity. In future some more properties are going to be discussed.

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