ERQ MODEL FOR IMPERFECT PRODUCTION SYSTEM UNDER ENHANCED PREVENTIVE MAINTENANCE STRATEGY

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Abstract

Product recycling has a significant impact on today's sustainable business initiatives. The goal of this research is to improve the quality of the products by incorporating a sustainable recycling method into a production model for a manufacturing system that is not flawless and has a fixed proportion of recyclable damaged products. A preventive maintenance strategy is implemented to slow down the production system's degrading process and lessen its effect on the calibre of output items. Under different circumstances, the constant demand rate of defect free products are taken into account for production uptime, production downtime with positive inventory and production downtime with shortages. Using this concept, two cases are examined according to the production process. First case does not take recycling procedures whereas second case collects all substandard products before recycling them during the production downtime. This study seeks to determine the optimal quantity and reduce overall costs. In comparison to the first case, the production process that includes recycling under preventive maintenance yields a better result. To formulate the objective function by utilizing the standard optimization method, theoretical derivations have been constructed. Finally, numerical example and the future research's scope are provided. Finally, a numerical example is provided to verify the study and the direction for forthcoming research were suggested.

Keywords: Economic recycle quantity, inventory, faulty production, maintenance strategies, preventive maintenance.

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1. Introduction

Inventory management decides the ideal inventory in order to reduce the overall inventory cost. Inventory of products is necessary to the production procedure to fulfil client demand. EPQ model usually assume flawless product quality and manufacturing process. However, seamless production systems are rare in reality and defective products will occur. Therefore, the imperfect items cannot be ignored during production. In a highly competitive world, enterprises are dealing with complex problems. In fact, retailers frequently employ economical tactics to enhance the system's profitability and sustainability. Both scholars and management agreed that strategies that separated complementary operations (maintenance, quality, production, and inventory) were ineffective. One of the secrets to success is to think of a global systemic strategy that incorporates the connection between all of these functions, or a portion of them. According to this perspective, the level of interconnection between these fields has piqued researcher's interest in creating policies that would integrate all or a portion of these different functions. Numerous sectors have been devoted to modelling and optimising these strategies. In today's fiercely competitive and open marketplace, maintaining product quality is one of the growing concerns. In this study, we may divide maintenance actions into two categories based on the extent to which the working conditions of an equipment are improved by maintenance: a) Perfect maintenance: Brings the system back to "better than new" condition. b) Imperfect maintenance: brings the system back to a functioning state that is somewhere between "better than new" and "worse than old". Here, we suggest starting the cycle with ineffective prevention methods and ending it with an effective preventive action that returns the production system to its original, faultless state.

The main contributions of this study:

(i) An inventory model is constructed based on the recycling of faulty products obtained from regular production following the appropriate screening and complete raw material recovery utilised for manufacturing in the respective item category.

(ii) This model is created for three different levels of constant demand with preventive maintenance policy that fluctuates under production uptime, production downtime with positive inventory and production downtime with shortages.

The remaining portions of the paper are structured as follows: In section 2, the literature on the models is covered. In section 3, assumptions and notations that define the symbols for the parameters, decision variables are introduced. In section 4, the model formulation is explained. In section 5, theoretical findings and the distinctiveness of the optimal solution are shown. In section 6, numerical example is provided. In section 7, the study's conclusion and forthcoming research were acknowledged.

Literature Review

Over the past 20 years, numerous scholars have examined inventory models with imperfect manufacturing systems. The classic EPQ model was modified by Salameh and Jaber considering low-quality items [8]. Defective products can sometimes decay and emit carbon into the atmosphere (Mashud et al.). Even though they deteriorate, some items are wasted throughout the whole supply chain during production [5]. Kang et al. created a model for a long-run single-stage manufacturing system with faulty products that included the rework operation, inspection process and planned backordering [4]. Hasan et al. established a model for agricultural products in which they employed product separation and gave several discount schemes to avoid product deterioration. The economic production quantity (EPQ) model at a single-stage production process with rework including planned backorders was revisited by Sarkar et al. [9]. AlArjani incorporated a sustainable economic recycle quantity model for imperfect production system with shortages [1]. Researchers' interest has been focused to the integration of maintenance, quality, and production. Ben-Daya addressed the problem of imperfect maintenance and economic production lot sizing for a flawed process with an increasing random failure rate. An age reduction of the system proportional to the PM level is a characteristic of preventive maintenance procedures. It has been discovered that PM can reduce quality related costs. It is also made clear that engaging in PM activity is not justifiable if the costs associated with maintaining quality are not reduced [2]. Radhoui et al. studied an integrated preventive maintenance and quality control strategy for a production system that produces both perfect and imperfect items [7]. Chelbi et al. suggested an integrated production-maintenance strategy for uncertain production systems producing conforming and non-conforming goods that integrates EMQ, quality and an age-based PM policy. It has been demonstrated that doing preventative maintenance results in lower costs for inventory and quality-related costs [3]. Wang suggested the related problem of production scheduling and ineffective preventive maintenance. The system revamp is viewed as perfect, whereas the PM is considered to be imperfect [10]. Nourelfath et al. looked into a combined manufacture, maintenance, and quality assurance method for a flawed process in the multi-product equipped lot sizing [6]. The proposed model assumes modification of faulty products at each manufacturing stage under preventive maintenance and as a result, the faulty items are flawless after reworking. The production model for three different levels of continual demand patterns are considered.

Assumptions and Notations

Assumptions

(i) The production rate is constant and finite.

(ii) The defect rate is fixed and well-known. These products are made alongside the perfect items.

(iii) The flawless product's demand rate is a piecewise constant function. The following describes the demand during production uptime, production downtime and stock-out phase:

$$\begin{cases} D_r\text{: during production uptime} \\ mD_r\text{: during production downtime with positive inventory} \\ nD_r\text{: during production downtime with shortages} \end{cases}$$

(iv) Duration of the process of preventive maintenance is known.

(v) Lead time is minimal and the lack of quantity is permitted since the model is deemed for only one kind of item.

(vi) Faulty products are recyclable and the recovered raw materials can be used to make the same product in the following cycle. Compared to materials that were purchased, recycled materials are more valuable.

(vii) The holding expense for each faulty item is identical to those of new items, whereas the cost to recycle each defective item is same regardless of its quantity.

Notations

Notation	Description
P_r	Rate of production
D_r	Demand rate for perfect products during production uptime
d_r	Rate of production of imperfect items
F_c	Setup cost
H_c	Holding cost
P_c	Production cost
S_c	Shortage cost
I_c	Inspection cost
RM_c	Raw material cost
R_c	Recycle cost for faulty items
C_{pm}	Faulty preventive maintenance process's unit cost
$C_{p,\beta}^{x}$	Flawless preventive maintenance process's unit cost done after $(x-1)$
	faulty preventive actions
g^{β}	Deterioration factor
$M_{imperfect}$	The total cost of faulty preventive maintenance
$M_{perfect}$	The total cost of flawless preventive maintenance
PMC	The total cost of preventive maintenance
TC_d	Inventory's total cost [case 1]
TC_R	Inventory's total cost [case 2]
T_1	Production uptime with positive inventory
T_2	Production downtime with positive inventory
T_3	Production downtime with negative inventory
T_4	Production uptime with negative inventory
m	Ratio of rate of demand for production downtime with positive inventory
	and production uptime
n	Ratio of rate of demand for production downtime with negative inventory

	and production uptime
k	Parameter for the precise average demand
Z	Number of faulty products produced per cycle
W	Quantity of the lot
W_s	Maximum level of shortage
W_d	Maximum quantity of perfect items in stock
T_c	Production time

Hence, $T_c = T_1 + T_2 + T_3 + T_4$

Model Formulation

During the manufacturing uptime of time period T_c, a quantity of W units is generated at a production rate P_r, and along with it, Z units of faulty products are generate at an imperfect rate d_r. Due to increased customer demand during the production downtime, the stock runs out. As soon as the stock reaches zero, shortage occurs. The demand rate D_r during production uptime is used to indicate current demand. During manufacturing downtime, when the stock is positive, the rate mD_r is computed using a ratio coefficient. When the stock is reduced during the period T_3, shortages also happen at the demand rate nD_r.

Over the replenishment phase, the inventory is positive and increases over time by $(P_r - D_r - d_r)$ units of flawless items during the period T_1. Maximum quantity of non-defective W_d units are obtained at the end of replenishment. Hence,

$$T_1 = \frac{W_d}{(P_r - D_r - d_r)} \quad \text{..... (1)}$$

At the completion of time interval T_1, production ceases, and during time period T_2, inventory drops at a different demand rate of mD_r units. The inventory runs out of available stock at the end of time interval T_2. In this scenario, the inventory reduces at the demand rate nD_r units if the demand rate stays at D_r. Hence

$$T_2 = \frac{W_d}{mD_r} \quad \text{….. (2)}$$

Thus, $T_1 + T_2 = \frac{\{P_r - (1-m)D_r - d_r\}W_d}{mD_r(P_r - D_r - d_r)}$ ...... (3)

In time T_3, a total of W_s units are lacking.

$$T_3 = \frac{W_s}{nD_r} \quad \text{….. (4)}$$

Production begins when the level of shortage hits W_s. The shortage quantity W_s is supplied at the rate of $(P_r - D_r - d_r)$ units which is satisfied at D_r during the time period T_4. Following this time period, again the stock resets to zero. Therefore,

$$T_4 = \frac{W_s}{(P_r - D_r - d_r)} \quad \text{….. (5)}$$

Thus, $T_3 + T_4 = \frac{\{P_r - (1-n)D_r - d_r\}W_s}{nD_r(P_r - D_r - d_r)}$ ….. (6)

In the time interval $T_1 + T_4$, W units are produced, of which Z are flawed.

$$T_1 + T_4 = \frac{Z}{d_r} = \frac{W}{P_r} \quad \text{….. (7)}$$

Adding equations (1) and (5),

$$W_d + W_s = (P_r - D_r - d_r)\frac{Z}{d_r} \quad \text{….. (8)}$$

If there is no damaged goods (i.e., d_r = 0) and demand (i.e., D_r = 0), the merchandise is refilled.

The total demand D_r during production downtime is modified by the ratios of demand rates m and n with positive and negative inventory. The average demand rate is $\left(\frac{(2+m+n)D_r}{4k}\right)$, requiring $(W - Z)$ units of new products throughout the cycle's duration T_c.

$$T_c = \frac{4k(W-Z)}{(2+m+n)D_r} = \frac{4k(P_r-d_r)Z}{(2+m+n)d_rD_r} \quad \ldots (9)$$

Hence $$\frac{T_1+T_2}{T_c} = \frac{(2+m+n)d_r\{P_r-(1-m)D_r-d_r\}W_d}{4km(P_r-D_r-d_r)(P_r-d_r)Z} \quad \ldots (10)$$

$$\frac{T_3+T_4}{T_c} = \frac{(2+m+n)d_r\{P_r-(1-n)D_r-d_r\}W_S}{4kn(P_r-D_r-d_r)(P_r-d_r)Z} \quad \ldots (11)$$

The main goal of this study is to reduce the overall inventory cost where imperfect goods are combined with flawless items, yet imperfect goods are recyclable, and recycled raw materials can be utilised to create the new items in the same type. The producer must first specify all expenses, production aspects, production capabilities must first specify all expenses, production aspects, production capabilities and their perspective on recycling. Each damaged item involved in the recycling process incurs a cost.

The following two situations are described concerning the recycling of defective products.

Case 1 [EPQ Model for imperfect items with three different levels of constant demand]

Fixed cost: $FC = \frac{1}{T_c}\frac{F_c}{4} = \frac{(2+m+n)d_rD_r}{4\times 4k(P_r-d_r)Z}F_c$

Production cost: $PC = \frac{1}{T_c}WP_c = \frac{(2+m+n)P_rD_r}{4k(P_r-d_r)}P_c$

Preventive maintenance cost: We perform $(x-1)$ defective preventive maintenance activities during a cycle and one flawless preventive maintenance action at the conclusion of that cycle. A flawless preventive maintenance activity which is performed after (x) flawed preventive activities will cost more than the one that is performed after $(x-1)$ flawed actions. Exactly, it will be multiplied by a deterioration factor g^{β}, where $\beta > 0$.

Then $C_{p,\beta}^{x} = C_{p,\beta}^{x-1} \times g^{\beta}$

$$PMC = \frac{1}{T_c}\left(M_{imperfect} + M_{perfect}\right)$$

where $M_{imperfect} = C_{pm} \times (h-1)$; $M_{perfect} = C_{p,\beta}^{1} \times g^{(x-1)\times\beta}$

Therefore $PMC = \frac{1}{T_c}\left(C_{pm} \times (h-1) + C_{p,\beta}^{1} \times g^{(x-1)\times\beta}\right)$

Raw material cost: $RMC = \frac{1}{T_c}WRM_c = \frac{(2+m+n)P_rD_r}{4k(P_r-d_r)}RM_c$

Inspection cost: $IC = \frac{1}{T_c}WI_c = \frac{(2+m+n)P_rD_r}{4k(P_r-d_r)}I_c$

Holding cost: In this situation, there is no holding cost for imperfect items. Thus,

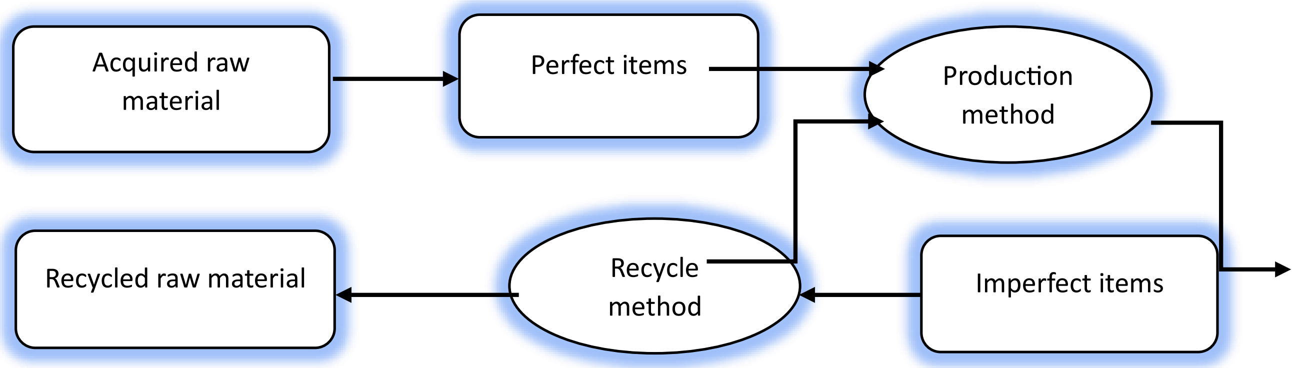
$$HC = \frac{1}{2}\times\frac{W_d}{4}\frac{(T_1+T_2)}{T_c}H_c$$
$$= \frac{d_r(2+m+n)\{P_r-(1-m)D_r-d_r\}}{4\times 8mk(P_r-d_r)(P_r-D_r-d_r)z}\left\{(P_r-D_r-d_r)\frac{Z}{d_r}-W_s\right\}^2 H_c$$

Shortage cost: $SC = \frac{W_s}{2}\times\frac{S_c}{4}\times\frac{T_3+T_4}{T_c} = \frac{(2+m+n)d_r\{P_r-(1-n)D_r-d_r\}}{4\times 4kn(P_r-d_r)(P_r-D_r-d_r)}\left(\frac{{W_s}^2}{2Z}\right)\times S_c$

Therefore, overall cost function is $TC_D = FC + PC + PMC + RMC + IC + HC + SC$

Case 2 [ERQ Model for imperfect items with three different levels of constant demand]

During production, the manufacturer accepts a certain amount of recyclable imperfect goods, and these products are shipped to be recycled during production downtime. Recycled materials are used as raw materials in the subsequent production cycle, additional raw materials must be bought from suppliers to meet customer demand. The study's goal is to manage the acquisition of raw materials, recycling quantity and the production process with preventive maintenance strategy to satisfy the demands while reducing the overall cost of inventories.



In this scenario, manufacturing cycle begins with a lack of quantity W_s, which increases at the rate $(P_r - D_r - d_r)$ over the time period T_4. At this time, the existing demand is satisfied at D_r. Production halts at the end of period T_1 and inventory increases at the rate $(P_r - D_r - d_r)$ over that time. Due to the demand rate mD_r over time T_2 and the demand rate nD_r over time T_3, the inventory level begins to decrease and shortages approach W_s. Then, Z units of damaged products are formed during production runtime $(T_1 + T_4)$, which are then recycled during production downtime $(T_2 + T_3)$ and then they are included to the raw materials for the following production cycle. As a result, the ERQ model's inventory purchases raw supplies for $(W - Z)$ units of products throughout each cycle and it repeats.

The following are the overall cost functions of the inventory management system.

Fixed cost: $FC = \frac{1}{T_c}\frac{F_c}{4} = \frac{(2+m+n)d_rD_r}{4\times 4k(P_r-d_r)Z}F_c$

Production cost: $PC = \frac{1}{T_c}WP_c = \frac{(2+m+n)P_rD_r}{4k(P_r-d_r)}P_c$

Preventive maintenance cost: $PMC = \frac{1}{T_c}\left(M_{imperfect} + M_{perfect}\right)$

where $M_{imperfect} = C_{pm} \times (h-1)$; $M_{perfect} = C^1_{p,\beta} \times g^{(x-1)\times\beta}$

$$PMC = \frac{1}{T_c}\left(C_{pm} \times (h-1) + C^1_{p,\beta} \times g^{(x-1)\times\beta}\right)$$

Raw material cost: Each production quantity is W, however raw materials are purchased for $(W - Z)$ units of items in each cycle time. Hence,

$$RMC = \frac{1}{T_c}\frac{(W-Z)}{2}RM_c = \frac{(2+m+n)D_r}{2\times 4k}RM_c$$

Holding cost: Before the products are sent for recycling, holding costs for damaged products are taken into account for the time period $(T_1 + T_4)$. Therefore,

$$HC = \frac{1}{2}\frac{W_d}{2}\frac{(T_1+T_2)}{T_c} + \frac{1}{2}\frac{d_r}{2}\frac{(T_1+T_4)^2}{T_c}H_c$$

$$= \frac{(2+m+n)}{4\times 4k(P_r-d_r)}\left[\frac{d_r\{P_r-(1-m)D_r-d_r\}}{2m(P_r-D_r-d_r)z}\left\{(P_r - D_r - d_r)\frac{Z}{d_r} - W_s\right\}^2 + \frac{D_rZ}{2}\right]H_c$$

Inspection cost: $IC = \frac{1}{T_c}WI_c = \frac{(2+m+n)P_rD_r}{4\times 4k(P_r-d_r)}I_c$

Shortage cost: $SC = \frac{W_s}{2} \times \frac{S_c}{4} \times \frac{T_3+T_4}{T_c} = \frac{(2+m+n)d_r\{P_r-(1-n)D_r-d_r\}}{4\times 4kn(P_r-d_r)(P_r-D_r-d_r)}\left(\frac{W_s{}^2}{2Z}\right) \times S_c$

Recycle cost: $RC = \frac{1}{T_c}\frac{Z}{4}R_c = \frac{(2+m+n)d_rD_r}{4\times 4k(P_r-d_r)}R_c$

Therefore, the overall cost function is $TC_R = FC + PC + PMC + RMC + HC + IC + SC + RC$. The aim is to determine the optimum level of shortage and the ideal number of faulty products to be recycled in order to reduce the overall inventory cost.

Theoretical Derivations

The Hessian matrix was used to establish the convexity of the overall cost functions TC_D and TC_R corresponding to the decision variables, enabling for the unique ideal solution to be determined.

5.1. Case 1 [EPQ Model for imperfect items with three different levels of constant demand]

Theorem 1. The overall cost function TC_D provides least value for Z and W_s simultaneously when

$(1-m)D_r + d_r > 0, \quad P_r - (1-n)D_r - d_r > 0$, thus TC_D discharges unique ideal solution Z and W_s.

Proof. Since TC_D is a variable of Z and W_s, partial derivatives of TC_D in the first order with respect to Z and W_s are

$$\frac{\partial(TC_D)}{\partial Z} = \frac{(2+m+n)}{4k(P_r-d_r)}\left[-\frac{d_rD_rF_c}{Z^2} - \frac{(P_r-D_r-d_r)\{P_r-(1-m)D_r-d_r\}H_c}{2md_r} + \frac{d_r\{P_r-(1-m)D_r-d_r\}H_cW_s^2}{2m(P_r-D_r-d_r)Z^2} - \frac{d_r\{P_r-(1-n)D_r-d_r\}S_cW_s^2}{2n(P_r-D_r-d_r)Z^2}\right]$$

$$\frac{\partial(TC_D)}{\partial W_s} = \frac{(2+m+n)}{4k(P_r-d_r)}\left[-\frac{d_r\{P_r-(1-m)D_r-d_r\}}{m(P_r-D_r-d_r)Z}\left\{(P_r-D_r-d_r)\frac{Z}{d_r} - W_s\right\}H_c + \frac{d_r\{P_r-(1-n)D_r-d_r\}S_c}{n(P_r-D_r-d_r)Z}W_s\right]$$

The partial derivatives of TC_D in second order are

$$\frac{\partial^2(TC_D)}{\partial Z^2} = \frac{d_rD_r(2+m+n)}{2k(P_r-d_r)Z^3}F_c + \frac{d_r(2+m+n)[n\{P_r-(1-m)D_r-d_r\}H_c + m\{P_r-(1-n)D_r-d_r\}S_c]W_s^2}{4kmn(P_r-d_r)(P_r-D_r-d_r)Z^3}$$

$$\frac{\partial^2(TC_D)}{\partial W_s^2} = \frac{d_r(2+m+n)[n\{P_r-(1-m)D_r-d_r\}H_c + m\{P_r-(1-n)D_r-d_r\}S_c]}{4kmn(P_r-d_r)(P_r-D_r-d_r)Z}$$

$$\frac{\partial^2(TC_D)}{\partial Z\partial W_s} = -\frac{d_r(2+m+n)[n\{P_r-(1-m)D_r-d_r\}H_c + m\{P_r-(1-n)D_r-d_r\}S_c]W_s}{4kmn(P_r-d_r)(P_r-D_r-d_r)Z^2}$$

$$\frac{\partial^2(TC_D)}{\partial W_s\partial Z} = -\frac{d_r(2+m+n)[n\{P_r-(1-m)D_r-d_r\}H_c + m\{P_r-(1-n)D_r-d_r\}S_c]W_s}{4kmn(P_r-d_r)(P_r-D_r-d_r)Z^2}$$

The Hessian matrix of TC_D is given as $H_{ij} = \begin{bmatrix} \frac{\partial^2(TC_D)}{\partial Z^2} & \frac{\partial^2(TC_D)}{\partial Z\partial W_s} \\ \frac{\partial^2(TC_D)}{\partial W_s\partial Z} & \frac{\partial^2(TC_D)}{\partial W_s^2} \end{bmatrix}$

The initial principal minor is

$$|H_{11}| = \frac{\partial^2(TC_D)}{\partial Z^2} = \frac{(2+m+n)}{4k(P_r-d_r)}\left[\frac{2d_rD_r}{Z^3}F_c + \frac{d_r\{P_r-(1-m)D_r-d_r\}H_c}{m(P_r-D_r-d_r)Z^3}W_s^2 + \frac{d_r\{P_r-(1-n)D_r-d_r\}S_c}{n(P_r-D_r-d_r)Z^3}W_s^2\right]$$

The presumption is that $P_r - D_r - d_r > 0$. Hence, $P_r-(1-m)D_r-d_r > 0, P_r-(1-n)D_r-d_r > 0 \; and \; P_r - d_r > 0$. As a result, $|H_{11}| > 0$.
The Hessian matrix for TC_D is positive definite because its initial principal minor is positive. Therefore, TC_D is least at unique ideal solutions with respect to decision variables Z and W_s. By solving the conditions $\frac{\partial(TC_D)}{\partial Z} = 0$ and $\frac{\partial(TC_D)}{\partial W_s} = 0$, the following outcomes are attained.
The ideal value is given by

$$Z = d_r\sqrt{\frac{2D_rF_c}{H_c}}\sqrt{\frac{m\{P_r-(1-n)D_r-d_r\}S_c + n\{P_r-(1-m)D_r-d_r\}H_c}{(P_r-D_r-d_r)\{P_r-(1-m)D_r-d_r\}\{P_r-(1-n)D_r-d_r\}S_c}}$$

$$W_s = n\sqrt{2D_rF_c}\sqrt{\frac{(P_r-D_r-d_r)\{P_r-(1-m)D_r-d_r\}}{\{P_r-(1-n)D_r-d_r\}}}\sqrt{\frac{H_c}{[m\{P_r-(1-n)D_r-d_r\}S_c + n\{P_r-(1-m)D_r-d_r\}H_c]S_c}}$$

The ideal production quantity is

$$W = P_r\sqrt{2D_rF_c}\sqrt{\frac{1}{(P_r - D_r - d_r)\{P_r - (1-m)D_r - d_r\}\{P_r - (1-n)D_r - d_r\}}}$$

$$\sqrt{\frac{m\{P_r - (1-n)D_r - d_r\}S_c + n\{P_r - (1-m)D_r - d_r\}H_c}{H_cS_c}}$$

The ideal cycle duration is given by

$$T_c = \sqrt{\frac{2F_c}{D_r}}\sqrt{\frac{\{P_r - (1-m)D_r - d_r\}\{P_r - (1-n)D_r - d_r\}}{(P_r - D_r - d_r)[m\{P_r - (1-n)D_r - d_r\}S_c + n\{P_r - (1-m)D_r - d_r\}H_c]}}.\frac{S_c + H_c}{\sqrt{H_cS_c}}$$

The maximum on-hand inventory is

$$W_d = m\sqrt{2D_rF_c}\sqrt{\frac{(P_r - D_r - d_r)\{P_r - (1-n)D_r - d_r\}}{\{P_r - (1-m)D_r - d_r\}}}\sqrt{\frac{S_c}{[m\{P_r - (1-n)D_r - d_r\}S_c + n\{P_r - (1-m)D_r - d_r\}H_c]H_c}}$$

Case 2 [ERQ Model for imperfect items with three different levels of constant demand]

Theorem 2. The overall cost function TC_R provides least value for Z and W_s, thus a unique solution exists for Z and W_s.

Proof. Since TC_R is a variable of Z and W_s, the connection between TC_R and TC_D is

$$TC_R = TC_D - \frac{(2+m+n)D_r}{4k(P_r - d_r)}\left[d_r(RM_c - R_c) - \frac{Z}{2}H_c\right]$$

From the partial derivatives of first and second orders, the following conclusions are drawn from the preceding equation.

$$\frac{\partial^2(TC_R)}{\partial Z^2} = \frac{\partial^2(TC_D)}{\partial Z^2};\frac{\partial^2(TC_R)}{\partial W_s^2} = \frac{\partial^2(TC_D)}{\partial W_s^2};\frac{\partial^2(TC_R)}{\partial Z\partial W_s} = \frac{\partial^2(TC_D)}{\partial Z\partial W_s};\frac{\partial^2(TC_R)}{\partial W_s\partial Z} = \frac{\partial^2(TC_D)}{\partial W_s\partial Z}$$

The Hessian matrix of TC_R is given as $H_{ij} = \begin{bmatrix}\frac{\partial^2(TC_R)}{\partial Z^2} & \frac{\partial^2(TC_R)}{\partial Z\partial W_s}\\ \frac{\partial^2(TC_R)}{\partial W_s\partial Z} & \frac{\partial^2(TC_R)}{\partial W_s^2}\end{bmatrix} = \begin{bmatrix}\frac{\partial^2(TC_D)}{\partial Z^2} & \frac{\partial^2(TC_D)}{\partial Z\partial W_s}\\ \frac{\partial^2(TC_D)}{\partial W_s\partial Z} & \frac{\partial^2(TC_D)}{\partial W_s^2}\end{bmatrix}$

From Theorem 1, the initial principal minor $|H_{11}| = \frac{\partial^2(TC_R)}{\partial Z^2} = \frac{\partial^2(TC_D)}{\partial Z^2} > 0$

The Hessian matrix for TC_R is positive definite because its initial principal minor is positive. Therefore, TC_R is least at unique ideal values with respect to decision variables Z and W_s. By solving the conditions $\frac{\partial(TC_R)}{\partial Z} = 0$ and $\frac{\partial(TC_R)}{\partial W_s} = 0$, the following outcomes are attained.

The ideal value is given by

$$Z = d_r\sqrt{\frac{2D_rF_c}{H_c}}\sqrt{\frac{m\{P_r - (1-n)D_r - d_r\}S_c + n\{P_r - (1-m)D_r - d_r\}H_c}{(P_r - D_r - d_r)\{P_r - (1-m)D_r - d_r\}\{P_r - (1-n)D_r - d_r\}S_c + d_rD_r[m\{P_r - (1-n)D_r - d_r\}S_c + n\{P_r - (1-m)D_r - d_r\}H_c]}}$$

The shortage level is provided by

$$W_s = \frac{n(P_r - D_r - d_r)\{P_r - (1-m)D_r - d_r\}H_c}{\sqrt{[m\{P_r - (1-n)D_r - d_r\}S_c + n\{P_r - (1-m)D_r - d_r\}H_c]H_c}}$$

$$\sqrt{\frac{2D_rF_c}{(P_r-D_r-d_r)\{P_r-(1-m)D_r-d_r\}\{P_r-(1-n)D_r-d_r\}S_c+d_rD_r[m\{P_r-(1-n)D_r-d_r\}S_c+n\{P_r-(1-m)D_r-d_r\}H_c]}}$$

The ideal production quantity is

$$W=P_r\sqrt{\frac{2D_rF_c}{H_c}}\sqrt{\frac{m\{P_r-(1-n)D_r-d_r\}S_c+n\{P_r-(1-m)D_r-d_r\}H_c}{(P_r-D_r-d_r)\{P_r-(1-m)D_r-d_r\}\{P_r-(1-n)D_r-d_r\}S_c+d_rD_r[m\{P_r-(1-n)D_r-d_r\}S_c+n\{P_r-(1-m)D_r-d_r\}H_c]}}$$

The ideal cycle duration is given by

$$T_c=\sqrt{\frac{2F_c}{D_r}}\frac{\{P_r-(1-m)D_r-d_r\}\{P_r-(1-n)D_r-d_r\}}{\sqrt{(P_r-D_r-d_r)\{P_r-(1-m)D_r-d_r\}\{P_r-(1-n)D_r-d_r\}S_c+d_rD_r[m\{P_r-(1-n)D_r-d_r\}S_c+n\{P_r-(1-m)D_r-d_r\}H_c]}}$$

$$\frac{H_c+S_c}{z\sqrt{\begin{bmatrix}m\{P_r-(1-n)D_r-d_r\}S_c+\\ n\{P_r-(1-m)D_r-d_r\}H_c\end{bmatrix}H_c}}$$

The maximum on-hand inventory is

$$W_d=\frac{m(P_r-D_r-d_r)\{P_r-(1-n)D_r-d_r\}S_c}{\sqrt{\begin{bmatrix}m\{P_r-(1-n)D_r-d_r\}S_c+\\ n\{P_r-(1-m)D_r-d_r\}H_c\end{bmatrix}H_c}}$$

$$\sqrt{\frac{2D_rF_c}{(P_r-D_r-d_r)\{P_r-(1-m)D_r-d_r\}\{P_r-(1-n)D_r-d_r\}S_c+d_rD_r[m\{P_r-(1-n)D_r-d_r\}S_c+n\{P_r-(1-m)D_r-d_r\}H_c]}}$$

Value of “k”:

The arbitrary constant "k" determines the average demand with accuracy and the overall cost of stocks, but it has no impact on the ideal solution. It is incorporated in the theoretical findings to avoid their intricacy.

Adding equations (3) and (6), the cycle time is

$$T_c=\frac{m\{P_r-(1-n)D_r-d_r\}W_s+n\{P_r-(1-m)D_r-d_r\}W_d}{mnD_r(P_r-D_r-d_r)}$$

From Equation (9),

$$k=\frac{d_r(2+m+n)[m\{P_r-(1-n)D_r-d_r\}W_s+n\{P_r-(1-m)D_r-d_r\}W_d]}{4mn(P_r-d_r)(P_r-D_r-d_r)Z}$$

Using the ideal values of W_s and W_d of overall cost function TC_D and TC_R, the value of k is calculated as

$$k=\frac{(2+m+n)\{P_r-(1-m)D_r-d_r\}\{P_r-(1-n)D_r-d_r\}(S_c+H_c)}{4(P_r-d_r)[m\{P_r-(1-n)D_r-d_r\}S_c+n\{P_r-(1-m)D_r-d_r\}H_c]}$$

Numerical Example

The suggested model's practical applications can be seen in a brick manufacturing enterprise. For the manufacturing time of brick’s raw supplies, before fire, for example, when the fixing and drying processes are in progress, certain damaged bricks are made. During production downtime, these damaged bricks are gathered, carefully stored, dispatched to be recycled. Then they are added to the following manufacturing cycle.

$P_r=5000,\ D_r=4500,\ d_r=100,\ F_c=1000,\ P_c=50,\ R_c=5,\ C_{pm}=100,\ C^1_{p,\beta}=400,\ H_c=10,\ RM_c=50,\ S_c=3,\ I_c=2,\ \beta=0.05,\ m=0.75$ and $n=0.5$

For Case 1

Ideal values for decision variables: Z = 136, W = 6822, W_s = 414, W_d = 131, T_c = 1.58

Ideal cycle time values: $T_1=0.3275,\ T_2=0.0388, T_3=0.184, T_4=1.035$

Both the ideal costs and the overall inventory cost: $FC=158,\ PC=214860,\ PMC=404,\ SC=120,\ HC=38,\ I_c=8594,\ RMC=214860$, and $TC_D=439{,}034$.

For Case 2

Ideal values for decision variable: Z = 98, W = 4910, W_s = 298, W_d = 94, T_c = 1.14
Ideal cycle time values: T_1 = 0.235, T_2 = 0.0278, T_3 = 0.1324, T_4 = 0.745
Both the ideal costs and the overall inventory cost: $FC = 219$, $PC = 214860$, $PMC = 404$, $SC = 86$, $RC = 108$, $I_c = 21056$, $RMC = 105282$, $HC = 530$ and $TC_R = 342{,}148$.

2. Conclusion

In this research, we suggested a production model with and without the recycling of imperfect products. This study describes when to begin and halt the production process. An enhanced preventative maintenance strategy is used to reduce the number of defects and their impact on output product quality. Our approach includes performing the cycle with ineffective prevention methods and ending it with an effective preventive action that returns the production system to its original, faultless state. This model takes output product quality, recycled raw materials into account and minimizing the overall cost. It is important to note that, in case of any uncertainty, recycled materials might serve as an alternative way of raw materials. Recycling imperfect products is beneficial to the environment in view of the fact that it reduces waste. From the results of numerical example, in comparison to the first case, the production process that includes recycling under preventive maintenance gives a better result. This paper is applicable to the manufacture of bricks, textiles, jute, etc. Furthermore, it is still possible to construct multi-item production systems by utilising the same recyclable raw materials. It would be interesting to continue this paper by taking environmental emissions into account. One can enhance the analytical model by employing new techniques to simulate faulty preventive maintenance.

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