TOPOLOGICAL INVARIANTS ASSOCIATED WITH SUDOKU NETWORKS

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Abstract

To study the quantitative structure-property relationship of molecules, the efficient tools are the topological indices. This is mainly due to the fact that these families of networks have topologies that reflect the communication pattern of a wide variety of natural problems. Mesh/torus-like low-dimensional networks have recently received a lot of attention for their better scalability to larger networks, as opposed to more complex networks such as hypercubes. There is a lot of relevant work on interdependent networks which can be reviewed. In particular, the failure of cooperation on dependent networks has been studied a lot recently. The topological indices of certain interconnection networks got the attention of many researchers in recent years. In this connection, we study the topological invariants associated with certain Sudoku networks.

Keywords: Sombor index; Modified Sombor index; Sudoku Network.

Subject Classification: 05C09; 05C69; 05C92.

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1 Introduction

All graphs considered in this paper are simple with vertex set V and edge set E. The order and size of G are denoted by $|V| = n$ and $|E| = m$ respectively. The degree of a vertex $v \in V$ is the number of edges incident to v and it is denoted by $d_G(v)$. The degree of an edge $e = uv$ is defined as $d_G(e) = d_G(u) + d_G(v) - 2$. For undefined terminology in this paper refer to [7].

There is really mathematics involved in electrical and electronic engineering. It depends on what area of electrical and electronic engineering, for example, there is a lot more abstract mathematics in communication theory and signal processing and networking, etc, Networks involve nodes communicating with each other. A lot of computers linked are from a network. Cell phone users form a network. Networking involves the study of the best way of implementing the work.

Multiprocessor interconnection networks are often required to connect thousands of homogeneously replicated processor-memory pairs, each of which is called a processing node. Instead of using shared memory, all synchronization and communication between processing nodes for program execution are often done via message passing. The design and use of multiprocessor interconnection networks have recently drawn considerable attention due to the availability of inexpensive, powerful microprocessors and memory chips. mesh networks have been recognized as versatile interconnection networks massively parallel computing. This is mainly due to the fact that these families of networks have topologies that reflect the communication pattern of a wide variety of natural problems.

Mesh/torus-like low-dimensional networks have recently received a lot of attention for their better scalability to larger networks, as opposed to more complex networks such as hypercubes. There are a lot of relevant works on interdependent networks which can be reviewed. In particular, the failure of cooperation on dependent networks has been studied a lot recently.

A number of hierarchical interconnection networks provide a framework for designing networks with a reduced link cost by designing networks with a reduced link cost by taking advantage of the locality of communication that exists in parallel applications. HIN employs multiple levels. Lower-level networks provide local communication while higher-level networks facilitate remote communication. Multistage networks have long been used as communication networks for parallel computing.

The topological index is simply a numeric associated with the molecular graph. So far, a large number of such quantities are put forward by many researchers right from 1972 [3]. According to Prof. Gutman (Personal Communication), a useful topological index is one which has good predicting power in QSPR studies. Therefore, topological indices can be categorized into two categories useful and not-so-useful TI's see []. One of the most useful topological index is the Sombor index $SO(G)$ which is put forward by I Gutman[3a]:

$$SO(G) = \sum_{uv \in E(G)} [\sqrt{deg(u)^2 + deg(v)^2}] \quad (1.1)$$

(1.2)

Motivated by the Sombor index, here we put forward the variants of the Sombor index as follows:

$$SO'(G) = \sum_{uv \in E(G)} [\sqrt{deg(u)^2 \times deg(v)^2}]$$

$$N_{SO}(G) = \sum_{uv \in E(G)} [\sqrt{deg(S_u)^2 + deg(S_v)^2}]$$

$$N'_{SO}(G) = \sum_{uv \in E(G)} [\sqrt{deg(S_u)^2 \times deg(S_v)^2}]$$

where $deg(S_u)(G)$ is the sum of the degrees of neighborhood vertices of u in G.

2 Sudoku Networks

Sudoku is a popular puzzle game. An $n \times n$ sudoku puzzle is a grid of cells partitioned into $n-$smaller blocks of n-element. Sudoku can also be viewed as a bipartite graph. The examples of sudoku graphs are depicted in the following figure.

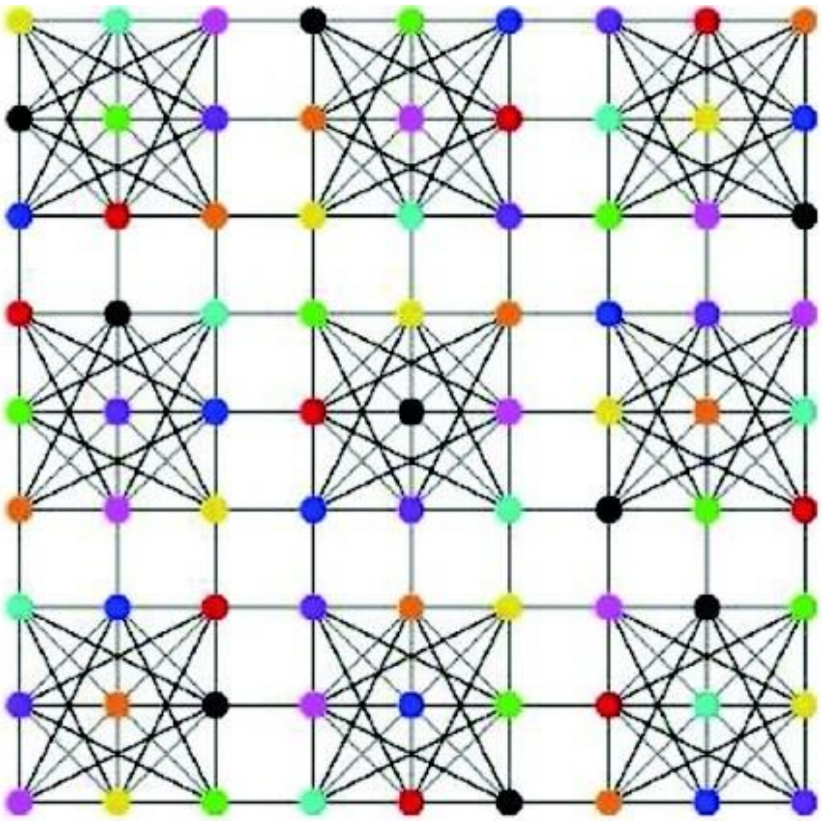

Figure 1. 3 × 3 sudoku network.

3 Results

Theorem 1. *Consider the* $(SK)_{n\times n}$ *sudoku network for* $n \geq 2$, *then its Sombor index and modified Sombor index is given by*
$SO(SK)_{n\times n} = (365.33)n^2 - (113.37)n + 5.9412.$
$SO'(SK)_{n\times n} =\backslash\ (1956)n^2 - (2288)n + 72.$

Proof. Let $(SK)_{n\times n}$ be a sudoku network. The number of vertices and edges of $(SK)_{n\times n}$ is given by $9n^2$ and $n(34n - 6)$ respectively. The edge partition of the sudoku network is based on the end vertices of each edge is can be calculated as follows:

· there are 8 edges having end vertex degrees $deg(u) = 5$, $deg(v) = 7$ respectively
· there are 12 edges having end vertex degrees $deg(u) = 5$, $deg(v) = 8$ respectively
· there are (4n-4) edges having end vertex degrees $deg(u) = 6$, $deg(v) = 6$ respectively · there are 8n edges having end vertex degrees $deg(u) = 6$, $deg(v) = 7$ respectively
· there are (32n-40) edges having end vertex degrees $deg(u) = 6$, $deg(v) = 8$ respectively
· there are $4n^2$ edges having end vertex degrees $deg(u) = 7$, $deg(v) = 7$ respectively
· there are $(20n^2-36n+20)$ edges having end vertex degrees $deg(u) = 7$, $deg(v) = 8$ respectively
· there are $(10n^2 - 14n + 4)$ edges having end vertex degrees $deg(u) = 8$, $deg(v) = 8$ respectively

By using the edge partition presented above, we have

$$
\begin{aligned}
SO(SK)_{n\times n} &= \sum_{uv\in E(G)} [\sqrt{deg(u)^2 + deg(v)^2}] \\
&= 8(\sqrt{5^2+7^2}) + 12(\sqrt{5^2+8^2}) + (4n-4)(\sqrt{6^2+6^2}) \\
&+ 8n(\sqrt{6^2+7^2}) + (32n-40)(\sqrt{6^2+8^2}) + 4n^2(\sqrt{7^2+7^2}) \\
&+ (20n^2-36n+20)(\sqrt{7^2+8^2}) + (10n^2-14n+4)(\sqrt{8^2+8^2}) \\
&= (365.33)n^2 - (113.37)n + 5.9412,
\end{aligned}
$$

as asserted. Now for $SO'(SK)_{n\times n}$ we have

$$\begin{aligned} SO'(SK)_{n\times n} &= \sum_{uv\in E(G)} [\sqrt{deg(u)^2 \times deg(v)^2}] \\ &= 8(\sqrt{5^2\times 7^2}) + 12(\sqrt{5^2\times 8^2}) + (4n-4)(\sqrt{6^2\times 6^2}) \\ &+ 8n(\sqrt{6^2\times 7^2}) + (32n-40)(\sqrt{6^2\times 8^2}) + 4n^2(\sqrt{7^2\times 7^2}) \\ &+ (20n^2-36n+20)(\sqrt{7^2\times 8^2}) + (10n^2-14n+4)(\sqrt{8^2\times 8^2}) \\ &= (1956)n^2 - (2288)n + 72, \end{aligned}$$

as asserted. □

Theorem 2. *Consider the* $(SK)_{n\times n}$ *sudoku network for* $n \geq 4$*, then its Sombor index and modified Sombor index is given by*
$SO(SK)_{n\times n} = (2768.24)n^2 + (7664.31)n - 31.12.$
$SO'(SK)_{n\times n} = (1956)n^2 - (2288)n + 72.$

Proof. Let $(SK)_{n\times n}$ be a sudoku network. The number of vertices and edges of $(SK)_{n\times n}$ is given by $9n^2$ and $n(34n - 6)$ respectively. For $n \geq 4$ the edge partition of the sudoku network is based on the end vertices of each edge is can be calculated as follows:

· there are 8 edges having end vertex degrees $deg(u) = 38$, $deg(v) = 47$ respectively
· there are 4 edges having end vertex degrees $deg(u) = 38$, $deg(v) = 54$ respectively
· there are 8 edges having end vertex degrees $deg(u)) = 38$, $deg(v) = 55$ respectively
· there are 8 edges having end vertex degrees $deg(u) = 44$, $deg(v) = 45$ respectively
· there are 16 edges having end vertex degrees $deg(u) = 44$, $deg(v) = 47$ respectively
· there are 8 edges having end vertex degrees $deg(u) = 44$, $deg(v) = 54$ respectively
· there are 16 edges having end vertex degrees $deg(u) = 44$, $deg(v) = 55$ respectively
· there are $(4n - 12)$ edges having end vertex degrees $deg(u) = 45$, $deg(v) = 45$ respectively
· there are $(8n - 16)$ edges having end vertex degrees $deg(u) = 45$, $deg(v) = 50$ respectively
· there are $(24n - 48)$ edges having end vertex degrees $deg(u) = 45$, $deg(v) = 57$ respectively
· there are $(8n - 16)$ edges having end vertex degrees $deg(u) = 45$, $deg(v) = 58$ respectively
· there are 4 edges having end vertex degrees $deg(u) = 47$, $deg(v) = 47$ respectively
· there are 8 edges having end vertex degrees $deg(u) = 47$, $deg(v) = 52$ respectively
· there are 8 edges having end vertex degrees $deg(u) = 47$, $deg(v) = 54$ respectively
· there are 8 edges having end vertex degrees $deg(u) = 47$, $deg(v) = 55$ respectively
· there are $(8n - 16)$ edges having end vertex degrees $deg(u) = 50$, $deg(v) = 53$ respectively
· there are $(12n - 24)$ edges having end vertex degrees $deg(u) = 50$, $deg(v) = 57$ respectively
· there are 8 edges having end vertex degrees $deg(u) = 52$, $deg(v) = 53$ respectively
· there are 4 edges having end vertex degrees $deg(u) = 52$, $deg(v) = 54$ respectively
· there are 8 edges having end vertex degrees $deg(u) = 52$, $deg(v) = 55$ respectively
· there are $(4n - 12)$ edges having end vertex degrees $deg(u) = 53$, $deg(v) = 53$ respectively
· there are $(8n - 16)$ edges having end vertex degrees $deg(u) = 53$, $deg(v) = 54$ respectively
· there are $(24n - 48)$ edges having end vertex degrees $deg(u) = 53$, $deg(v) = 57$ respectively
· there are $(8n - 16)$ edges having end vertex degrees $deg(u) = 53$, $deg(v) = 58$ respectivelythere are $(4n^2-20n+24)$ edges having end vertex degrees $deg(u) = 54$, $deg(v) = 54$ respectively
· there are 8 edges having end vertex degrees $deg(u) = 54$, $deg(v) = 55$ respectively
· there are $(20n^2-80n+80)$ edges having end vertex degrees $deg(u) = 54$, $deg(v) = 60$ respectively
· there are 4 edges having end vertex degrees $deg(u) = 55$, $deg(v) = 55$ respectively
· there are 8 edges having end vertex degrees $deg(u) = 55$, $deg(v) = 57$ respectively
· there are $(12n - 28)$ edges having end vertex degrees $deg(u) = 57$, $deg(v) = 57$ respectively
· there are $(12n - 24)$ edges having end vertex degrees $deg(u) = 57$, $deg(v) = 58$ respectively
· there are $(4n - 8)$ edges having end vertex degrees $deg(u) = 58$, $deg(v) = 60$ respectively
· there are $(10n^2-42n+44)$ edges having end vertex degrees $deg(u) = 60$, $deg(v) = 60$ respectively

By using the edge partition presented above, we have

$$
\begin{aligned}
SO(SK)_{n\times n} &= \sum_{uv\in E(G)}[\sqrt{deg(u)^2+deg(v)^2}]\\
&= 8(\sqrt{38^2+47^2})+4(\sqrt{38^2+54^2})+8(\sqrt{38^2+55^2})\\
&+ 8(\sqrt{44^2+45^2})+16(\sqrt{44^2+47^2})+8(\sqrt{44^2+54^2})\\
&+ 16(\sqrt{44^2+55^2})+(4n-12)(\sqrt{45^2+45^2})\\
&+ (8n-16)(\sqrt{45^2+50^2})+(24n-48)(\sqrt{45^2+57^2})\\
&+ (8n-16)(\sqrt{45^2+58^2})+4(\sqrt{47^2+47^2})+8(\sqrt{47^2+52^2})\\
&+ 8(\sqrt{47^2+54^2})+8(\sqrt{47^2+55^2})+(8n-16)(\sqrt{50^2+53^2})\\
&+ (12n-24)(\sqrt{50^2+57^2})+8(\sqrt{52^2+53^2})+4(\sqrt{52^2+54^2})\\
&+ 8(\sqrt{52^2+55^2})+(4n-12)(\sqrt{53^2+53^2})+(8n-16)(\sqrt{53^2+54^2})\\
&+ (24n-48)(\sqrt{53^2+57^2})+(8n-16)(\sqrt{53^2+58^2})+(4n^2-20n+24)(\sqrt{54^2+54^2})\\
&+ 8(\sqrt{44^2+45^2})+16(\sqrt{44^2+47^2})+8(\sqrt{44^2+54^2})\\
&+ 8(\sqrt{54^2+55^2})+(20n^2-80n+80)(\sqrt{54^2+60^2})+4(\sqrt{55^2+55^2})\\
&+ 8(\sqrt{55^2+57^2})+(12n-28)(\sqrt{57^2+57^2})+(12n-24)(\sqrt{57^2+58^2})\\
&+ (4n-8)(\sqrt{58^2+60^2})+(10n^2-42n+44)(\sqrt{60^2+60^2})\\
&= (365.33)n^2-(113.37)n+5.9412,
\end{aligned}
$$

as asserted. Now for $SO'(SK)_{n\times n}$ we have

$$
\begin{aligned}
SO'(SK)_{n\times n} &= \sum_{uv\in E(G)}[\sqrt{deg(u)^2+deg(v)^2}]\\
&= 8(\sqrt{38^2\times 47^2})+4(\sqrt{38^2\times 54^2})+8(\sqrt{38^2\times 55^2})\\
&+ 8(\sqrt{44^2\times 45^2})+16(\sqrt{44^2\times 47^2})+8(\sqrt{44^2\times 54^2})\\
&+ 16(\sqrt{44^2\times 55^2})+(4n-12)(\sqrt{45^2\times 45^2})\\
&+ (8n-16)(\sqrt{45^2\times 50^2})+(24n-48)(\sqrt{45^2\times 57^2})\\
&+ (8n-16)(\sqrt{45^2\times 58^2})+4(\sqrt{47^2\times 47^2})+8(\sqrt{47^2\times 52^2})\\
&+ 8(\sqrt{47^2\times 54^2})+8(\sqrt{47^2\times 55^2})+(8n-16)(\sqrt{50^2\times 53^2})\\
&+ (12n-24)(\sqrt{50^2\times 57^2})+8(\sqrt{52^2\times 53^2})+4(\sqrt{52^2\times 54^2})\\
&+ 8(\sqrt{52^2\times 55^2})+(4n-12)(\sqrt{53^2\times 53^2})+(8n-16)(\sqrt{53^2\times 54^2})\\
&+ (24n-48)(\sqrt{53^2\times 57^2})+(8n-16)(\sqrt{53^2\times 58^2})+(4n^2-20n+24)(\sqrt{54^2\times 54^2})\\
&+ 8(\sqrt{44^2\times 45^2})+16(\sqrt{44^2\times 47^2})+8(\sqrt{44^2\times 54^2})\\
&+ 8(\sqrt{54^2\times 55^2})+(20n^2-80n+80)(\sqrt{54^2\times 60^2})+4(\sqrt{55^2\times 55^2})\\
&+ 8(\sqrt{55^2\times 57^2})+(12n-28)(\sqrt{57^2\times 57^2})+(12n-24)(\sqrt{57^2\times 58^2})\\
&+ (4n-8)(\sqrt{58^2\times 60^2})+(10n^2-42n+44)(\sqrt{60^2\times 60^2})\\
&= (112464)n^2-(80984)n-12664,
\end{aligned}
$$

as asserted. □

Theorem 3. *Consider the Sudoku* $(SK)_{2\times 2}$, *then*
$N_{SO}(SK)_{2\times 2} = 8662.1704$
$N_{SO'}(SK)_{2\times 2} = 300036.$

Proof. Let $(SK)_{2\times 2}$ be a sudoku network. The edge partition of the sudoku network is based on the end vertices of each edge is can be calculated as follows:

there are 8 edges having end vertex degrees $S_u = 38$, $S_v = 47$ respectively · there are 4 edges having end vertex degrees $S_u = 38$, $S_v = 54$ respectively

· there are 8 edges having end vertex degrees $S_u = 38$, $S_v = 55$ respectively
· there are 4 edges having end vertex degrees $S_u = 44$, $S_v = 44$ respectively

· there are 16 edges having end vertex degrees $S_u = 44$, $S_v = 47$ respectively · there are 8 edges having end vertex degrees $S_u = 44$, $S_v = 54$ respectively
· there are 16 edges having end vertex degrees $S_u = 44$, $S_v = 55$ respectively
· there are 4 edges having end vertex degrees $S_u = 47$, $S_v = 47$ respectively
· there are 8 edges having end vertex degrees $S_u = 47$, $S_v = 52$ respectively
· there are 8 edges having end vertex degrees $S_u = 47$, $S_v = 54$ respectively
· there are 8 edges having end vertex degrees $S_u = 47$, $S_v = 55$ respectively
· there are 4 edges having end vertex degrees $S_u = 52$, $S_v = 52$ respectively
· there are 4 edges having end vertex degrees $S_u = 52$, $S_v = 54$ respectively
· there are 8 edges having end vertex degrees $S_u = 52$, $S_v = 55$ respectively
· there are 8 edges having end vertex degrees $S_u = 54$, $S_v = 55$ respectively
· there are 8 edges having end vertex degrees $S_u = 55$, $S_v = 55$ respectively

By using the edge partition presented above, we have

$$\begin{aligned}
N_{SO}(SK)_{2\times 2} &= \sum_{uv\in E(G)} [\sqrt{S_u^2 + S_v^2}] \\
&= 8(\sqrt{38^2 + 47^2}) + 4(\sqrt{38^2 + 54^2}) + 8(\sqrt{38^2 + 55^2}) \\
&+ 4(\sqrt{44^2 + 44^2}) + 16(\sqrt{44^2 + 47^2}) + 8(\sqrt{44^2 + 54^2}) \\
&+ 16(\sqrt{44^2 + 55^2}) + 4(\sqrt{47^2 + 47^2}) \\
&+ 8(\sqrt{47^2 + 52^2}) + 8(\sqrt{47^2 + 54^2}) \\
&+ 8(\sqrt{47^2 + 55^2}) + 4(\sqrt{52^2 + 52^2}) + 4(\sqrt{52^2 + 54^2}) \\
&+ 8(\sqrt{52^2 + 55^2}) + 8(\sqrt{54^2 + 55^2}) + 8(\sqrt{55^2 + 55^2}) \\
&= 8662.1704
\end{aligned}$$

as asserted. Now for $N_{SO}'(SK)_{3\times 3}$ we have

$$\begin{aligned}
N_{SO}(SK)_{2\times 2} &= \sum_{uv\in E(G)} [\sqrt{S_u^2 \times S_v^2}] \\
&= 8(\sqrt{38^2 \times 47^2}) + 4(\sqrt{38^2 \times 54^2}) + 8(\sqrt{38^2 \times 55^2}) \\
&+ 4(\sqrt{44^2 \times 44^2}) + 16(\sqrt{44^2 \times 47^2}) + 8(\sqrt{44^2 \times 54^2}) \\
&+ 16(\sqrt{44^2 \times 55^2}) + 4(\sqrt{47^2 \times 47^2}) \\
&+ 8(\sqrt{47^2 \times 52^2}) + 8(\sqrt{47^2 \times 54^2}) \\
&+ 8(\sqrt{47^2 \times 55^2}) + 4(\sqrt{52^2 \times 52^2}) + 4(\sqrt{52^2 \times 54^2}) \\
&+ 8(\sqrt{52^2 \times 55^2}) + 8(\sqrt{54^2 \times 55^2}) + 8(\sqrt{55^2 \times 55^2}) \\
&= 300036.
\end{aligned}$$

as asserted. □

Theorem 4. *Consider the Sudoku* $(SK)_{3\times 3}$*, then*
$N_{SO}(SK)_{3\times 3} = 21249.9012$
$N_{SO}'(SK)_{3\times 3} = 741688.$

Proof. Let $(SK)_{3\times 3}$ be a sudoku network. The edge partition of the sudoku network is based on the end vertices of each edge is can be calculated as follows:
· there are 8 edges having end vertex degrees $S_u = 38$, $S_v = 47$ respectively
· there are 4 edges having end vertex degrees $S_u = 38$, $S_v = 54$ respectively
· there are 8 edges having end vertex degrees $S_u = 38$, $S_v = 55$ respectively

· there are 8 edges having end vertex degrees $S_u = 44$, $S_v = 45$ respectively
· there are 16 edges having end vertex degrees $S_u = 44$, $S_v = 47$ respectively · there are 8 edges having end vertex degrees $S_u = 44$, $S_v = 55$ respectively

· there are 16 edges having end vertex degrees S_u = 44, S_v = 55 respectively
· there are 8 edges having end vertex degrees S_u = 45, S_v = 50 respectively.
. there are 24 edges having end vertex degrees S_u = 45, S_v = 57 respectively
· there are 8 edges having end vertex degrees S_u = 45, S_v = 58 respectively
· there are 4 edges having end vertex degrees S_u = 47, S_v = 47 respectively
· there are 8 edges having end vertex degrees S_u = 47, S_v = 52 respectively
· there are 8 edges having end vertex degrees S_u = 47, S_v = 54 respectively
· there are 8 edges having end vertex degrees S_u = 47, S_v = 55 respectively
· there are 8 edges having end vertex degrees S_u = 50, S_v = 53 respectively
· there are 8 edges having end vertex degrees S_u = 50, S_v = 57 respectively
· there are 8 edges having end vertex degrees S_u = 52, S_v = 53 respectively
· there are 4 edges having end vertex degrees S_u = 52, S_v = 54 respectively
· there are 8 edges having end vertex degrees S_u = 52, S_v = 55 respectively
· there are 8 edges having end vertex degrees S_u = 53, S_v = 54 respectively
· there are 24 edges having end vertex degrees S_u = 53, S_v = 57 respectively
· there are 8 edges having end vertex degrees S_u = 53, S_v = 58 respectively
· there are 8 edges having end vertex degrees S_u = 54, S_v = 55 respectively
· there are 20 edges having end vertex degrees S_u = 54, S_v = 66 respectively
· there are 4 edges having end vertex degrees S_u = 55, S_v = 55 respectively
· there are 8 edges having end vertex degrees S_u = 55, S_v = 57 respectively
· there are 8 edges having end vertex degrees S_u = 57, S_v = 57 respectively
· there are 12 edges having end vertex degrees S_u = 57, S_v = 58 respectively
· there are 4 edges having end vertex degrees S_u = 58, S_v = 60 respectively
· there are 8 edges having end vertex degrees S_u = 60, S_v = 60 respectively By using the edge partition presented above, we have

$$\begin{aligned}
N_{SO}(SK)_{3\times 3} &= \sum_{uv\in E(G)} [\sqrt{S_u^2+S_v^2}] \\
&= 8(\sqrt{38^2+47^2}) + 4(\sqrt{38^2+54^2}) + 8(\sqrt{38^2+55^2}) \\
&+ 8(\sqrt{44^2+45^2}) + 16(\sqrt{44^2+47^2}) + 8(\sqrt{44^2+54^2}) \\
&+ 16(\sqrt{44^2+55^2}) + 8(\sqrt{45^2+50^2}) \\
&+ 24(\sqrt{45^2+57^2}) + 8(\sqrt{45^2+58^2}) \\
&+ 4(\sqrt{47^2+47^2}) + 8(\sqrt{47^2+52^2}) + 8(\sqrt{47^2+54^2}) \\
&+ 8(\sqrt{57^2+55^2}) + 8(\sqrt{50^2+53^2}) + 12(\sqrt{50^2+57^2}) \\
&+ 8(\sqrt{52^2+53^2}) + 4(\sqrt{52^2+54^2}) + 8(\sqrt{52^2+55^2}) \\
&+ 8(\sqrt{53^2+54^2}) + 24(\sqrt{53^2+57^2}) + 8(\sqrt{53^2+58^2}) \\
&+ 8(\sqrt{54^2+55^2}) + 20(\sqrt{54^2+60^2}) + 4(\sqrt{55^2+55^2}) \\
&+ 8(\sqrt{55^2+57^2}) + 8(\sqrt{57^2+57^2}) + 12(\sqrt{57^2+58^2}) \\
&+ 4(\sqrt{58^2+60^2}) + 8(\sqrt{60^2+60^2}) \\
&= 21249.9012
\end{aligned}$$

as asserted. Now $N'_{SO}(SK)_{3\times 3}$ we have

$$
\begin{aligned}
N'_{SO}(SK)_{3\times3} &= \sum_{uv\in E(G)} [\sqrt{S_u^2 \times S_v^2}] \\
&= 8(\sqrt{38^2\times 47^2}) + 4(\sqrt{38^2\times 54^2}) + 8(\sqrt{38^2\times 55^2}) \\
&+ 8(\sqrt{44^2\times 45^2}) + 16(\sqrt{44^2\times 47^2}) + 8(\sqrt{44^2\times 54^2}) \\
&+ 16(\sqrt{44^2\times 55^2}) + 8(\sqrt{45^2\times 50^2}) \\
&+ 24(\sqrt{45^2\times 57^2}) + 8(\sqrt{45^2\times 58^2}) \\
&+ 4(\sqrt{47^2\times 47^2}) + 8(\sqrt{47^2\times 52^2}) + 8(\sqrt{47^2\times 54^2}) \\
&+ 8(\sqrt{57^2\times 55^2}) + 8(\sqrt{50^2\times 53^2}) + 12(\sqrt{50^2\times 57^2}) \\
&+ 8(\sqrt{52^2\times 53^2}) + 4(\sqrt{52^2\times 54^2}) + 8(\sqrt{52^2\times 55^2}) \\
&+ 8(\sqrt{53^2\times 54^2}) + 24(\sqrt{53^2\times 57^2}) + 8(\sqrt{53^2\times 58^2}) \\
&+ 8(\sqrt{54^2\times 55^2}) + 20(\sqrt{54^2\times 60^2}) + 4(\sqrt{55^2\times 55^2}) \\
&+ 8(\sqrt{55^2\times 57^2}) + 8(\sqrt{57^2\times 57^2}) + 12(\sqrt{57^2\times 58^2}) \\
&+ 4(\sqrt{58^2\times 60^2}) + 8(\sqrt{60^2\times 60^2}) \\
&= 741688.
\end{aligned}
$$

as asserted.

Conclusion:

The study of split domination number of list subdivision graph of a graph. Regarding the concept of split domination, we have studied this domination parameter for the list subdivision graph and obtained some bounds.

- $(SK)_{n\times n}$ be a sudoku network. The number of vertices and edges of $(SK)_{n\times n}$ is given by $9n^2$ and $n(34n - 6)$ respectively. The edge partition of the sudoku network is based on the end vertices of each edge is can be calculated.
- $(SK)_{n\times n}$ be a sudoku network. The number of vertices and edges of $(SK)_{n\times n}$ is given by $9n^2$ and $n(34n - 6)$ respectively. For $n \geq 4$ the edge partition of the sudoku network is based on the end vertices of each edge is can be calculated.
- $(SK)_{2\times2}$ be a sudoku network. The edge partition of the sudoku network is based on the end vertices of each edge is can be calculated.
- $(SK)_{3\times3}$ be a sudoku network. The edge partition of the sudoku network is based on the end vertices of each edge is can be calculated

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