WEIGHTED MEAN LABELING OF STANDARD GRAPHS

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ABSTRACT

The labeling of a graph G is said to be Weighted Mean Labeling(WML), if its vertices are labelled from $\{0,1,2,\ldots,q\}$, where q is the number of edges of G such that the edges of G, can be labelled with $\left\lceil\frac{f(\delta)\deg(\delta)+f(\mu)\deg(\mu)}{\deg(\delta)+\deg(\mu)}\right\rceil$ the resulting edge labels are distinct from $\{1,2,\ldots,q\}$. If a graph G admits WML, then G is said to be Weighted Mean Graph (WMG). In this paper, we have discussed the WML of some Standard Graphs.

Keywords. Weighted Mean Labeling, Weighted Mean Graph.

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INTRODUCTION

Rosa introduced the graceful labeling method in 1967 [4] further S. Somasundaram and R. Ponraj developed the concept of mean labeling of graphs [5],[6]. A. Duraibasker et.al developed the concept of geometric mean labeling of graphs and studied the behavior [1]. In this article, the new concept of Weighted mean labeling being introduced and studied their behavior some standard graphs. The labeling of a graph G is said to be Weighted Mean Labeling(WML), if its vertices are labelled from$\{0,1,2,\dots,q\}$, where q is the number of edges of G such that the edges of G, canbe labelled with $\left\lceil\frac{f(\delta)\deg(\delta)+f(\mu)\deg(\mu)}{\deg(\delta)+\deg(\mu)}\right\rceil$ the resulting edge labels are distinct from $\{1,2,\dots,q\}$. If a graph G admits WML, then G is said to be Weighted Mean Graph (WMG).

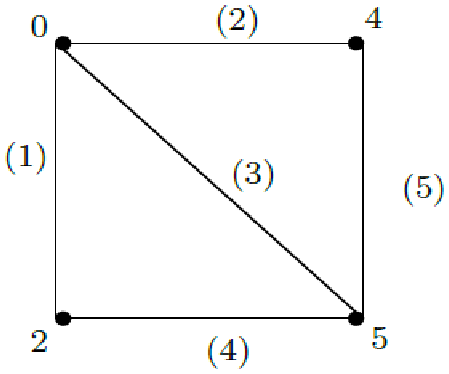

Figure 1

For standard terminology and notation the reader can be refer to [3] and the study the graph labeling the reader can be refer to Gallian (2022) [2].

PRELIMINARIES

Definition 2.1. The H-graph is obtained from two paths $\delta_1,\delta_2,\dots,\delta_n$ and $\mu_1,\mu_2,\dots,\mu_n$ of equal length by joining an edge $\delta_{\frac{n+1}{2}}\mu_{\frac{n+1}{2}}$ when n is odd and $\delta_{\frac{n}{2}}+1\mu_{\frac{n}{2}}$ when n is even.

RESULTS

Theorem 3.1. Every path P_n is a WMG.

Proof. Let the vertices of P_n be $\delta_1,\delta_2,\dots,\delta_n$ and the $E(P_n)=\{\delta_\alpha\delta_{\alpha+1}: 1\le\alpha\le n-1\}$.

Define $\pi: V(P_n)\to\{0,1,2,..,n-1\}$ as follows:

$\pi(\delta_\alpha)=\alpha-1$, if $1\le\alpha\le n$,

we attain the following edge labeling as:

$\pi^*(\delta_\alpha\delta_{\alpha+1})=\alpha$, if $1\le\alpha\le n-1$.

Thus P_n is labelled with WML and hence we conclude that P_n is a WMG.

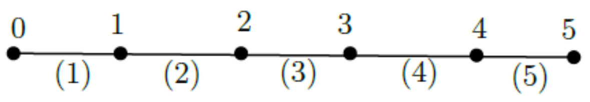

Figure 2: A WML of P_6

Theorem 3.2. Every ladder L_n is a WMG.

Proof. Let $\delta_1,\delta_2,\dots,\delta_n$ and $\eta_1,\eta_2,..,\eta_n$ be the vertices of L_n. $E(L_n)=\{\delta_\alpha\delta_{\alpha+1},\eta_\alpha\eta_{\alpha+1}: 1\le\alpha\le n-1\}\cup\{\delta_\alpha\eta_\alpha: 1\le\alpha\le n\}$.

Define $\pi: V(L_n)\to\{0,1,2,..,3n-2\}$ as follows:

$\pi(\delta_\alpha)=3\alpha-3$, if $1\le\alpha\le n$ and

$\pi(\eta_\alpha)=3\alpha-2$, if $1\le\alpha\le n$

we attain the following edge labeling as:

$\pi^*(\delta_\alpha\delta_{\alpha+1})=3\alpha-1$, if $1\le\alpha\le n-1$,

$\pi^*(\delta_\alpha\eta_\alpha)=3\alpha-2$, if $1\le\alpha\le n$ and

$\pi^*(\eta_\alpha\eta_{\alpha+1})=3\alpha$, if $1\le\alpha\le n-1$.

Thus L_n is labelled with WML and hence we conclude that L_n is a WMG.

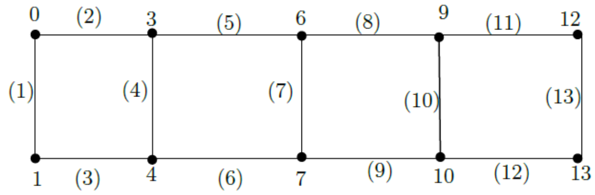

Figure 3: A WML of L_5

Theorem 3.3. The middle graph of the path is a WMG.

Proof. Let $\delta_1,\delta_2,\dots,\delta_n$ be the vertices of P_n. Then $V(M(P_n))=\{\delta_1,\delta_2,\dots,\delta_n,\eta_1,\eta_2,..,\eta_{n-1}\}$ *and* $E(M(P_n))=\{\delta_\alpha\eta_\alpha,\eta_\alpha\delta_{\alpha+1}: 1\le\alpha\le n-1\}\cup\{\eta_\alpha\eta_{\alpha+1}: 1\le\alpha\le n-2\}$.

Define $\pi: V(L_n)\to\{0,1,2,..,3n-4\}$ as follows:

$\pi(\delta_1)=0$,

$\pi(\delta_\alpha)=3\alpha-4$, if $2\le\alpha\le n$ and

$\pi(\eta_\alpha)=3\alpha-2$, if $1\le\alpha\le n-1$.

we attain the following edge labeling as:

$\pi^*(\delta_\alpha\eta_\alpha)=3\alpha-2$, if $1\le\alpha\le n-1$,

$\pi^*(\eta_\alpha\delta_{\alpha+1})=3\alpha-1$, if $1\le\alpha\le n-1$ *and*

$\pi^*(\eta_\alpha\eta_{\alpha+1})=3\alpha$, if $1\le\alpha\le n-2$.

Thus $M(P_n)$ is labelled with WML and hence we conclude that $M(P_n)$ is a WMG.

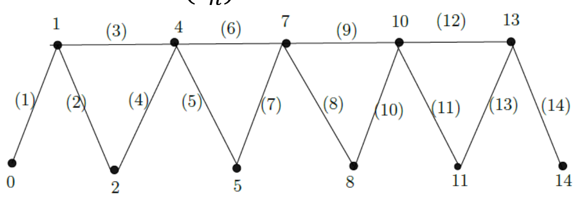

Figure 4: A WML of $M(P_6)$

Theorem 3.4. $S(L_n)$ is a WMG.

Proof. Let $V(S(L_n))=\{\delta_\alpha,\eta_\alpha: 1\le\alpha\le n\}$ and $E(S(L_n))=\{\eta_\alpha\delta_{\alpha+1},\eta_\alpha\eta_{\alpha+1},\delta_\alpha\delta_{\alpha+1}: 1\le\alpha\le n-1\}$.

Define $\pi: V(S(L_n))\to\{0,1,2,..,3n-3\}$ as follows:

$\pi(\delta_1)=0$,

$\pi(\delta_\alpha)=3\alpha-5$, if $2\le\alpha\le n$,

$\pi(\eta_\alpha)=3\alpha-1$, if $1\le\alpha\le n-1$ and

$\pi(\eta_n)=3n-3$

we attain the following edge labeling as:
$\pi^*(\delta_1\delta_2) = 1,$
$\pi^*(\delta_\alpha\delta_{\alpha+1}) = 3\alpha - 3$, if $2 \le \alpha \le n-1$,
$\pi^*(\eta_\alpha\eta_{\alpha+1}) = 3\alpha + 1$, if $1 \le \alpha \le n-2$,
$\pi^*(\eta_{n-1}\eta_n) = 3n - 3$ and
$\pi^*(\eta_\alpha\delta_{\alpha+1}) = 3\alpha - 1$, if $1 \le \alpha \le n-1$.

Thus $S(L_n)$ is labelled with WML and hence we conclude that $S(L_n)$ is a WMG.

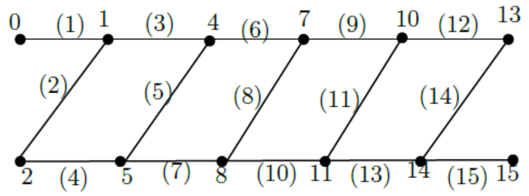

Figure 5: A WML of $S(L_6)$

Theorem 3.5. The Open Ladder $O(L_n)$ is a WMG.
Proof: Let $V(O(L_n)) = \{\delta_\alpha, \eta_\alpha : 1 \le \alpha \le n\}$ *and* $E(O(L_n)) = \{\delta_\alpha\delta_{\alpha+1}, \eta_\alpha\eta_{\alpha+1} : 1 \le \alpha \le n-1\} \cup \{\delta_\alpha\eta_\alpha : 2 \le \alpha \le n-1\}$.
Define $\pi: V(O(L_n)) \to \{0,1,2,..,3n-4\}$ as follows:
$\pi(\delta_\alpha) = \alpha - 1$, if $1 \le \alpha \le n$ and
$\pi(\eta_\alpha) = 2n + \alpha - 4$, if $1 \le \alpha \le n$.
we attain the following edge labeling as:
$\pi^*(\delta_\alpha\delta_{\alpha+1}) = \alpha$, if $1 \le \alpha \le n-1$,
$\pi^*(\delta_\alpha\eta_\alpha) = n + \alpha - 2$, if $2 \le \alpha \le n-1$ and
$\pi^*(\eta_\alpha\eta_{\alpha+1}) = 2n + \alpha - 3$, if $1 \le \alpha \le n-1$.
Thus $O(L_n)$ is labelled with WML and hence we conclude that $O(L_n)$ is a WMG.

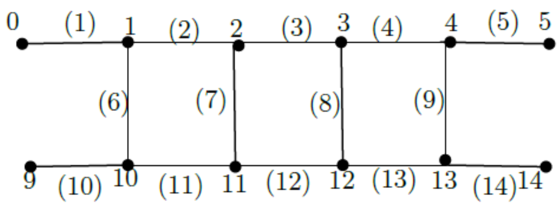

Figure 6: A WML of $O(L_6)$

Theorem 3.6. Any H-graph H_n is a WMG.
Proof: Let $\delta_1, \delta_2, ..., \delta_n$ and $\eta_1, \eta_2, .., \eta_n$ be the vertices of the path of equal length on 2n vertices on H_n.
Case (i) n is odd
Define $\pi: V(H_n) \to \{0,1,2,..,2n-1\}$ as follows:
$\pi(\delta_\alpha) = \alpha - 1$, if $1 \le \alpha \le n$ and
$\pi(\eta_\alpha) = n + \alpha - 1$, if $1 \le \alpha \le n$.
we attain the following edge labeling as:
$\pi^*(\delta_\alpha\delta_{\alpha+1}) = \alpha$, if $1 \le \alpha \le n-1$,
$\pi^*(\delta_\alpha\eta_\alpha) = n$, if $\alpha = \left\lfloor \frac{n}{2} \right\rfloor + 1$ and
$\pi^*(\eta_\alpha\eta_{\alpha+1}) = n + \alpha$, if $1 \le \alpha \le n-1$.
Thus H_n is labelled with WML and hence we conclude that H_n is a WMG.

Case (ii) n is even
Define $\pi: V(H_n) \to \{0,1,2,..,2n-1\}$ as follows:
$\pi(\delta_\alpha) = \alpha - 1$, if $1 \le \alpha \le n$ and
$\pi(\eta_\alpha) = n + \alpha - 1$, if $1 \le \alpha \le n$.
we attain the following edge labeling as:
$\pi^*(\delta_\alpha\delta_{\alpha+1}) = \alpha$, if $1 \le \alpha \le n-1$,
$\pi^*(\delta_{\alpha+1}\eta_\alpha) = n$, if $\alpha = \frac{n}{2}$ and
$\pi^*(\eta_\alpha\eta_{\alpha+1}) = n + \alpha$, if $1 \le \alpha \le n-1$.
Thus H_n is labelled with WML and hence we conclude that H_n is a WMG.

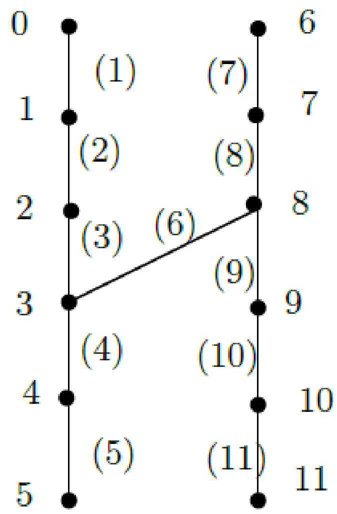

Figure 7: A WML of H_6

Theorem 3.7. A Tadpole $T(n,k)$ is a WMG.
Proof: Let $V(T(n,k)) = \{\delta_1, \delta_2, ..., \delta_n = \eta_1, \eta_2, .., \eta_n\}$ and $E(T(n,k)) = \{\delta_\alpha\delta_{\alpha+1} : 1 \le \alpha \le n-1\} \cup \{\delta_1\delta_n\} \cup \{\eta_\alpha\eta_{\alpha+1} : 1 \le \alpha \le n-1\}$
Define $\pi: V(T(n,k)) \to \{0,1,2,..,n+k-1\}$ as follows:
$\pi(\delta_\alpha) = \alpha - 1$, if $1 \le \alpha \le \left\lceil \frac{3n}{5} \right\rceil - 1$,
$\pi(\delta_\alpha) = \alpha$, if $\left\lceil \frac{3n}{5} \right\rceil \le \alpha \le n$ and
$\pi(\eta_\alpha) = n + \alpha - 1$, if $2 \le \alpha \le k$.
we attain the following edge labeling as:

$$\pi^*(\delta_\alpha\delta_{\alpha+1}) = \begin{cases} \alpha & \text{if } 1 \le \alpha \le \left\lceil \frac{3n}{5} \right\rceil - 1 \\ \alpha + 1 & \text{if } \left\lceil \frac{3n}{5} \right\rceil \le \alpha \le n-1, \end{cases}$$

$\pi^*(\delta_1\delta_n) = \left\lceil \frac{3n}{5} \right\rceil$ and
$\pi^*(\eta_\alpha\eta_{\alpha+1}) = n + \alpha$, if $1 \le \alpha \le k-1$.
Thus $T(n,k)$ is labelled with WML and hence we conclude that $T(n,k)$ is a WMG.

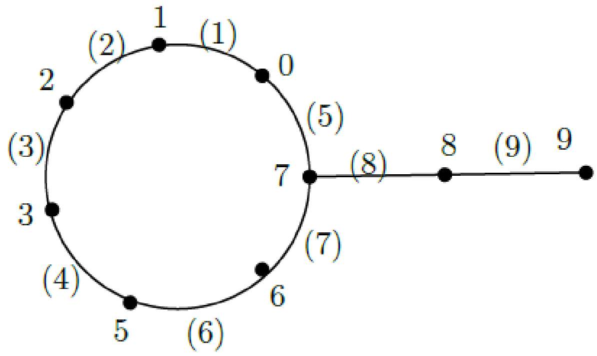

Figure 8: A WML of T(7,3)

Theorem 3.8. Q_n is WMG

Proof: Let $V(Q_n) = \{\delta_1, \delta_2, \dots, \delta_n, \eta_1, \eta_2, \dots, \eta_{n-1}, \mu_1, \mu_2, \dots, \mu_{n-1}\}$ *and* $E(Q_n) = \{\delta_\alpha\delta_{\alpha+1}, \delta_\alpha\eta_\alpha, \eta_\alpha\mu_\alpha, \mu_\alpha\delta_{\alpha+1} : 1 \le \alpha \le n-1\}$.

Define $\pi: V(Q_n) \to \{0,1,2,\dots,4(n-1)\}$ as follows:

$\pi(\delta_1) = 3$,
$\pi(\delta_\alpha) = 4\alpha - 4$, if $2 \le \alpha \le n$,
$\pi(\eta_1) = 0$,
$\pi(\eta_\alpha) = 4\alpha - 2$, if $2 \le \alpha \le n$,
$\pi(\mu_1) = 1$ and
$\pi(\mu_\alpha) = 4\alpha - 1$, if $2 \le \alpha \le n$.

we attain the following edge labeling as:
$\pi^*(\delta_1\delta_2) = 4$,
$\pi^*(\delta_\alpha\delta_{\alpha+1}) = 4\alpha - 2$, if $2 \le \alpha \le n-1$,
$\pi^*(\delta_1\eta_1) = 2$,
$\pi^*(\delta_\alpha\eta_\alpha) = 4\alpha - 3$, if $2 \le \alpha \le n-1$,
$\pi^*(\eta_1\mu_1) = 1$,
$\pi^*(\eta_\alpha\mu_\alpha) = 4\alpha - 1$, if $2 \le \alpha \le n-1$,
$\pi^*(\mu_1\delta_2) = 3$ and
$\pi^*(\mu_\alpha\delta_{\alpha+1}) = 4\alpha$, if $2 \le \alpha \le n-1$.

Thus Q_n is labelled with WML and hence we conclude that Q_n is a WMG.

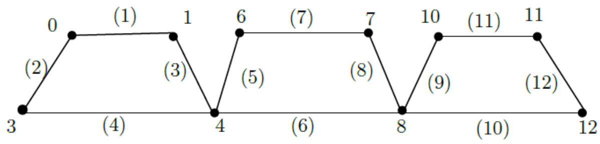

Figure 9: A WML of Q_4

Theorem 3.9. Every comb is an WMG.

Proof: Let $V(P_n \odot K_1) = \{\delta_1, \delta_2, \dots, \delta_n, \eta_1, \eta_2, \dots, \eta_n\}$ *and* $E(P_n \odot K_1) = \{\delta_\alpha\delta_{\alpha+1} : 1 \le \alpha \le n-1\} \cup \{\delta_\alpha\eta_\alpha : 1 \le \alpha \le n\}$.

Define $\pi: V(G) \to \{0,1,2,\dots,2n-1\}$ as follows:
$\pi(\delta_\alpha) = 2\alpha - 1$, if $1 \le \alpha \le n$ and
$\pi(\eta_\alpha) = 2\alpha - 2$, if $1 \le \alpha \le n$.

we attain the following edge labeling as:
$\pi^*(\delta_\alpha\delta_{\alpha+1}) = 2\alpha$, if $1 \le \alpha \le n-1$, and
$\pi^*(\delta_\alpha\eta_\alpha) = 2\alpha - 1$, if $1 \le \alpha \le n$,

Thus $P_n \odot K_1$ is labelled with WML and hence we conclude that $P_n \odot K_1$ is a WMG.

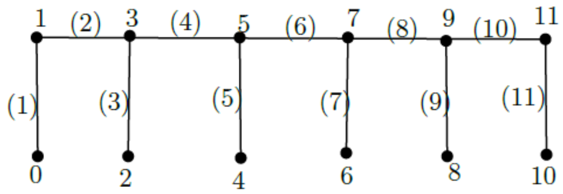

Figure 10: A WML of $P_6 \odot K_1$

Theorem 3.10. TL_n is a WMG.

Proof: Let $V(TL_n) = \{\delta_1, \delta_2, \dots, \delta_n, \eta_1, \eta_2, \dots, \eta_n\}$ *and* $E(TL_n) = \{\delta_\alpha\delta_{\alpha+1} : 1 \le \alpha \le n-1\} \cup \{\eta_\alpha\eta_{\alpha+1} : 1 \le \alpha \le n-1\} \cup \{\delta_\alpha\eta_{\alpha+1} : 1 \le \alpha \le n-1\} \cup \{\delta_\alpha\eta_\alpha : 1 \le \alpha \le n\}$.

Define $\pi: V(TL_n) \to \{0,1,2,\dots,4n-3\}$ as follows:

$$\pi(\delta_\alpha) = \begin{cases} 4n - 3\alpha & \text{if } \alpha = 1 \\ 4n - 4\alpha + 2 & \text{if } 2 \le \alpha \le n-2 \\ 4n - 4\alpha - 4 & \text{if } \alpha = n-1 \\ 2 & \text{if } \alpha = n \end{cases}$$

$$\pi(\eta_\alpha) = \begin{cases} 4n - 3\alpha - 1 & \text{if } \alpha = 1 \\ 4n - 4\alpha & \text{if } 2 \le \alpha \le n-2 \\ 4n - 4\alpha + 2 & \text{if } \alpha = n-1 \\ 4 & \text{if } \alpha = n \end{cases}$$

we attain the following edge labeling as:

$$\pi^*(\delta_\alpha\delta_{\alpha+1}) = \begin{cases} 4n - 4\alpha & \text{if } 1 \le \alpha \le n-3 \\ 4n - 4\alpha - 3 & \text{if } n-2 \le \alpha \le n-1 \end{cases}$$

$$\pi^*(\eta_\alpha\eta_{\alpha+1}) = \begin{cases} 4n - 4\alpha - 2 & \text{if } 1 \le \alpha \le n-3 \\ 4n - 4\alpha - 1 & \text{if } \alpha = n-2 \\ 6 & \text{if } \alpha = n-1 \end{cases}$$

$$\pi^*(\delta_\alpha\eta_\alpha) = \begin{cases} 4n - 4\alpha + 1 & \text{if } 1 \le \alpha \le n-2 \\ 4n - 4\alpha - 1 & \text{if } \alpha = n-1 \\ 4 & \text{if } \alpha = n \end{cases}$$

$$\pi^*(\delta_\alpha\eta_{\alpha+1}) = \begin{cases} 4n - 4\alpha - 1 & \text{if } 1 \le \alpha \le n-3 \\ 4n - 4\alpha & \text{if } \alpha = n-2 \\ 2 & \text{if } \alpha = n-1 \end{cases}$$

Hence π is a WML of TL_n graphs. Thus the graph TL_n is a WMG.

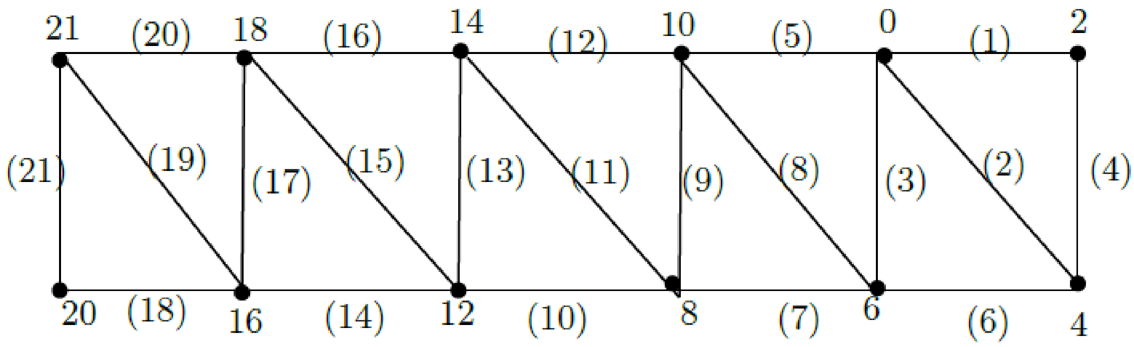

Figure 11: A WML of TL_6

Theorem 3.11. $T(P_n)$ is an WMG.
Proof: Let $V(T(P_n)) = \{\delta_1, \delta_2, \dots, \delta_n, \eta_1, \eta_2, \dots, \eta_{n-1}\}$ *and* $E(T(P_n)) = \{\delta_\alpha\delta_{\alpha+1}, \eta_\alpha\delta_\alpha, \eta_\alpha\delta_{\alpha+1} : 1 \le \alpha \le n-1\} \cup \{\eta_\alpha\eta_{\alpha+1} : 1 \le \alpha \le n-2\}$.
Define $\pi: V(T(P_n)) \to \{0,1,2,\dots,4(n-1)\}$ as follows:

$$\pi(\delta_\alpha) = \begin{cases} 0 & \text{if } \alpha = 1 \\ 3 & \text{if } \alpha = 2 \\ 4\alpha - 4 & \text{if } 3 \le \alpha \le n-1 \\ 4n-6 & \text{if } \alpha = n \end{cases}$$

$$\pi(\eta_\alpha) = \begin{cases} 5\alpha - 4 & \text{if } 1 \le \alpha \le 2 \\ 4\alpha - 2 & \text{if } 3 \le \alpha \le n-3 \\ 4\alpha - 3 & \text{if } \alpha = n-2 \\ 4\alpha - 1 & \text{if } \alpha = n-1 \end{cases}$$

we attain the following edge labeling as:
$\pi^*(\delta_\alpha\delta_{\alpha+1}) = 4\alpha - 2$, if $1 \le \alpha \le n-2$,
$\pi^*(\delta_{n-1}\delta_n) = 4n - 3$,
$\pi^*(\eta_\alpha\eta_{\alpha+1}) = 4\alpha$, if $1 \le \alpha \le n-2$,
$\pi^*(\eta_\alpha\delta_\alpha) = 4\alpha - 3$, if $1 \le \alpha \le n-2$,
$\pi^*(\eta_{n-1}\delta_n) = 4n - 6$, and
$\pi^*(\eta_\alpha\delta_{\alpha+1}) = 4\alpha - 1$, if $1 \le \alpha \le n-1$.
Thus $T(P_n)$ is labelled with WML and hence we conclude that $T(P_n)$ is a WMG.

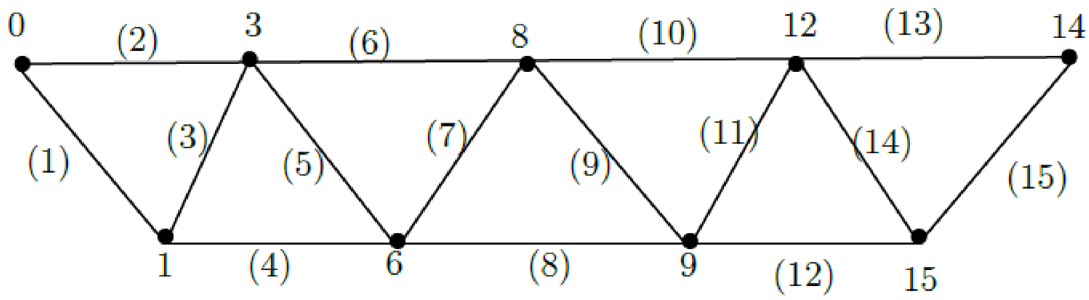

Figure 12: A WML of $T(P_5)$

Theorem 3.12. P_n^2 is an WMG.
Proof: Let $V(P_n^2) = \{\delta_1, \delta_2, \dots, \delta_n\}$ *and* $E(P_n^2) = \{\delta_\alpha\delta_{\alpha+1} : 1 \le \alpha \le n-1\} \cup \{\delta_\alpha\delta_{\alpha+2} : 1 \le \alpha \le n-2\}$.

Case (i): $n \ge 8$

Subcase (a): $n \cong 1\ (or)\ 2\ (or)\ 3\ (mod\ 4)$
Define $\pi: V(P_n^2) \to \{0,1,2,\dots,(2n-3)\}$ as follows:

$$\pi(\delta_\alpha) = \begin{cases} 0 & \text{if } \alpha = 1 \\ 2\alpha - 3 & \text{if } 2 \le \alpha \le 3 \\ 2\alpha - 2 & \text{if } 4 \le \alpha \le n-4 \\ 2\alpha - 3 & \text{if } \alpha = n-3 \\ 2\alpha - 2 & \text{if } \alpha = n-2 \\ 2\alpha - 1 & \text{if } \alpha = n-1 \\ 2\alpha - 4 & \text{if } \alpha = n \end{cases}$$

we attain the following edge labeling as:

$$\pi^*(\delta_\alpha\delta_{\alpha+1}) = \begin{cases} 2\alpha - 1 & \text{if } 1 \le \alpha \le n-3 \\ 2\alpha & \text{if } \alpha = n-2 \\ 2\alpha - 1 & \text{if } \alpha = n-1 \end{cases}$$

$$\pi^*(\delta_\alpha\delta_{\alpha+2}) = \begin{cases} 2\alpha & \text{if } 1 \le \alpha \le n-3 \\ 2\alpha - 1 & \text{if } \alpha = n-2 \end{cases}$$

Thus P_n^2 is labelled with WML and hence we conclude that P_n^2 is a WMG.

Case (b): $n \cong 0 \ (mod\ 4)$

Define $\pi: V(P_n^2) \to \{0,1,2,..,(2n-3)\}$ as follows:

$$\pi(\delta_\alpha) = \begin{cases} 2\alpha & \text{if } 1 \le \alpha \le 2 \\ \alpha - 3 & \text{if } \alpha = 3 \\ 2\alpha - 2 & \text{if } \alpha = 4 \\ 2\alpha & \text{if } 5 \le \alpha \le n-3 \ and\ \alpha\ is\ odd \\ 2\alpha - 4 & \text{if } 6 \le \alpha \le n-4 \ and\ \alpha\ is\ even \\ 2\alpha - 1 & \text{if } \alpha = n-1 \\ 2n - 4 & \text{if } \alpha = n \end{cases}$$

we attain the following edge labeling as:

$$\pi^*(\delta_\alpha\delta_{\alpha+1}) = \begin{cases} 4 & \text{if } \alpha = 1 \\ \alpha & \text{if } 2 \le \alpha \le 3 \\ 8 & \text{if } \alpha = 4 \\ 2\alpha - 1 & \text{if } 5 \le \alpha \le n-1 \end{cases}$$

$$\pi^*(\delta_\alpha\delta_{\alpha+2}) = \begin{cases} 1 & \text{if } \alpha = 1 \\ 6 & \text{if } \alpha = 2 \\ 2\alpha - 1 & \text{if } 3 \le \alpha \le 4 \\ 2\alpha + 2 & \text{if } 5 \le \alpha \le n-1 \ and\ \alpha\ is\ odd \\ 2\alpha - 2 & \text{if } 6 \le \alpha \le n-1 \ and\ \alpha\ is\ even \end{cases}$$

Thus P_n^2 is labelled with WML and hence we conclude that P_n^2 is a WMG.

Case ii. $n \ge 7$

The resulting the graph shown in the Figure 13.

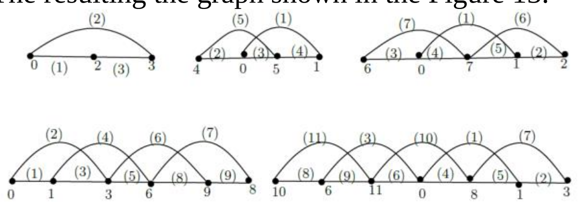

Figure 13: A WML of $P_3^2, P_4^2, P_5^2, P_6^2, P_7^2$

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