LMI approach to Asymptotic Stability of Linear Systems with Interval Time-Varying Delays

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Abstract

The LMI approach to asymptotic stability of linear interval time delay differential system is examined in this research article. The augmented Lyapunov-Krasovskii functional (LKF) with free matrix variables and linear matrix inequalities (LMIs) are used to construct the stability requirements. To show that the suggested approach is effective, numerical examples are provided in terms of LMI

KEYWORDSAsymptotic stability,(LMI) Linear Matrix Inequality, LKF(Lyapunov-Krasovski functional).

1. INTRODUCTION

Over the past few decades, time delay dynamical systems have received some exciting academic interest. In particular, the synchronisation of neutral systems with delays, which is a fruitful idea in the fields of bioengineering, circuit theory, automatic control, population dynamics, and other research fields. Recent efforts have increased the interest in the area of the stability or synchronisation of linear time delay dynamical systems.

The asymptotic synchronizationof various systems was proposed by numerous scholars in [1, 6,7, 13,15]. In addition, many researchers [12,13] found the sufficient conditions for delay dependent dynamical systems.

In [3] the exponential synchronizationof time varying nonlinear uncertainties of neutral delay (time) differential system with was proved by Mr. Syed Ali by using the new way of approach generalized eigen value problem and he minimized the delay very effectively using an LMI approach.

In [3,8,10,11] the various synchronization of time varying nonlinear uncertainties with time delay of neutral delay (time) differential system with was proved by various researchers using an LMI approach

In [15] the exponential synchronization oftime varying neutral delay differential system using an LMI approach. is proved by the researcher. Umesha V More specifically researchers Balasubramanian, Syed ali, Saravanan, Mathiyalagan … are showed necessary sufficient conditionsof dynamical systems with delays and uncertainties…

In [9] the LMI approach to exponential stability of linear systems with interval time-varying delaysis proved by the researcher V.N. Phata and he showed the stability of it.

Moreoverthe improved asymptotic stability of linear time delay systems is an open problem that makes us to more interest on it and to get the stability on it. Using Jensen's inequality [5] and the LKF, we ensured the improved asymptoticstability and we discovered a new LMI condition in this paper.

Continuously,MATLAB Linear Matrix Inequality -control toolbox is used to find the maximum permitted length of delays is derived using LMI.

2. PROBLEM STATEMENT AND PRELIMINARIES

The linear system with interval time varying delay is of the form

$$\dot{x} = A\,x(t) + B\,x(t-h) \qquad (1)$$

with the initial condition

$$x(t) = \phi(t) \quad \forall t \in (-h_2, 0),$$

where "A, $B \in R^{n*n}$"be the constant and real matrices, "$x(t) \in R^n$"be the state vector and the positive timevarying delay is h(t) Here let us execute the asymptotic stability of above system described by (1) with h(t) satisfies the condition

$$0 \leq h_1 \leq h(t) \leq h_2 \quad \text{where } t \in R^+$$

In this section we show about the definitions, lemmas and results related to LMI and stabilities of delay differential systems.

Lemma2. 1(Schur complement [1])
Let $E= E^T$, $F= F^T$, $G=G^T$ be the given matrices such that G > 0, which depends affinely on x then$\begin{pmatrix} F & E^T \\ E & -G \end{pmatrix} < 0 \leftrightarrow F + E^T G^{-1} E < 0$

Lemma 2.2
For any two vectors y,x$\in R^n$and a scalar $\in > 0$, then

$$2x^T y \leq \in x^T x + \in^{-1} y^T y$$

Lemma 2.3
For a scalar ρ> 0, a vector function y: $[0, \rho] \rightarrow R^n$, any constant matrix $G \in R^{n*n} > 0$, $G = G^T > 0$, and such that the integrations are well defined, then

$$\left(\int_0^{\rho} r(s)ds\right) G\left(\int_0^{\rho} r(s)ds\right)^T \leq \rho \int_0^{\rho} r^T(s)\, G\, r(s)ds$$

Definition 2.4
Let an equilibrium point of$\dot{X}$ = f(X)be x=0 is asymptotically stable if there exists a differentiable function be V: $R^n \rightarrow R$, such that

(i) $V(0) = 0$

(ii) $V(X) > 0$.

(iii) $\dot{V}(X) < 0$.

3.MAIN RESULT

Theorem 3.1 Linear System (1) is asymptotically stable if there exists a symmetric positive definite matrices appropriate dimensions of $P_j > 0, j = 1,2,3.4. \in R^{n*n}$and matrices $S_i > 0, i = 1,2,3.4, 5$ matrices such that LMI in the following holds.

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} \\ \Xi_{21} & \Xi_{22} & 0 & \Xi_{24} & 0 \\ \Xi_{31} & 0 & \Xi_{33} & \Xi_{34} & 0 \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} & \Xi_{45} \\ \Xi_{51} & \Xi_{52} & \Xi_{53} & \Xi_{54} & \Xi_{55} \end{bmatrix} < 0 \qquad (2)$$

Where

$$\Xi_{11} = (AP_1 + A^T P_1 + 2P_2 + 2P_3 - S_1)\ ]\Xi_{12} = -P_3 - S_2 A$$

$$\Xi P_{13} = -P_3 - S_3 A\ ;\ \Xi_{14=} B^T P_1 - S_4 A$$

$$\Xi_{15=} P_3 {h_1}^2 - S_5 A + P_3 {h_2}^2 ; \Xi_{21=} P_3$$

$$\Xi_{22=} P_3 + P_4 - P_2(1-d)\ ;\ \Xi_{23=} 0$$

$$\Xi_{24=} P_4\ ;\ \Xi_{25=} 0; \Xi_{31=} P_3; \Xi_{32=} 0$$

$$\Xi_{33=} -P_3 + P_4 - P_2(1-d)\ ; \Xi_{34=} P_4$$

$$\Xi_{35=} 0\ ;\ \Xi_{41=} BS_1 + BP_1$$

$$\Xi_{42=} P_4 - S_2 B$$

$$\Xi_{43=} P_4 - S_3 B \quad \Xi_{44=} P_4 - S_4 B$$

$$\Xi_{45=}\ - S_5 B; \Xi_{51=} S_1$$

$$\Xi_{52=} S_2\ ;\ \Xi_{53=} S_3$$

$$\Xi_{54=} S_4$$

$$\Xi_{55=} S_5 + P_4\,(h_2 - h_1)^2$$

Proof:

This theorem can be proved by considering the Lyapunov functions are

$$V(X) = \sum_{i=1}^{6} V_i(t)$$

are as follows. (2)

We define theLyapunov functions as follows

$V_1(t) = x(t) P_1 x^T(t)$

$V_2(t) = \int_{-h_1}^{0} x^T(t+s) P_2\, x(t+s)\, ds$

$V_3(t) = \int_{-h_2}^{0} x^T(t+s) P_2\, x(t+s)\, ds$

$V_4(t) = h_1 \int_{t+s}^{t} \int_{-h_1}^{0} \dot{x}^T(t+s) P_3 \dot{x}(t+s)\, dt ds$

$V_5(t) = h_2 \int_{t+s}^{t} \int_{-h_2}^{0} \dot{x}^T(t+s) P_3 \dot{x}(t+s)\, dt ds$

$V_6(t) = (h_2 - h_1) \int_{t+s}^{t} \int_{-h_2}^{-h_1} \dot{x}^T(t+s) P_4 \dot{x}(t+s)\, dt ds$

Let us define the differentiations of the Lyapunov functions is as follows:

$$\dot{V}(X) = \sum_{i=1}^{6} \dot{V}_i(t)$$

$$\dot{V}_1 = P_1\,[\dot{x}(t)\mathrm{x}^T(t) + x(t)\dot{\mathrm{x}}^T(t)]$$

$$\dot{V}_1 = [AP_1 + A^T P_1]\,x(t)\mathrm{x}^T(t) + \mathrm{B}\;P_1\mathrm{x}\,(t-h)\mathrm{x}^T(t) + B^T P_1\mathrm{x}^T(t-h)x(t)$$

$$+\mathrm{B}^T P_1\mathrm{x(t)}\mathrm{x}^T(t-h) + \mathrm{C}^T P_1\dot{x}^T(\mathrm{t-h})P_1 x(t)$$

$$\dot{V}_2 = x(t)P_2\mathrm{x}^T(t) - (1-d)\mathrm{x}^T(t-h_1)P_2\mathrm{x}\,(t-h_1)\text{-}\,2\alpha V_2$$

$$\dot{V}_3 = x(t)P_2\mathrm{x}^T(t) - (1-d)\mathrm{x}^T(t-h_2)P_2\mathrm{x}\,(t-h_2)\text{-}\,2\alpha V_3$$

$$\dot{V}_4 = {h_1}^2 x(t)P_3\dot{\mathrm{x}}^T(t) - \mathrm{x}^T(t)P_3\mathrm{x}\,(t) + \mathrm{x}^T(t)P_3\mathrm{x}\,(t-h_1) - \mathrm{x}^T(t-h_1)P_3\mathrm{x}\,(t) + \mathrm{x}^T(t-h_1)P_3\mathrm{x}\,(t-h_1)$$

$$\dot{V}_5 = {h_2}^2 x(t)P_3\dot{\mathrm{x}}^T(t) - \mathrm{x}^T(t)P_3\mathrm{x}\,(t) + \mathrm{x}^T(t)P_3\mathrm{x}\,(t-h_2) - \mathrm{x}^T(t-h_2)P_3\mathrm{x}\,(t) + \mathrm{x}^T(t-h_2)P_3\mathrm{x}\,(t-h_2)$$

$$\dot{V}_6 = (h_2-h_1)^2\dot{x}(t)P_4\dot{x}^T(t) - (h_2-h_1)\int_{t-\hbar_2}^{t-\hbar_1}\dot{x}^T(s)P_4\dot{x}(s)\,ds$$

$$\dot{V}_6 = (h_2-h_1)^2\dot{x}(t)P_4\dot{\mathrm{x}}^T(t) - \mathrm{x}^T(t-h)P_4 x(t-h) + \mathrm{x}^T(t-h)P_4 x(t-h_2) + \mathrm{x}^T(t-h_2)P_4 x(t-h) + \mathrm{x}^T(t-h_2)P_4 x(t-h_2) - \mathrm{x}^T(t-h_1)P_4 x(t-h_1) + \mathrm{x}^T(t-h_1)P_4 x(t-h) + \mathrm{x}^T(t-h)P_4 x(t-\hbar_1)$$

By using the below identity

$$\dot{x} - A\,x(t) - B\,x(t-h) = 0$$

We have

$$\mathrm{x}^T(\mathrm{t})S_1\dot{x}(t) - \mathrm{x}^T(\mathrm{t})S_1 A\,x(t) - \mathrm{x}^T(\mathrm{t})\,S_1 B\,x(t-h) = 0$$

$$\mathrm{x}^T(\mathrm{t}-h_1)S_2\dot{x}(t) - \mathrm{x}^T(\mathrm{t}-h_1)S_2 A\,x(t) - \mathrm{x}^T(\mathrm{t}-h_1)S_2 B\,x(t-h) = 0$$

$$\mathrm{x}^T(\mathrm{t}-h_2)S_3\dot{x}(t) - \mathrm{x}^T(\mathrm{t}-h_2)S_3 A\,x(t) - \mathrm{x}^T(\mathrm{t}-h_2)S_3 B\,x(t-h) = 0$$

$$\mathrm{x}^T(\mathrm{t-h})S_4\dot{x}(t) - \mathrm{x}^T(\mathrm{t-h})S_4 A\,x(t) - \mathrm{x}^T(\mathrm{t-h})\,S_4 B\,x(t-h) = 0$$

$$\dot{\mathrm{x}}^T(\mathrm{t})S_5\dot{x}(t) - \dot{\mathrm{x}}^T(\mathrm{t})S_5 A\,x(t) - \dot{\mathrm{x}}^T(\mathrm{t})\,S_5 B\,x(t-h) = 0$$

By combining all the differentials, we get

$$\dot{\boldsymbol{V}}(\boldsymbol{X}) \le [A\dot{P}_1 + A^T P_1 + 2P_2 + 2P_3 - S_1]x(t)\mathrm{x}^T(t) + [-P_3 - S_2 A]\mathrm{x}^T(t-h_1)\,\mathrm{x(t)} + [-P_3 - S_3 A]\,\mathrm{x}^T(t-h_2)\,\mathrm{x(t)} + [B^T P_1 - S_4 A]x(t)\mathrm{x}^T(t-h) + [\,{h_1}^2 P_3 + {h_2}^2 P_3 - S_5 A]\;\dot{\mathrm{x}}^T(t)\,x(t) + [P_3]\,\mathrm{x}^T(t)\,\mathrm{x}\,(t-h_1)$$

$$+[P_3+P_4-(1-d)P_2]\,\mathrm{x}^T(t)\,\mathrm{x}\,(t-h_1)+[P_4]\,\mathrm{x}\,(t-h_1)\mathrm{x}\,(t-h)$$
$$+[P_3]\,\mathrm{x}^T(t)\,\mathrm{x}\,(t-h_2)+[-P_3+P_4-(1-d)P_2]\ \mathrm{x}^T(t-h_2)\,\mathrm{x}\,(t-h_1)$$
$$[P_4]\,\mathrm{x}\,(t-h_2)\,\mathrm{x}^T(t-h)+[-P_1B-S_1B\,]\mathrm{x}^T(t-h)\mathrm{x}^T(t)$$
$$[P_4-S_2B]\,\mathrm{x}^{\,T}(t-h_1)\mathrm{x}\,(t-h)+[P_4-S_3B]\,\mathrm{x}^{\,T}(t-h_2)\mathrm{x}\,(t-h)$$
$$[P_4-S_4B]\,\mathrm{x}^{\,T}(t-h)\mathrm{x}\,(t-h)+[-S_5B]\,\mathrm{x}\,(t-h)\,\dot{\mathrm{x}}^T(t)$$
$$+[S_1]\dot{\mathrm{x}}(t)\mathrm{x}^T(t)+[S_2]\dot{\mathrm{x}}(t)\mathrm{x}^T(t-h_1)+[S_2]+\dot{\mathrm{x}}(t)\mathrm{x}^T(t-h_2)+$$
$$[S_4]\dot{\mathrm{x}}(t)\mathrm{x}^T(t-h)+[S_5+(h_2-h_1)^2P_4]\dot{\mathrm{x}}(t)\dot{\mathrm{x}}^T(t)$$

Furthermore, using the Schur-complement lemma, the above rewriting in terms of LMI's

$$\Xi=\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15}\\ \Xi_{21} & \Xi_{22} & 0 & \Xi_{24} & 0\\ \Xi_{31} & 0 & \Xi_{33} & \Xi_{34} & 0\\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} & \Xi_{45}\\ \Xi_{51} & \Xi_{52} & \Xi_{53} & \Xi_{54} & \Xi_{55}\end{bmatrix}$$

$$\boldsymbol{Where}\ \eta=[\ \mathrm{x(t)},\mathrm{x}\,(t-h_1),\mathrm{x}\,(t-h_2),\mathrm{x}\,(t-h),\dot{\mathrm{x}}(t)]$$

$$\dot{\boldsymbol{V}}(\boldsymbol{X})\le\ \eta^T\Xi\ \eta$$

By applying the previous lemmas with some effort $\Xi<0$

By using the lemma 2.2, we achieve the following result as follows

$$\dot{\boldsymbol{V}}(\boldsymbol{X})\le\ V(t)$$

Remark 3.1. Here in Theorem 3.1 the LMI is solvable by using the **MATLAB - LMI toolbox**.

Remark 3.2. In many research article [6] the delays are more smaller than our results.

Remark 3.3. The tmin for the LMI is**-0.0186** which is very efficient numerical value in which that the LMI is negative definite where the output values are positive definite.

4. NUMERICAL EXAMPLES

Example 4.1

Consider the uncertain linear system with interval time-varying delay (1) with any time delay function h(t) with $\mathbf{h_1=0.2, h_2=1.5}$

A=$\begin{bmatrix}-0.075 & 1\\ -0.005 & -1\end{bmatrix}$; B=$\begin{bmatrix}-0.065 & 0.004\\ 0.003 & -0.005\end{bmatrix}$

Applying the above theorem, the possible and feasible solutions of the above LMI areP_1 =$\begin{bmatrix}0.1954 & -1.5206\\ -1.5206 & 0.4723\end{bmatrix}$; P_2 =$\begin{bmatrix}1.8867 & -0.0103\\ -0.0103 & 1.9128\end{bmatrix}$;

P_3= $\begin{bmatrix}-0.0056 & -0.0009\\ -0.0009 & -0.0032\end{bmatrix}$ P_4=$\begin{bmatrix}-0.0624 & -0.0009\\ -0.0009 & -0.0607\end{bmatrix}$

$S_1 = \begin{bmatrix} -0.0526 & -0.1236 \\ 2.7073 & 0.8814 \end{bmatrix}$; $S_2 = \begin{bmatrix} -0.0052 & 0.0021 \\ -0.0033 & -0.0025 \end{bmatrix}$;

$S_3 = \begin{bmatrix} -0.0052 & 0.0021 \\ -0.0033 & -0.0025 \end{bmatrix}$ $S_4 = \begin{bmatrix} -0.0255 & -0.0705 \\ -0.0168 & -0.0316 \end{bmatrix}$

Hence the given system is asymptotically stable in which our delays both h_1 and h_2 are bigger than the previous research article [9].

5. CONCLUSION

In this study, the necessary conditions for the asymptotic synchronisation of "time-delayed linear systems are examined". The Schur complement lemma , Newton-Leibniz formula and two examples with comparisons are used to describe the adequate criteria in terms of LMI and showed our result using the MATLAB LMI toolbox.

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