ON LIGHTLY FUZZY G˜- CLOSED SETS AND IT'S PROPERTIES

R.Prabhakaran[1], K.Balasubramaniyan[2]

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Abstract

We introduce the idea of **Lf**g̃-closed sets in fuzzy topological spaces as the major goal of this research. We also look into the relationships between the linked qualities and other connected sets.

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[1]Department of Mathematics, Annamalai University, Annamalainagar-608002, Chidambaram, Tamil Nadu, India.
Email: pkraviprabha@gmail.com

[2]Department of Mathematics, Arignar Anna Government Arts College, Vadachennimalai-636121, Attur, Salem District, Tamil Nadu, India.
Email: kgbalumaths@gmail.com

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1. INTRODUCTION

The concept of a fuzzy subset is the very first thing we take into consideration. Zadeh[18] introduced and analysed it in 1965. The subsequent study in this field and related fields has found use in many fields of science and engineering. In 1968, Chang [4] developed fuzzy topological spaces as a generalization of topological spaces.The construction of fuzzy topological spaces has been made possible by numerous academics, including Azad [1], Sinha [3], Wong[16] and many more. There are several operations on fuzzy sets, including union, intersection, complementation of fuzzy sets, and associated attributes. A fuzzy set's inverse image under a function is included, along with the properties established by Chang[4]. On fuzzy topological spaces fundamental ideas and findings from the works of Chang[4], Wong[16] and Malghan [10] are also discussed. We introduce the idea of **Lf**g̃-closed sets in fuzzy topological spaces as the major goal of this research. We also look into the relationships between the linked qualities and other connected sets.

2. PRELIMINARIES

Definition 2.1. A fuzzy subset A of a fuzzy topological space (X, τ) is called:

(1) Fuzzy semi-open set [1] if A ≤ cl(int(A)).
(2) Fuzzy α-open set [3] if A ≤ int(cl(int(A))).
(3) Fuzzy semi-pre open set [15] if A ≤ cl(int(cl(A))).
(4) Fuzzy regular open set [1] if A = int(cl(A)).

The complements of the above mentioned fuzzy open sets are called their respective fuzzy closed sets.

Definition 2.2. A fuzzy subset A of a fuzzy topological space (X, τ) is called:

(1) A fuzzy generalized closed (briefly fg-closed) set [2] if cl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fg-closed set is called fg – open set;
(2) A fuzzy semi-generalized closed (briefly fsg-closed) set [9] if scl(A) ≤ U whenever A ≤ U and U is fuzzy semi-open in (X, τ). The complement of fsg-closed set is called fsg-open set;
(3) A fuzzy generalized semi-closed (briefly fgs-closed) set [9] if scl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fgs-closed set is called fgs-open set;

(4) A fuzzy α-generalized closed (briefly fαg-closed) set [12] if αcl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fαg-closed set is called fαg-open set;
(5) A fuzzy generalized semi-preclosed (briefly fgsp-closed) set [5] if spcl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fgsp-closed set is called fgsp-open set;
(6) A fuzzy pre-semi-generalized closed (briefly fpsg-closed) set [13] if spcl(A) ≤ U whenever A ≤ U and U is fuzzy semi-open in (X, τ). The complement of fpsg-closed set is called fpsg-open set;

Definition 2.3. Let A be a fuzzy set of a fuzzy topological space (X, τ). Then,
(1) α int(A) = ∨ {G : G ≤ A and G is a fuzzy α open of X}, is called a fuzzy α interior of A [11];
(2) sint(A) = ∨ {G : G ≤ A and G is a fuzzy semi-open of X }, is called a fuzzy semi-interior of A [17];
(3) pint(A) = ∨ {G : G ≤ A and G is a fuzzy preopen set of X}, is called a fuzzy preinterior of A [14];
(4) spint (A) = ∨ {G : G ≤ A and G is a fuzzy semi preopen set of X}, is called a fuzzy semi preinterior of A [6].

Definition 2.4. Let A be a fuzzy set of a fuzzy topological space (X, τ). Then,
(1) pint (A) ≤ A ∧ int(cl(A)) [7];
(2) pcl (A) ≥ A ∨ cl(int(A)) [7];
(3) sint (A) = A ∧ cl(int(A)) [8];
(4) scl (A) = A ∨ int(cl(A)) [8];
(5) αint (A) = A ∧ int(cl(int(A))) [11];
(6) αcl (A) = A ∨ int(cl(int(A))) [11].

Properties of lightly f$\tilde{g}$ - closed sets

We define lightly f$\tilde{g}$ - closed sets in fts and investigate the connections between them.

Definition 3.1. A fuzzy sub set H of a fts (X, F_τ) is said to be lightly f$\tilde{g}$ - closed (shortly Lf$\tilde{g}$-closed) set if H ≤ U, fsg-open ⇒ c(i(H)) ≤ U.
The complement of Lf$\tilde{g}$ - closed set is said to be Lf$\tilde{g}$ - open.

Theorem 3.2. In a space (X, F_τ), for a fuzzy subset H of X, the following implications are true.

(1) H is fuzzy closed set ⇒ H is $\mathbf{Lf}\tilde{g}$ - closed.
(2) H is $\mathbf{f}\tilde{g}$- closed set ⇒ H is $\mathbf{Lf}\tilde{g}$ - closed.
(3) H is fuzzy regular closed set ⇒ H is $\mathbf{Lf}\tilde{g}$ - closed.
(4) H is $\mathbf{Lf}\tilde{g}$ - closed set ⇒ H is $\mathbf{f}$gsp - closed.
(5) H is $\mathbf{f}$g – closed set ⇒ H is $\mathbf{Lf}\tilde{g}$ - closed.

Proof.

(1) Assuming H is any fuzzy closed set and U is any fsg - open set in (X, F_τ) such that $H \leq U$, then $c(H) = H$, $c(i(H)) \leq c(H) = H$. We have $c(i(H)) \leq H \leq U$ whenever $H \leq U$ and U is fsg-open. In this way H is $\mathbf{Lf}\tilde{g}$ - closed.
(2) Assuming H is a $\mathbf{f}\tilde{g}$ - closed set in (X, F_τ) and U is any $\mathbf{f}$sg-open set such that $H \leq U$, then $c(i(H)) \leq c(H) \leq U$. Therefore H is $\mathbf{Lf}\tilde{g}$ - closed.
(3) If H is any fuzzy regular closed set in (X, F_τ) and U is any fsg-open set such that $H \leq U$, then $H = c(i(H)) \leq U$. This proves that H is $\mathbf{Lf}\tilde{g}$ - closed.
(4) Assuming that $H \leq U$ and U is fuzzy open in (X, F_τ). Since any fuzzy Open set is $\mathbf{f}$sg – open set and H is $\mathbf{Lf}\tilde{g}$ - closed set in (X, F_τ). Likewise sp- $c(H) \leq c(H) \leq U$. Which proves that H is fgsp - closed.
(5) Assuming that $H \leq U$ and U is fuzzy open in (X, F_τ). Since any fuzzy open set is fsg – open set and H is fg – closed set, $c(i(H)) \leq c(H) \leq U$. Which proves that H is $\mathbf{Lf}\tilde{g}$ - closed.

Remark 3.3. The converse of Theorem 3.2 is untrue in (X, F_τ) is shown in the case below.

Example 3.4.

(1) Consider $X = \{m, n\}$ and $F_\tau = \{0_X, H, 1_X\}$, where H is a fuzzy set in X specified by $H(m) = 1$ and $H(n) = 0$. As a result (X, F_τ) is a fts. $K(m) = 0.5$ and $K(n) = 1$ define exactly K however $K(n) = 1$ is not fuzzy closed but is a $\mathbf{Lf}\tilde{g}$ - closed set.
(2) Let $X = \{m, n\}$ with $F_\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by $\alpha(m) = 0.6$ and $\alpha(n) = 0.5$. Then (X, F_τ) is a fts. Clearly β defined by $\beta(m)=0.4$ and $\beta(n)=0.4$. This verifies that is not $\mathbf{f}\tilde{g}$ - closed set but is a $\mathbf{Lf}\tilde{g}$ - closed set.
(3) Let $X = \{m, n\}$ with $F_\tau = \{0_X, H, 1_X\}$ where H is a fuzzy set in X defined by $H(a) = 1$ and $H(b) = 0$. Then (X, F_τ) is a fts. Clearly K defined by $K(a) = 0.5$ and $K(b) = 1$. It is follows that $\mathbf{Lf}\tilde{g}$ - closed set but not fuzzy regular closed.
(4) Let $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ where α is a fuzzy set in X defined by $\eta(m) = 0.4$ and $\eta(n) = 0.5$. Then (X, F_τ) is a fts. Clearly μ defined by $\mu(m) = 0.5$ and $\mu(n) = 0.5$ which is not $\mathbf{Lf}\tilde{g}$ - closed set but is $\mathbf{f}$gsp - closed.

(5) Let X = {m, n} with $F_\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by α(m) = 0.6 and α(n) = 0.5. Then (X, F_τ) is a fts. Clearly β defined by β(m) = 0.5 and β(n) = 0.5. It is follows that not fg-closed set but is $\mathbf{L}f\tilde{g}$-closed.

Remark3.5. The family of $\mathbf{L}f\tilde{g}$ - closed sets and the family of fuzzy semi-closed sets are independent in (X, F_τ) is shown in the case below.

Example 3.6.

(1) Let X = {m, n} with $F_\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by α(m) = 0.6 and α(n) = 0.5. Then (X, F_τ) is a fts. Clearly β defined by β(m) = 0.5 and β(n) = 0.5. It is not fuzzy semi-closed set but is $\mathbf{L}f\tilde{g}$-closed.

(2) Let X = {m, n} with $F_\tau = \{0_X, \eta, 1_X\}$ where η is a fuzzy set in X defined by η(m) = 0.4 and η(n) = 0.5. Then (X, F_τ) is a fts. Clearly μ defined by μ(m) = 0.5 and μ(n) = 0.5. It is not $\mathbf{L}f\tilde{g}$-closed set but is fuzzy semi-closed.

Remark 3.7. We have the following relations for a fuzzy sub set in the space (X, F_τ) from the results stated above.

Fuzzy - closed → fuzzy regular - closed → fg - closed

↓ ↓ ↓

$f\tilde{g}$ - closed → $\mathbf{L}f\tilde{g}$ - closed $\nLeftrightarrow$ Fuzzy semi closed

Theorem 3.8. Let H be a fuzzy subset of a space (X, F_τ), if both fuzzy closed and fαg-closed, then $\mathbf{L}f\tilde{g}$ - closed.

Proof. For H is a fαg - closed set and U is fuzzy open set in (X, F_τ) such that H ≤ U, since H is fuzzy closed, we have $c(i(H)) \leq c(i(c(H))) \leq H \vee c(i(c(H))) = \alpha\text{-}c(H) \leq U$ This verifies that H is $\mathbf{L}f\tilde{g}$-closed.

Corollary 3.9. For a fuzzy subset H of a space (X, F_τ), if both fuzzy open and $\mathbf{L}f\tilde{g}$ - closed set, then fuzzy closed.

Proof. Assuming that H is both fuzzy open and $\mathbf{L}f\tilde{g}$ - closed set in (X, F_τ), $H \geq c(i(H)) = c(H)$. Which proves that H is fuzzy closed.

Theorem 3.10. For a fuzzy subset H of a space (X, F_τ), if both fuzzy open and $\mathbf{L}f\tilde{g}$ - closed set then both fuzzy regular open and fuzzy regular closed.

Proof. Assuming H is both fuzzy open and $Lf\tilde{g}$ - closed set in (X, F_τ), and from the Corollary 3.9, we have $c(i(H)) = H$ and $i(c(H)) = H$. This verifies that H is both fuzzy regular open and fuzzy regular closed.

4. Other properties

Proposition 4.1. In a space (X, F_τ), each fuzzy open set is $Lf\tilde{g}$-open.

Proof. Assuming H is a fuzzy open set and H^c is fuzzy closed in (X, F_τ).From the Theorem 3.2(1), which follows that H^c is $Lf\tilde{g}$ - closed in (X, F_τ). This proves that H is $Lf\tilde{g}$ - open.

Remark 4.2. The converse of Proposition 4.1 is untrue in (X, F_τ) is shown in the case below.

Example 4.3. Let $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ where η is a fuzzy set in X defined by $\eta(m) = 0.6$ and $\eta(n) = 0.5$. Then (X, F_τ) is a fts. Clearly ξ defined by $\xi(m) = 0.5$ and $\xi(n) = 0.5$is not fuzzy open set but is $Lf\tilde{g}$ - open.

Proposition 4.4. In a space (X, F_τ), for a fuzzy subset Hof X, the following implications are true.

(1) H is $f\tilde{g}$ - open set ⇒ H is $Lf\tilde{g}$ - open.
(2) H is fuzzy regular open set ⇒ H is $Lf\tilde{g}$ - open.
(3) H is fg - open set ⇒ H is $Lf\tilde{g}$ - open.
(4) H is $Lf\tilde{g}$ - open set ⇒ H is fgsp - open.

Proof.

(1) For H is a $f\tilde{g}$- open set and H^c is $f\tilde{g}$ - closed set in (X, F_τ). From the Theorem 3.2(1), which implies that H^c is $Lf\tilde{g}$ - closed set in (X, F_τ). Thus H is $Lf\tilde{g}$ - open.
(2) Since H is a fuzzy regular open set and H^c is fuzzy regular closed set in (X, F_τ). From the Theorem 3.2(3), it follows that H^c is $Lf\tilde{g}$ - closed set in (X, F_τ). This verifies that H is $Lf\tilde{g}$ - open.
(3) H is a fg - open set and H^c is fg - closed set in (X, F_τ). From the Theorem 3.2(2), it follows that H^c is $Lf\tilde{g}$ - closed set in (X, F_τ). Therefore H is $Lf\tilde{g}$ - open.
(4) H is a $Lf\tilde{g}$ - open set and H^c is $Lf\tilde{g}$ - closed set in (X, F_τ). From the Theorem 3.2(4), it follows that H^c is fgsp-closed set in (X, F_τ). Therefore H is fgsp-open.

Remark 4.5. These implications are shown in the diagram.

fuzzy regular open

↓

f$\tilde{g}$ - open → Lf$\tilde{g}$ - open → fgsp - open

↑

fg - open

The following example shows that converse of each relations in Proposition 4.4 are not true

Example 4.6.

(1) Let X = {m, n} with $F_\tau = \{0_X, \eta, 1_X\}$ where η is a fuzzy set in X defined by η(m) = 0.6 and η(n) = 0.5. Then (X, F_τ) is a fts. Clearly ξ defined by ξ(m) = 0.5 and ξ(n) = 0.5 is Lf$\tilde{g}$ - open set but not f$\tilde{g}$-open set. Also it is neither fuzzy regular open set nor fg - open.

(2) Let X = {m, n} with $F_\tau = \{0_X, \mu, 1_X\}$ where μ is a fuzzy set in X defined by μ(m) = 0.4 and μ(n) = 0.5. Then (X, F_τ) is a fts. Clearly α defined by α(m) = 0.5 and α(n) = 0.5 is not Lf$\tilde{g}$-open set but is fgsp- open.

Theorem 4.7. For fuzzy subset H of a space (X, F_τ) is Lf$\tilde{g}$ - open if $U \leq i(c(H))$ Whenever $U \leq H$ and U is fsg - closed.

Proof. Let H be any Lf$\tilde{g}$ - open set. Then H^c is Lf$\tilde{g}$ - closed set. Let U be a fsg-closed set such that $U \leq H$. Then U^c is a fsg-open set such that $H^c \leq U^c$. Since H^c is Lf$\tilde{g}$-closed, we have $c(i(H^c)) \leq U^c$. Therefore $U \leq i(c(H))$.

Conversely, we assume that $U \leq i(c(H))$ whenever $U \leq H$ and U is fsg - closed set. Then U^c is a fsg - open set such that $H^c \leq U^c$ and $U^c \geq (i(c(H)))^c$. It follows that $U^c \geq c(i(H^c))$. Hence H^c is Lf$\tilde{g}$ - closed set and so H is Lf$\tilde{g}$ - open set.

5. REFERENCES

[1] K.K.Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity, J.Math.Anal., 82(1)(1981),14-32.

[2] G.Balasubramanian and P.Sundaram, On some generalizations of fuzzy continuous functions, Fuzzy Sets and Systems, 86(1)(1997),93-100.

[3] A.S.Bin Shahna, On fuzzy strong semi continuity and fuzzy pre continuity, Fuzzy Sets and Systems, 44(2)(1991),303-308.

[4] C.L.Chang, Fuzzy topological spaces, J.Math.Anal.Appl., 24(1968),182-190.

[5] M.E.El-Shafei and A.Zakari, Semi-generalized continuous mappings fuzzy topological spaces, J.Egypt Math.Soc.,15(1)(2007),57-67.

[6] A.Hakeem Othman, On fuzzy sp-open sets, Advances in Fuzzy Systems, (2011), 5 pages.

[7] B.Krsteska, Fuzzy strongly

pre open sets and fuzzy strongly pre continuity, Mat.Vesnik, 50(1998),111-123.

[8] B.Krsteska, Anoteonthearticle "Fuzzy less strongly semi-open sets and fuzzy less strong semi-continuity", Fuzzy Sets and Systems, 107(1)(1999),107-108

[9] H.Maki, T.Fukutaka, M.Kojima and H.Harade, Generalized closed sets in fuzzy topological spaces, I.Meeting on topological Spaces, Theory and Appl., (1998),23-26.

[10] S.R.Malghan and S.S.Benchalli, Open maps, closed maps and logal compactness in fuzzy topological spaces, J.Math.Anal.Appl.,99(2)(1984),74-79.

[11] M.Rajamani and V.Inthumathi, A decomposition of a weaker form of fuzzy continuity, (pre print).

[12] R.K.Saraf, M.Caldas and S.Mishra, Results via Fgα-Closed sets and Fαg-closed sets, pre print.

[13] R.K.Saraf, G.Navalagi and Meena Khanna, On fuzzy semi-pre-generalized closed sets, Bull.Malays.Math.Sci.Soc.,(2)28 (1)(2005),19-30.

[14] M.K.Singal and N.Prakash, Fuzzy pre open sets and fuzzy pre separation axioms, Fuzzy Sets and Systems, 44(1991),273-281.

[15] S.S.Thakur and S.Singh, On fuzzy semi pre-open sets and fuzzy semi pre continuity, Fuzzy Sets and Systems, 98(3)(1998),383-391.

[16] C.K.Wong, Fuzzy points and local properties of fuzzy topology, J.Math. Anal. Appl., 46(1974), 316-328.

[17] T.H.Yalvac, semi-interior and Semi-closure of a fuzzy set, J.Math.Anal.Appl., 132(2)(1988),356-364.

[18] L.A.Zadeh, Fuzzy sets, Information and Control, 8(1965),338-353.