THE SPLIT DETOUR DOMINATION NUMBER OF A GRAPH

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ABSTRACT :

A detour dominating set $S \subseteq V(G)$ is casual detour dominating set of G. If the induced subgraph < V- S > is disconnected. The split detour dominating number $\gamma_{sd}(G)$ of G is the minimum cardinality of a detour dominating set. In this paper we study Split detour dominating set and we found the Split detour domination number for some standard graphs.

Keywords : Domination, Domination number, Detour Domination ,Detour Domination number, Split domination, Split domination number, Split detour domination, Split detour domination number.

Introduction

Let G be a finite , undirected *g*raph without multiple ed*g*es or loops. The order and size of *G* is denoted by to end of respectively. In this paper we consider for all connected *g*raph *G* with at least two vertices.

For vertices u and v in a connected *g*raph *G* with at least two vertices, the detour distance D(u, v) is the len*g*th of the lar*g*est u–v path in *G*. A u–v path of len*g*th D(u, v) B caused as u–v detour. It is known that the detour distance is a metric and the vertex set v(*G*). The detour eccentricity $e_p(v)$ of a vertex V in *G* is the minimum detour distance from v to a vertex of *G*. The detour radius, rads of *G* is the minimum detour eccentricity amon*g* the vertices of the *G* while detour diameter diams of *G* is the maximum detour eccentricity among the vertices of *G*. A vertex W is said to lie on a u–v detour path P if W is a vertex of u–v detour path P including the vertices u & v.

A set D ⊆V(G) is called a detour set if every vertex u in *G* lie on a detour joining C pair of vertices of D. The detour number dn(*G*) of *G* is the minimum order of a detour set and any detour set of order dn(*G*) is called a minimum detour set of *G*.

Let *G* = (V, E) be a connected graph with at least two vertices. A set D ⊆V(G) is called a dominating set of *G*, if every vertex in V(G)–D is adjacent to any one vertices in D. The domination number γ(G) of *G* is the minimum order of its dominating set and any dominating set of order γ(G) is called a γ set of *G*.

A set $S \subseteq V(G)$ is called a detour dominating set of G, if S is a detour set and dominating set of G. The detour domination number γ_d(G) of G is the minimum order of its detour dominating sets and any detour dominating set of order γ_d(G) is called a γ_d - set of G.

Theorem: 1

Every end vertex of a non trivial connected graph *G* belongs to every detour set of *G* moreover,if the set S of all end-vertices of *G* is a detour set, then S is the unique minimum detour set of *G*.

Theorem: 2

Every end vertex of *G* belongs to every detour domination set of *G*.

Theorem: 3

For a cycle G = C_p (ρ ≥ 3), γ(*G*)= $\left\lceil \frac{p}{3} \right\rceil$

Theorem: 4

For any cycle C_p with (p≥4), vertices $\gamma_s(C_p) = \left\lceil \frac{p}{3} \right\rceil$

Definition : 5

A detour Dominating set S⊆V(*G*) is called a split detour domination set of G if the induced sub graph <V-S> is disconnected. The split detour dominating number $\gamma_{sd}(G)$ of *G* is the minimum cardinality of a detour dominating set.

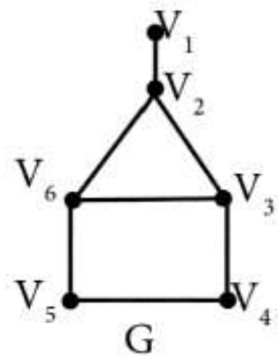

Figure 1

Example : 6

For the graph *G* given in Figure 1

$S_1=\{V_1, V_4\}$ $S_2=\{V_1, V_5\}$ are be only two minimum detour sets of *G*. So that dn(*G*)=2.

Also $S_3=\{V_2, V_4\}$, $S_4=\{V_2, V_5\}$ are be only two minimum dominating sets of *G* so that $\gamma(G)=2$.

Also $S_5=\{V_1, V_3, V_4\}$, $S_6=\{V_1, V_5, V_6\}$ to be only two detour dominating set of *G*.

And finally $S_7=\{V_1, V_3, V_6\}$, $S_8=\{V_1, V_3, V_5\}$, $S_9=\{V_1, V_4, V_6\}$ are the split detour dominating set of *G*.

Remark: 7

There can be more than one γ_{sd} set of *G* for the graph G given in figure 1

$S_8=\{V_1, V_3, V_5\}$ & $S_9 = \{V_1, V_4, V_6\}$ are be another γ_{sd} set of *G*.

Remark: 8

For the graph *G* given in figure 1 the detour domination set of *G*& Split detour dominating set of *G* have same 3 elements subset of *G*. In figure 1 $S_5= S_6= S_7= S_8= S_9$ have same elements. but the induced sub graph $<V_1 - S_5>$ $<V- S_6>$ is connected & the induced sub graph $<V- S_7>$, $<V- S_8>$ & $<V- S_9>$ is disconnected.

Theorem: 9

Every end vertex of *G* belongs to the every split detour domination set of *G*.

Proof:

Every split detour dominating set is a detour dominating set of *G* & also every detour dominating set of *G* is a detour set of *G*, the result follows from theorem 1 & 2.

Theorem: 10

Let *G* be a connected graph with p vertices & u be the internal vertex all other vertices are end vertex say S. then S is a unique minimum split detour dominating set of *G*.

Proof:

Let S be the set of end vertices of *G* which is a split detour dominating set of *G* then $\gamma_{sd}(G) \leq |S|$. By theorem 5, $\gamma_{sd}(G) \geq |S|$. Therefore$\gamma_{sd}(G) = |S|$.By the statement $u \notin S$,S is a subset of every split detour dominating set . Therefore S is the unique minimum split detour dominating set of *G*.

In the following we determine the split detour domination number of some standard graphs.

Theorem: 11

For the star $G = K_{1,P-1}$,$\gamma_{sd}(G) = p - 1$ $(p \geq 3)$

Proof:

Let $G = K_{1,\rho-1}$ and S be the set of all end vertices of *G*. Then by theorem 9, S is a subset of every split detour dominating set of *G* & so $\gamma_{sd}(G) \geq p - 1$.Then by theorem 10,It is clear that S is a unique minimum split detour domination set of *G*. So that $\gamma_{sd}(G) = p - 1$. Hence the theorem is proved.

Theorem: 12

If *G* is a double star ,$\gamma_{sd}(G) = p - 2$

Proof:

The Proof is final to that of proof of Theorem 11

Theorem: 13

For the path $G = P_p(p \geq 3), \gamma_{sd}(G) = \left\{\lceil\frac{p-4}{3}\rceil + 2 \ if \ p \geq 5 \ 2 \qquad if \ p = 3 \ or \ 4\right\}$

Proof:

Let $P_p$be $V_1, V_2, \ldots \ldots, V_P$. If p =3 or 4, then { V_1 , V_P } is a minimum split detour dominating set of *G*. So that γ_{sd} $P_3 or P_4$)=2.Let P ≥ 5. Let S be a minimum dominating set of P_{p-4}. Then S U {V1, Vρ} is a minimum split detour dominating set of *G* so that $\gamma_{sd}(G) = \lceil\frac{p-4}{3}\rceil + 2$.

Hence the proof.

Theorem: 14

For any cycle Cp with $p \geq 4$ vertices,$\gamma_{sd}(Cp) = \lceil\frac{p}{3}\rceil$

Proof:

For the cycle Cp($p \geq 4$), every dominating set of G is a detour dominating set G & every detour domination set of G is a split detour dominating set of G and so

$\gamma_{sd}(C\rho) \leq \gamma_d(C\rho) \leq \gamma(C\rho)$. Then it follows from Theorem 3 & 4 that

$\gamma(C\rho) = \gamma_d(C\rho) = \gamma_{sd}(C\rho) = \left\lceil \frac{\rho}{3} \right\rceil$.

Theorem: 15

For the wheel $W_p = C_{p-1}+K$, $p \geq 5$,$\gamma_{sd}(G)=2$.

Proof:

Let V(Wp)={V , W_1 ,W_2,W_3,.......W_{P-1}} with V as its central vertex. Then S={V,W_1}. It's a split detour dominating set of G. So that $\gamma_{sd}(G)=2$.

Theorem: 16

For any complete bipartite graph $G = K_{m,n}$ with $2 \leq m \leq n$. Then γ_{sd} $(G)=2$

Proof:

Let U & V be the two bipartite set of G.

let U={u_1 ,u_2 ,u_3,..........,u_m} and V={V_1,V_2,V,......,V_n}

then S={U_1,V_n} is a minimum detour domination set of G but induced < S > (U-U_1 & V-V_n) is connected. So we take $S = |U| = m$ is a split detour dominating set of G so that induced<S> is disconnected &$\gamma_{sd}(G)$ or $\gamma_{sd}(K_{m,n})=2$.

We state without proof a straight forward result that characterizes dominating detour sets of G that are split detour dominating sets.

Theorem: 17

A detour dominating set S of G is a split detour domination set iff ∋ a two vertices u, v Є (V–S) such that every u–v path contain a vertex of S.

Observation : 18

For a connected graph G, $2 \leq Max\ \{d_n(G), \gamma(G)\} \leq \gamma_d(G) \leq \gamma_{sd}(G)$

Theorem: 19

Let G be a graph with K support vertices $(K \geq 5)$ & l end vertices then $\gamma_{sd}(G) = \ l + \lceil \frac{p-2}{3} \rceil$

Proof:

Let L denote the set of all end vertices of G & K denote the set of all support vertices. Let $|L| = l$ &$|K| = k$, Let $S = l + \left\lceil \frac{p-2}{3} \right\rceil$. Then S is a split detour dominating set of G so that

$$\gamma_{sd}(G) \le l + \left\lceil \frac{p-2}{3} \right\rceil.$$

CONCLUSION

Our aim is to detour the importance of graph theoretical ideas in various use of Science and Engineering for research that they can use domination concept of graph in image processing, security purpose. Mobile network etc., Researchers will get ideas and some information related to graph theory and its application in various flied and also get ideas to their field of research.

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