TEXTURES OF NEUTRINO MASS MATRIX

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Abstract

We examine the current level of knowledge on neutrino masses and mixings. The study directs upon the seesaw mechanism, that falls into a larger picture of particle physics, like supersymmetry and grand unification, and provides a unified solution to the quark and lepton flavour issue. The neutrino mass matrix, thereby encoding neutrino characteristic having several mathematical abstractions are undefined. The perceptual repercussions of lepton mass matrices derived from family permutation symmetry and its appropriate breakings are investigated. In the case of charged lepton mass matrix and the Majorana neutrino mass matrix, thereby adopting the recently suggested revised mass matrix. In this review we further study the textures of different types of mixing matrices and how based upon their mixing they possess different values of mixing angles, and their deviation from the global fit value. Here, we try to study and understand 7 types of mixing textures including Bimaximal (BM), Trimaximal (TM), Tribimaximal (TBM), Magic, $\mathrm{TM}_2, \mathrm{TM}_2$ and zero texture matrix. We see how the prediction value is deviated from the global fit value.

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1. INTRODUCTION

The Standard Model has proved to be tremendously effective in delivering experimental findings and is thought as conceptually self-consistent [16], still it remains unaccounted for some processes and therefore lacks in becoming the full-fledged theory of all the basic processes. It is incapable of entirely explicating the baryon asymmetry, nor does it integrate the whole theory of gravitation [31] like defined by general relativity, nor does it provide sufficient explanation for the Universe's rapid expansion, which may be answered by dark energy. There is no feasible dark matter particle in the model that has all of the needed attributes based on empirical cosmology. Neutrino oscillations and their non-zero masses are likewise left out.

Much work has gone into identifying the structure of the neutrino mass matrix (M_ν), since the Super-Kamiokande experiment discovered neutrino oscillation in 1998 [32]. Numerous neutrino oscillation studies have calculated the neutrino oscillation parameters with reasonable precision, including three neutrino mixing angles (solar, atmospheric, and reactor) and two mass-squared differences (Δm^2_{ij} and $|\Delta m^2_{ij}|$). Recently neutrino oscillation investigations have pointed to a non-maximal atmospheric mixing angle [13] and a Dirac-type CP-violating phase at 270°[19]. Despite of the above mentioned tremendous breakthroughs, several additional concerns such as neutrino mass ordering, leptonic CP-violation, the origin of lepton flavour structure, neutrino nature (Dirac or Majorana), and the absolute neutrino mass scale remain unanswered.

Neutrino characteristics are encoded with the help of neutrino mass matrix, which composes of various physical constants that are unknown. Particular textures of neutrino mass matrices with fewer independent parameters can be obtained using Abelian or non-Abelian flavour symmetry based phenomenological models. Various common models based on these techniques, like texture zeros [3, 25], vanishing cofactors [20], equalities across elements/cofactors [28], hybrid textures [18, 29], and others, are immensely effective in describing the currently known neutrino oscillation data.

The findings of a global examination of neutrino data released till the middle of 2006 are used. The evaluation hypothesised that firstly, only three mixed, active neutrinos are present; secondly, so as to make sure that the masses and mixing angles in the neutrino and antineutrino channels coincide, the CPT is conserved; and lastly, neutrino masses and mixings have a pure "vacuum origin," i.e., due to interactions with Higgs field(s) as it develops a VEV on a scale much greater when being compared to neutrino mass. We further discuss what may happen if any of these assumptions are proven false. The oscillation parameters are mass-squared differences $\Delta m^2_{ij} \equiv m^2_i - m^2_j$, mixing angles θ_{ij} and the Dirac CP-violating phase δ. The absolute mass scale, associated with the mass of the heaviest neutrino, and two Majorana CP-violating phases are non oscillation parameters.

2. TYPES OF NEUTRINO

Left-handed neutrinos, as per the standard model (SM), form EW doublets L with charged leptons, contains no electric charge, and are colourless. By design, the right-handed parts, v R, are not included. The notion that right-handed neutrinos are absent in this paradigm accounts for the neutrinos' masslessness at the tree level. Owing to the inclusion of an accurate B-L symmetry in the system, even when B+L is breached by weak sphaleron configurations, this conclusion applies for all orders in perturbation theory, as well as for the ones wherein non-perturbative effects are put into consideration. Non-zero neutrino masses, it appears, must be linked to the presence of right-handed neutrinos and/or the violation of B-L symmetry, these two indicate new physics outside the standard model. The difference between the neutrinos and other fermions of the SM lies because of vanishing conserved charges (electric and color). This distinction opens up a slew of potential capabilities for neutrino masses, which altogether require new physics:

(i) The neutrino masses might be Majorana type, causing L to be broken by two units.
(ii) Neutrinos can combine with singlets from the SM symmetry group, namely singlet fermions in additional dimensions.

Consider several standard model extensions that might result in non-zero neutrino masses.
1). If there is existence of right-handed neutrinos exist, the Yukawa coupling can be used.

$$Y_\nu \bar{L} H \nu_R + \text{h.c.}$$

This leads to the Dirac neutrino mass following electroweak (EW) symmetry breakdown

$$m_D = Y_\nu \langle H \rangle$$

2). Majorana masses are permitted for right-handed neutrinos.

$$M_R \nu_R^T C^{-1} \nu_R + \text{h.c.}$$

where C is the charge-conjugation matrix of Dirac. M_R might show as a bare mass component in the Lagrangian or be formed by interactions involving singlet scalar σ field since ν_R represent singlets underneath the SM gauge group.

3). The Majorana masses m_L can likewise be acquired by left-handed neutrinos. The lepton number is violated by two units because of the associated mass having the weak isospin I = 1. As a result, they may be produced either by coupling along the Higgs triplet Δ or by non-renormalizable operators and two Higgs doublets or both:

$$f_\Delta L^T \Delta + \text{h.c.}$$
$$m_L = f_\Delta \langle \Delta \rangle.$$

Given by, the non-zero vacuum expectation value (VEV) of Δ

The mass parameters m_D, M_R and m_L should be regarded 3 × 3 (non-diagonal) matrices for three neutrino species. The general mass matrix in the basis (ν_L, N_L), may be expressed as by inserting the charge-conjugate left handed component $N_L \equiv (\nu_R)^C$.

$$M_\nu = \begin{pmatrix} m_L & m_D^T \\ m_D & M_R \end{pmatrix}$$

The Majorana neutrinos having varying Majorana masses are the eigenstates of this matrix.

The diagonalization of the mass matrix, for $M_R \gg m_D$, tends to the following estimated form for the mass matrix with light neutrinos m_ν:

$$\mathcal{M}_\nu = -m_D^T M_R^{-1} m_D$$

As previously stated, because M_R might be substantially greater than m_D, $m_\nu \ll m_{e,\mu,d}$ occurs rather naturally. This mechanism is regarded as the seesaw (type I) mechanism [26], and it explains why neutrino masses are so tiny.

We may express the resultant light neutrino mass matrix in the form if members of the matrix m_L are non-zero but substantially less than that of the other members of M_ν,

$$\mathcal{M}_\nu = m_L - m_D^T M_R^{-1} m_D$$

This is called the mixed seesaw [14, 11], and we call it a type II seesaw if the first term prevails. The m_L and m_R entries in the matrix could have magnitudes substantially less than m_D. The neutrinos in this situation are mostly Dirac type, with a little amount of Majorana mass. This instance is known as the pseudo-Dirac [21].
Flavor neutrinos- The flavour neutrinos, ν_e, ν_μ, ν_τ - particles created in conjunction with certain charged leptons, are referred to as electron, muon, and tau, respectively. The electron or muon neutrinos, for example, are neutrinos released in weak processes like beta decay or pion decay with electron or muon. Because the detectors are sensitive to charged lepton flavours such as (e, μ, τ), the flavour neutrino is selected out during the detection process. Flavor mixing occurs when flavour neutrinos ν_α (α = e, μ, τ) do not correspond with definite mass neutrinos ν_i (i = 1, 2, 3). Although the electron, muon, and tau neutrinos are without any fixed masses, still they are found to be coherent combinations of mass states. The neutrino mass states are mixed in the weak charged current operations.

This is the physical neutrino mass matrix

$$U_{PMNS} = U_l^\dagger U_\nu$$

Parameterizing the mixing matrix as

$$U_{PMNS} = U_{23}(\theta_{23}) U_{13}(\theta_{13}, \delta) U_{12}(\theta_{12}) I_\phi$$

The U_{PMNS} is a Pontecorvo - Maki - Nakagawa - Sakata lepton mixing matrix, a 3 × 3 unitary matrix [2, 34]. Where U_{ij} are matrices representing rotations in the ij plane with angle θ_{ij}, and δ thereby depicting Dirac CP violating phase associated with 1-3 rotation. Quite often the mixing matrix regarding Majorana neutrinos is expressed as $U'_{PMNS} = U_{PMNS} I_\phi$, where $I_\phi \equiv \mathrm{diag}(1, e^{i\phi_1}, e^{i\phi_2})$ is the diagonal matrix of the Majorana CP-violating phases.

3. NEUTRINO MIXING MATRIX

Let's start by looking at the mixing matrix. The following matrices can play a major part indicated by small 1-3 mixing, large 1-2 mixing and maximal 2-3 mixing,

1). The bimaximal mixing matrix [10]:

$$U_{bm} = U_{23}^{m} U_{12}^{m}$$

Here U_{12}^{m} and U_{23}^{m} represents the matrices having the maximal (π/4) rotations in 1-2 and 2-3 subspaces respectively. Now we have

$$U_{bm} = \frac{1}{2}\begin{pmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -1 & 1 & \sqrt{2} \\ 1 & -1 & \sqrt{2} \end{pmatrix}$$

Due to a large divergence of the 1-2 mixing from maximal, $U_{bm} = U_{PMNS}$ cannot be identified.

2). The tri-bimaximal mixing matrix [22]

$$U_{tbm} = U_{23}^{m} U_{12}(\theta_{12}), \qquad \sin^2\theta_{12} = 1/3$$

or explicitly,

$$U_{tbm} = \frac{1}{\sqrt{6}}\begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix}$$

is consistent with data that includes 1-2 mixing. Here, ν_2 is mixed trimaximally : the three flavors blend maximally in the second column, whereas ν_3 is mixed bi-maximally. The Clebsch-Gordon factors are basic numbers like 0, 1/3, and 1/2 that occur as mixing parameters.

4. NEUTRINO MASS MATRIX

Since the mass matrix contains information regarding masses and mixings, it is perhaps more fundamental when being compared to mixing angles and mass eigenvalues. Mass matrices, rather than their eigenstates and eigenvalues, can be used to illustrate dynamics and symmetry. The physical characteristics known in Majorana neutrinos are the components of this mass matrix. These may be directly measured, for example, in neutrinoless double beta decay and, in theory, in other processes.

The kind of mass spectrum may influence the response to the issue as to whether which is more fundamental: mass matrices or observables (Δm^2, θ).

In the flavour basis, the neutrino mass matrix is diagonalized by $U_\nu = U_{PMNS}$ as the charged leptons have diagonal mass matrix. As a result, the neutrino mass matrix in the flavour basis may be expressed as

$$\mathcal{M}_\nu = U^*{}_{PMNS} \mathcal{M}_\nu^{d} U_{PMNS}^{\dagger}$$

where

$$\mathcal{M}_\nu^{d} \equiv \mathrm{diag}(m_1, m_2 e^{-2i\phi_2}, m_3 e^{-2i\phi_3})$$

The Majorana phases are represented by ϕ_i and consider $\phi_1 = 0$. Reconstruction results demonstrate [23] that relying substantially on the unknown m_1, kind of mass hierarchy, and Majorana phases, a wide range of mass matrix configurations are feasible. The reliance on sin θ_{13} and δ is comparatively minor. This indicates that mass matrices are highly degenerate today, and maybe in the future, since it is impossible to quantify all parameters, including CP-violating phases, in reality. Even though all other parameters are known, changes in one Majorana phase might cause significant structural changes.

4.1 Texture of Bimaximal mass matrix

The solar mixing angle is singled out for specific consideration across many suggested mixing matrices. Having the similar atmospheric and reactor neutrino mixing angles as tri-bimaximal mixing or μ–τ symmetry, in the case of bimaximal mixing (BM), $\sin^2\theta_{12} = \frac{1}{2}$,. As a result, the mixing matrix takes the form [9]

$$U_{BM} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \\ \frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

The mass matrix for the same,

$$M_{BM} = \begin{pmatrix} a & b & b \\ b & c & a-c \\ b & a-c & c \end{pmatrix}$$

It was demonstrated in [27] that one may generate such a mixing matrix using the discrete symmetry S_3. Despite the fact that $\sin^2\theta_{12} = \frac{1}{2}$ is ruled out by a factor of almost 10σ, this mixing possibility has lately been reintroduced like a model predicated upon S_4 [12]. Charged lepton corrections can rectify bimaximal mixing, resulting in QLC (Quark-lepton complementarity) circumstances.

4.2 Texture of Tribimaximal mass matrix

One sort of mixing matrix suggested by Harison, Perkins, and Scott has ν_2 trimaximally mixed and ν_3 bimaximally mixed components. Tribimaximal mixing was born as a result. Here,

$$U_{TBM} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix}$$

The mass matrix M_{tbm} for U_{tbm} is,

$$M_{TBM} = \begin{pmatrix} a & b & b \\ b & a+d & b-d \\ b & b-d & a+d \end{pmatrix}$$

$\sin^2\theta_{13} = 0$, $\sin^2\theta_{12} = \frac{1}{3}$ and $\sin^2\theta_{23} = \frac{1}{2}$ are predicted by the TBM mixing matrix. The experimental results $\sin^2\theta_{12} = 0.306^{+0.012}_{-0.012}$ and $\sin^2\theta_{23} = 0.441^{+0.027}_{-0.027}$, for mixing angles θ_{12} and θ_{23} are in accord at 3σ with the latest global fit of the neutrino experimental data [15].

4.3 Texture of Trimaximal mass matrix

The second neutrino mass eigenstates ν_2 has trimaximal feature in Harrison, Perkins, and Scott's mixing scheme because it comes from maximal mixing of the three flavor eigenstates (ν_e, ν_μ and ν_τ). Furthermore, the third mass eigenstate, ν_3 is bimaximal since it is formed by the maximum mixing of two flavour eigenstates, namely ν_μ and ν_τ.

As a result, this mixing technique is known as tri-bimaximal (TBM) mixing, and it produces vanishing θ_{13} and maximal θ_{23}. Because the mixing angle θ_{13} evaporates in TBM mixing due to the bimaximal nature of ν_3, this characteristic must be dropped to provide for a non-vanishing θ_{13} and, therefore, the possibility of CP violation. In this expansion of TBM mixing, however, there is no need to sacrifice the trimaximal feature of the ν_2. Trimaximal (TM) [6], is the name given to such a mixing matrix.

$$U_{TM} = \begin{pmatrix} U_{11} & \frac{1}{\sqrt{3}} & U_{13} \\ U_{21} & \frac{1}{\sqrt{3}} & U_{23} \\ U_{31} & \frac{1}{\sqrt{3}} & U_{33} \end{pmatrix}$$

From the perspective of the symmetry groups associated with these mixing schemes, the link between TBM and TM mixing must also be explored. A neutrino mass matrix that may be parametrized as follows the TM mixing.

$$M_{TM} = \begin{pmatrix} a+2d & c-d & b-d \\ c-d & b+2d & a-d \\ b-d & a-d & c+2d \end{pmatrix}$$

When unitarity requirements and trimaximality are coupled, a concept of maximal CP failure arises naturally. Whenever θ_{23} becomes maximal, the Jarlskog rephasing invariant J [5] assumes the maximum $\frac{1}{2\sqrt{3}}|U_{11}||U_{13}|$. Likewise, the variance of maximal 2-3 mixing is proportional to θ_{13} and is inherently constrained by unitarity to be inside the current range of observations for values of θ_{13} inside its own observational boundaries [4]. As CP violation is maximal, this divergence is zero, and if there is no CP violation, it is maximal.

4.4 Texture of Magic mass matrix

$\mu - \tau$ exchange symmetry states that such neutrino mass matrix is unchanged under the concurrent swapping of its second and third (μ-τ) indices, while magic symmetry signifies that the summation of the elements of every row and column of the mass matrix retains the similarity. Maximal value of θ_{23} and vanishing θ_{13} are predicted by a neutrino mass matrix that seems to be invariant within magic symmetry and exchange symmetry.

One might tear MTBM by enraging the TBM mass matrix with an additional matrix in a quite manner that just one of the two symmetries it has is broken. Because $\mu - \tau$ symmetry anticipates vanishing θ_{13}, maintaining magic symmetry is a viable option. If the neutrino mass matrix remains intact after a G_j transformation of the neutrino fields, then

$$G_j^T M_v G_j = M_v$$

A symmetry of mass matrix M_v. is the transformation G_j. $G_j = 1 - u_j u_j^T$ (j = 1, 2, 3) may be used to determine the transformation matrix, where u_j is the column of the matrix correlating to G_j. The transformation matrix for such magic symmetry may be seen as

$$G_2 = \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

As a result, a mass matrix M_{magic} retaining the magic symmetry satisfies $G_2^T M_{magic} G_2 = M_{magic}$. The centre column of the mixing matrix correlating to certain mass matrices is just similar as U_{TBM} (tribimaximal) and may be expressed in the form of 2 independent variables θ and ϕ

$$U_{Magic} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\theta \\ \frac{e^{i\phi}\sin\theta - \frac{\cos\theta}{\sqrt{3}}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{-e^{i\phi}\cos\theta - \frac{\sin\theta}{\sqrt{3}}}{\sqrt{2}} \\ \frac{-\frac{\cos\theta}{\sqrt{3}} - e^{i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{e^{i\phi}\cos\theta - \frac{\sin\theta}{\sqrt{3}}}{\sqrt{2}} \end{pmatrix}$$

Trimaximal mixing is the name given to a mixing matrix with a trimaximally mixed column. M_{Magic} can indeed be expressed as

$$M_{Magic} = \begin{pmatrix} a & b & c \\ b & a+d & c-d \\ c & c-d & a+b-c-d \end{pmatrix}$$

Finding the mixing angles from U= U_{Magic} in terms of θ and ϕ,

$$\sin^2\theta_{12} = \frac{|U_{12}|^2}{1-|U_{13}|^2},\ \sin^2\theta_{23} = \frac{|U_{23}|^2}{1-|U_{13}|^2}, \sin^2\theta_{13} = |U_{13}|^2$$

we get,

$$\sin^2\theta_{12} = \frac{1}{3-2\sin^2\theta},$$
$$\sin^2\theta_{23} = \frac{1}{2}\left(\frac{\sqrt{3}\sin2\theta\cos\phi}{3-2\sin^2\theta}\right),$$
$$\sin^2\theta_{13} = \frac{2\sin^2\theta}{3},$$
$$\csc^2\delta = \csc^2\phi - \frac{3\sin^2 2\theta\cot^2\phi}{3-2\sin^2\theta}$$

4.5 Textures of TM_1 mass matrix

Customisation of TM_1as [7, 8, 17, 30, 33]:

$$U_{TM_1} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}}\cos\theta & \frac{1}{\sqrt{3}}\sin\theta \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\cos\theta - \frac{e^{i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}}\sin\theta + \frac{e^{i\phi}\cos\theta}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\cos\theta + \frac{e^{i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}}\sin\theta - \frac{e^{i\phi}\cos\theta}{\sqrt{2}} \end{pmatrix}$$

The first column for the neutrino mixing matrix is similar as the TBM mixing matrix, whereas the remaining two columns have indeed been factorized in the form of 2 free parameters (θ and ϕ) once the unitarity requirements over the mixing matrix have been taken into account. In the case of TM_1 mixing, the relevant neutrino mass matrix is created as

$$M_{TM_1} = \begin{pmatrix} a & 2b & 2c \\ 2b & 4b+d & a-b-c-d \\ 2c & a-b-c-d & 4c+d \end{pmatrix}$$

The mixing angles in terms of θ and ϕ,

$$s_{12}^2 = \frac{|U_{12}|^2}{1-|U_{13}|^2}, s_{23}^2 = \frac{|U_{23}|^2}{1-|U_{13}|^2}, s_{13}^2 = |U_{13}|^2$$

We get,

$$s_{12}^2 = 1 - \frac{2}{3-\sin^2\theta},$$
$$s_{23}^2 = \frac{1}{2}\left(1 + \frac{\sqrt{6}\sin 2\theta\cos\phi}{3-\sin^2\theta}\right),$$
$$s_{13}^2 = \frac{\sin^2\theta}{3}$$

The solar mixing angle is θ_{12}, which is less than the TBM value of $s_{12}^2 = 1/3$. In contrary, the θ_{12} for TM_2 mixing is higher than the TBM value. Phases θ and ϕ having nearly similar results, that is analogous to the TM_2 situation. In contrast to the TM_2 situation, the θ_{12} here diminishes as the number increases. This is a property of TM_1 mixing in general. This closes the gap between θ_{12} and its best-fit experimental value. TM_1 mixing is somewhat more enticing than TM_2 mixing that's because the experimental best fit value of θ_{12} is on the lower end of the TBM value.

4.6 Textures of TM_2 mass matrix

Customisation of TM_2 as [7, 8, 17, 30, 33]:

$$U_{TM_2} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\theta \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{e^{-i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\sin\theta}{\sqrt{6}} - \frac{e^{-i\phi}\cos\theta}{\sqrt{2}} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{e^{-i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}}\cos\theta & -\frac{\sin\theta}{\sqrt{6}} + \frac{e^{-i\phi}\cos\theta}{\sqrt{2}} \end{pmatrix}$$

The TM_2 mixing matrix has its middle column fixed to the TBM value (u), which leaves only two free parameters (θ and ϕ) in U_{TM_2} after we take into account the unitarity constraints. The neutrino mass matrix corresponding to TM_2 mixing is given as

After taking into consideration the unitarity restrictions, the middle column of the The TM_2 mixing matrix is set to the TBM value (u), leaving just 2 free parameters (θ and ϕ) in U_{TM_2}. The neutrino mass matrix for TM_2 mixing is calculated as

$$M_{TM_2} = \begin{pmatrix} a & b & c \\ b & d & a+c-d \\ c & a+c-d & b-c+d \end{pmatrix}$$

The mixing angles in terms of θ and ϕ,

$$s_{12}^2 = \frac{|U_{12}|^2}{1-|U_{13}|^2}, s_{23}^2 = \frac{|U_{23}|^2}{1-|U_{13}|^2}, s_{13}^2 = |U_{13}|^2$$

We get,

$$s_{12}^2 = \frac{1}{3-2\sin^2\theta},$$
$$s_{23}^2 = \frac{1}{2}\left(1 + \frac{\sqrt{3}\sin 2\theta\cos\phi}{3-2\sin^2\theta}\right),$$
$$s_{13}^2 = \frac{2\sin^2\theta}{3}$$

Because the TBM value of θ_{12} is already higher than the experimental best fit value, an increase in pushes θ_{12} farther away from the experimental best fit value. With a mixing angle of θ_{12}, TM_2 mixing causes some stress.

4.7 Texture zero mass matrix

To examine if any components can be precisely zero or equal is another way to look at potential mass matrices. This might be helpful in revealing dominating structures as well as underlying symmetries. The advantage here is that the number of free parameters are reduced and so allows for more precise predictions. Remember that there are six independent elements in the Majorana mass matrix for three neutrinos. We looked at mass matrices with varied numbers of zeros and zeros in numerous places across the matrix. Textures with three zeroes and two zeroes are the most commonly explored two instances in the literature. It's simple to believe that three zeros can't be in any of the 2 × 2 submatrices or along the off-diagonal entries and still suit the existing data. All mixings disappear in the first instance, and in the second situation, one cannot meet the criterion that $\Delta m_{12}^2 \ll \Delta m_{23}^2$ if θ_{23} and θ_{12} are huge, as seen. When all zeros are on the diagonal [1] (or two on the diagonal and the third off diagonal), the situation is far more delicate since one may now meet the conditions of significant solar and atmospheric mixings along with $\Delta m_{12}^2 \ll \Delta m_{23}^2$. Nevertheless, only three (real) factors may be calculated from Δm_{12}^2, Δm_{23}^2 and θ_{23} in this scenario. The solar mixing angle, $\sin^2 2\theta_{12} = 1 - r_\Delta/16$, is then predicted, which is inconsistent with data.

In terms of textures having two zeros, these contain five free parameters: with four real ones and a complex phase, making them promising candidates for neutrino mass matrices [24]. This has been examined to determine their typical forecasts. Now there are seven (total of fifteen) alternatives that are in agreement with evidence and provide estimations for numerous parameters like neutrinoless double beta decay and θ_{13}. See the matrix as an intriguing example.

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & X \\ 0 & X & X \\ X & X & X \end{pmatrix}$$

Here, non-zero items are indicated by X. As a result, a hierarchical mass matrix containing the assumption is produced

$$\sin^2\theta_{13} \sim \frac{r_\Delta}{\tan^2\theta_{12} - \cot^2\theta_{12}} \sim 0.01$$

and giving zero amplitude for the case of neutrinoless double beta decay.
One more workable texture is

$$\mathcal{M}_\nu = \begin{pmatrix} X & X & 0 \\ X & 0 & X \\ 0 & X & X \end{pmatrix}$$

This results in a degenerate mass spectrum with a neutrinoless double beta decay effective mass above 0.1 eV.

5. CONCLUSION

Recent neutrino physics revelations have created a new realm of physics outside the standard model. We have sought to give a glimpse of what we have learnt from the discoveries in this analysis, as well as what future studies may promise in terms of where this learning may go. The areas we tried to focus on are (i) the flavor structure of leptons and their mixing and the workable symmetries by them, (ii) types of neutrinos (LH and RH) and how they can be majorana type or dirac type or both, (iii) the different type of neutrino mixings and their corresponding mixing matrices. The aim of this review is to study textures of different neutrino mixing mass matrices. Studied how mixing different flavor eigenstates can give numerous mixing matrices predicting different values of mixing angles with some deviations in each case. Data for undiscovered neutrino species mixing with existing neutrinos, in contrast to the enormous mixing surprise, will be a fresh surprising fact and a new change. It can prompt queries like: Are there newer quark species that correlate to the newfound neutrinos? Could there be a mirror sector to the cosmos or are there more dimensions? What function do additional neutrinos play in the evolution of the universe? For the evolvement of universe in what way do the additional neutrinos will participate?

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