# The connected circular metric dimension of a graph

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**Abstract**

Let $G = (V, E)$ be a simple graph and $u, v$ be any two vertices of G. Then the circular distance between $u$ and $v$ denoted by $D^c(u, v)$ and is defined by

$$D^c(u,v) = \begin{cases} D(u,v) + d(u,v) & \text{if } u \neq v \\ 0 & \text{if } u = v \end{cases}$$

where $D(u, v)$ and $d(u, v)$ are detour distance and distance between $u$ and $v$ respectively. Let $W = \{w_1, w_2, \ldots, w_k\} \subset V(G)$ and $v \in V(G)$. The representation $cr(v/W)$ of $v$ with respect to $W$ is the $k$-tuple $\left(D^c(v, w_1), D^c(v, w_2), \ldots, D^c(v, w_k)\right)$.Then $W$ is called a circular resolving set if different vertices of $G$ have different representations with respect to $W$. A circular resolving set $W$ is called connected circular resolving set, $G[W]$ is connected . The minimum cardinality of a connected circular resolving set in a graph G is its connected circular metric dimension of $G$ and is denoted by $cdim_c(G)$. The connected circular metric dimension of some standard graphs are determined. Some general properties satisfies by this concept are studied. Connected graphs of order $n \geq 3$ with connected circular metric dimension 1 are characterized. Necessary condition for the connected circular metric dimension to be $n - 1$ is given.

**Keywords:** distance, detour distance, circular distance, circular resolving set, connected circular metric dimension.

**AMS Subject Classification:** 05C12.

**1. Introduction and Preliminaries**

Let $G$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$. The order of a graph $G$ is $|V(G)|$, its number of vertices denoted by $n$. The size of a graph $G$ is $|E(G)|$, its number of edges denoted by $m$. For basic graph theory terminology, we refer [3]. The *degree*, $\deg(v)$ of a vertex $v \in V(G)$ is the number of edges incident to $v$. We denote $\Delta(G)$ the maximum degree of a graph $G$. The *distance* $d(u,v)$ between two vertices $u, v \in V(G)$ is the length of a shortest path between them. These concepts were studied in [1,2,5,10-14,17-24,26,27]. The detour *distance* $D(u,v)$ between two vertices $u, v \in V(G)$ is the length of a longest path between them. These concepts were studied in [6-8,15]. Let $W = \{w_1, w_2, \ldots, w_k\} \subset V(G)$ and $v \in V(G)$. The representation $r(v/W)$ of $v$ with respect to $W$ is the $k$-tuple $\left(d(v,w_1), d(v, w_2), \ldots, d(v,w_k)\right)$.Then $W$ is called a *resolving set* if different vertices of $G$ have different representations with respect to $W$. A resolving set of minimum number of elements is called a *basis* for $G$ and the cardinality of the basis is known as the *metric dimension* of $G$, represented by $dim(G)$. These concepts were studied in [4,28]. The representation $Dr(v/W)$ of $v$ with respect to $W$ is the $k$-tuple $\left(D(v,w_1), D(v, w_2), \ldots, D(v,w_k)\right)$.Then $W$ is called a *detour resolving set* if different vertices of $G$ have different representations with respect to $W$. A detour resolving set of minimum number of elements is called a *detour basis* for $G$ and the cardinality of the basis is known as the *detour metric dimension* of $G$, represented by $Ddim(G)$.

The circular distance between $u$ and $v$ is denoted by $D^c(u,v)$ and is defined by

$$D^c(u,v) = \begin{cases} D(u,v) + d(u,v) & \text{if } u \neq v \\ 0 & \text{if } u = v \end{cases}$$

An $u - v$ path of length $D^c(u,v)$ is called a $u - v$ circular. The circular diameter is the maximum circular distance be a pair of vertices of $G$. It is denoted by the $D^c(G)$. A circular path of length $D^c(G)$ is called the circular diametral path. These concepts were studied in [16].

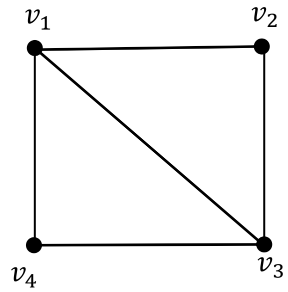
Let $W = \{w_1, w_2, \ldots, w_k\} \subset V(G)$ and $v \in V(G)$. The representation $cr(v/W)$ of $v$ with respect to $W$ is the $k$-tuple $\left(D^c(v,w_1), D^c(v, w_2), \ldots, D^c(v,w_k)\right)$.Then $W$ is called a *circular resolving set* if different vertices of $G$ have different representations with respect to $W$. A circular resolving set of minimum number of elements is called a circular basis for $G$ and the cardinality of the basis is known as the *circular metric dimension* of $G$, represented by $cdim(G)$. These concepts were studied in [25]. In this article, we study a new metric dimension called the connected circular metric dimension of a graph.The following theorem is used in the sequel.

**Theorem 1.3. [28]]** For the complete graph $G = K_n$ $(n \geq 2)$, $cdim(G) = n - 1$.

## 2. The connected circular metric dimension of a graph

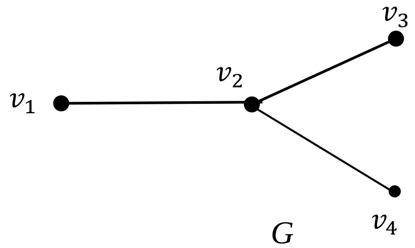
**Definition 2.1.** A circular resolving set $W$ is called a *connected circular resolving set* of $G$ if $G[W]$ is connected. The minimum cardinality of a connected circular resolving set in a graph $G$ is its *connected circular metric dimension* of $G$ and is denoted by $cdim_c(G)$.

**Example 2.2.** For the graph $G$ given in Figure 2.1, let $W$= $\{v_1, v_2\}$. Then $cr(v_1/W) = (0, 4)$, $cr(v_2/W) = (4,0)$, $cr(v_3/W) = (3,4)$, $cr(v_4/W) = (4,5)$. Since $cr(v/W)$ are distinct for all $v \in V(C_n)$, it follows that $W$ is a circular resolving set of $G$. Since $G[W]$ is connected, $W$ is a connected circular resolving set of $G$ and so $cdim_c(G) \leq 2$. Also since no singleton subset of $V(G)$ is a circular resolving set of $G$, we have $cdim_c(G) = 2$.




$G$
Figure 2.1

**Example 2.3.** For the graph $G$ given in Figure 2.2, no singleton subset of $V(G)$ is a circular resolving set of $G$, we have $cdim_c(G) \geq 2$. Let $W = \{v_1, v_3\}$. Then $cr(v_1/W) = (0,4)$, $cr(v_2/W) = (2,2)$, $cr(v_3/W) = (4,0)$, $cr(v_4/W) = (4,4)$. Since $cr(v/W)$ are distinct for all $v \in V(C_n)$, it follows that $W$ is a circular resolving set of $G$. Since $G[W]$ is not connected, $W$ is not a connected circular resolving set of $G$. It is easily verified that no two-element subset of $V(G)$ is not a circular resolving set of $G$ and so $cdim_c(G) \geq 3$. Let $W_1 = \{v_1, v_2, v_3\}$. Then $W_1$ is a connected circular resolving set of $G$ so that $cdim_c(G) = 3$.




$G$

Figure 2.2

**Observation 2.4. (i)** Let $G$ be a connected graph of order $n \geq 2$. Then $1 \leq cdim_c(G) \leq n-1$.
(ii) Each cut vertex of $G$ belongs to every connected resolving set of $G$.

In the following we determine the connected circular metric dimension of some standard graphs.

**Theorem 2.5.** For the graph $G = P_n$ $(n \geq 2)$, $cdim_c(G) = 1$.

**Proof.** Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and let $W = \{v_1\}$. Then $D^c(v_1, v_i) = 2(i-1), (1 \leq i \leq n)$. Since $cr(v/W)$ is distinct for all $v \in V(P_n)$, it follows that $W$ is a circular resolving set of $G$. Also, $G[W]$ is connected, Hence $W$ is a connected circular resolving set of $G$ so that $cdim_c(G) = 1$.

**Theorem 2.6.** For the cycle $G = C_n, n \geq 3$, $cdim_c(G) = n-1$.

**Proof.** Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ and let $W = \{v_1, v_2, \ldots, v_{n-1}\}$, The circular representations of $(n-1)$ tuples are as follows

$$cr(v_1/W) = (0, n, n, \ldots, n),$$
$$cr(v_2/W) = (n, 0, n, n, \ldots, n)$$
$$cr(v_3/W) = (n, n, 0, n, n, \ldots, n)$$

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$$cr(v_{n-1}/W) = (n, n, n, \ldots, n, 0)$$
$$cr(v_n/W) = (n, n, n, n, \ldots, n).$$

Since $cr(v/W)$ are distinct for all $v \in V(C_n)$, it follows that $W$ is a circular resolving set of $G$. Since $G[W]$ is connected, $W$ is a connected circular resolving set of $G$. Therefore $cdim_c(G) \leq n-1$. We substantiate that $cdim_c(G) = n-1$.Consider, however, that $cdim_c(G) \leq n-2$.

Then, a set $S'$exists such that $|S'| \leq n-2$. As a result, there are at least two vertices $u, v$ that satisfy the contradiction $cr(u/S') = cr(v/S') = (n, n, \dots, n)$.Consequently, $cdim_c(G) = n-1$.

**Theorem 2.7.** For the complete graph $G = K_n, n \geq 2,\ cdim_c(G) = n-1$.

**Proof.** The proof is similar to the Theorem 2.6.

**Theorem 2.8.** For the wheel graph $G = W_n, n \geq 3,\ cdim_c(G) = n-4$.

**Proof.** Let $V(G) = \{v_1, v_2, \dots, v_{n-1}, u\}$ and $W = \{v_1, v_2, \dots, v_{n-4}\}$, Then the circular representations of $(n-4)$ tuples are as follows

$$cr(v_1/W) = (0, n, n+1, \dots, n+1, n+1)$$
$$cr(v_2/W) = (n, 0, n, n+1, \dots, n+1, n+1)$$
$$cr(v_3/W) = (n+1, n, 0, n, n+1, \dots, n+1)$$

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$$cr(v_{n-4}/W) = (n+1, n+1, n+1, \dots, n, 0)$$
$$cr(v_{n-3}/W) = (n+1, n+1, n+1, \dots, n+1, n)$$
$$cr(v_{n-2}/W) = (n+1, n+1, n+1, \dots, n+1, n+1)$$
$$cr(v_{n-1}/W) = (n, n+1, n+1, \dots, n+1, n+1)$$
$$cr(u/W) = (n, n, n, n, \dots, n, n).$$

Since $cr(v/W)$ are distinct for all $v \in V(W_n)$, it follows that $W$ is a circular resolving set of $G$. Since $G[W]$ is connected, $W$ is a connected circular resolving set of $G$. Therefore $cdim_c(G) \leq n-4$. We substantiate that $cdim_c(G) = n-4$. Consider, however, that $cdim_c(G) \leq n-5$. Then, a set $W'$exists such that $|W'| \leq n-5$. As a result, there are at least two vertices $u, v$ that satisfy the contradiction

$$cr(u/S') = cr(v/S') = (n+1, n+1, \dots, n+1).$$

Consequently, $cdim_c(G) = n-4$.

**Theorem 2.9.** For the complete bipartite graph $G = K_{r,s}$, $(1 \leq r \leq s)$,

$$cdim_c(G) = \begin{cases} 1; r = 1, 1 \leq s \leq 2, \\ r+s-2;\ r = 1, s \geq 3. \\ r+s-1; 2 \leq r \leq s \end{cases}$$

**Proof.** Let $X = \{x_1, x_2, \dots, x_r\}$ and $Y = \{y_1, y_2, \dots, y_s\}$ be the two bipartite sets of $G$. We have the three cases.

**Case (i):**$r = 1, 1 \leq s \leq 2$. The result follows from Theorem 2.5.

**Case (ii):** $r = 1, s \geq 3$.

Let $W = V(G) - \{x_1, y_s\}$. Then the circular metric representations $(n-2)$ tuples are as follows:

$$cr(x_1/W) = (2,2,2,\dots,2,2)$$
$$cr(y_1/W) = (0,4,4,\dots,4,4)$$
$$cr(y_2/W) = (4,0,4,\dots,4,4)$$

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$$cr(y_{s-1}/W) = (4,4,4,\dots,4,0)$$
$$cr(y_s/W) = (4,4,4,\dots,4,4).$$

Since the representation are distinct and *G[W]* is connected, $W$ is a connected circular resolving set of $G$ so that $cdim_c(G) \le r+s-2$. We demonstrate that $cdim_c(G) = r+s-2$. On the other hand, imagine that $cdim_c(G) \le r+s-3$. Then there exists a circular resolving set $W'$ such that. $|W'| \le r+s-3$. As a result, there are at least two end vertices $u, v \in V\backslash W'$ such that $cr(u/W') = cr(v/W') = (4,4,4,\dots,4,4)$, which is incoherent. As a result, $cdim_c(G) = r+s-2$.

**Cases (iii):** $2 \le r \le s$.

Let $W = V(G) - \{y_s\}$. Then the circular metric representations $(r+s-1)$ tuples are as follows:

$$cr(x_1/W) = (0, r+s-1, r+s-1, \dots, r+s-1)$$
$$cr(x_2/W) = (r+s-1, 0, r+s-1, \dots, r+s-1)$$

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$$cr(x_r/W) = (r+s-1, r+s-1, \dots, 0, r+s-1, \dots, r+s-1)$$

→ $r^{\text{th}}$ place

$$cr(y_1/W) = (r+s-1, r+s-1, \dots, r+s-1, 0, r+s-1 \dots, r+s-1)$$

→ $(r+1)^{\text{th}}$ place

$$cr(y_2/W) = (r+s-1, r+s-1, \dots, r+s-1, 0, \ r+s-1 \ \dots, r+s-1)$$

→ $(r+2)^{\text{th}}$ place

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$$cr(y_{s-1}/W) = (r+s-1, r+s-1, r+s-1, \dots, r+s-1, r+s-1, \dots, 0)$$

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$(r+s-1)^{\text{th}}$ place

$$cr(y_s/W)=(r+s-1,r+s-1,r+s-1,\dots,r+s-1,r+s-1,\dots,r+s-1).$$

Since the representation are distinct and *G[W]* is connected, $W$ is a connected circular resolving set of $G$ so that $cdim_c(G)\le r+s-1$. We demonstrate that $cdim_c(G)=r+s-1$. Consider however, that $cdim_c(G)\le r+s-2$. If so, a circular resolving set $W'$ exists such that $|W'|\le r+s-2$ As a result, there are at least two vertices, $u,v\in V\backslash W'$such that $cr(u/W')=cr(v/W')=(r+s-1,r+s-1,r+s-1,\dots,r+s-1)$, which is incoherent. As a result, $cdim_c(G)=r+s-1$.

**Theorem 2.10.** Let $G$ be the graph obtained from $K_{1,n-1}$, $(n\ge 3)$, by subdividing the end edges exactly once. Then $cdim_c(G)=n-1$.

**Proof.** Let $x$ be the central vertex of $K_{1,n-1}$ $(n\ge 4)$ and $\{v_1,v_2,\dots,v_{n-1}\}$ be the set of end vertices of $G$. $G$ is the graph obtained form $K_{1,n-1}$, $(n\ge 4)$, by subdividing $xv_i$ $(1\le i\le n-1)$ by $u_i$ $(1\le i\le n-1)$. Let $W=\{x,u_1,u_2,\dots,u_{n-2}\}$. Then

$$cr(x/W)=(0,2,2,\dots,2,2)$$
$$cr(u_1/W)=(2,0,4,4,\dots,4,4)$$
$$cr(u_2/W)=(2,4,0,4,\dots,4,4)$$

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$$cr(u_{n-3}/W)=(2,4,4,4,\dots,0,4,4)$$
$$cr(u_{n-2}/W)=(2,4,4,4,\dots,4,4,0)$$
$$cr(u_{n-1}/W)=(2,4,4,\dots,4,4,4)$$
$$cr(v_1/W)=(4,2,6,6,6,\dots,6,6)$$
$$cr(v_2/W)=(4,6,2,6,6,\dots,6,6)$$

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$$cr(v_{n-3}/W)=(4,6,6,\dots,2,6)$$
$$cr(v_{n-2}/W)=(4,6,6,6,\dots,6,2)$$
$$cr(v_{n-1}/W)=(4,6,6,6,\dots,6,6,).$$

Due to the distinctness of the representations, $W$ is a circular resolving set of $G$. Also $G[W]$ is connected, $W$ is a connected circular resolving set of $G$ so that $cdim_c(G)\le n-1$. We substantiate that $cdim_c(G) = n - 1$. Consider, however, that $cdim_c(G) \leq n - 2$. If so, a circular resolving set $W'$ exists such that $|W'| \leq n - 2$ and $G[W']$ is disconnected. Consequently, $cdim_c(G) = n - 1$.

**Theorem 2.11.** Let $G$ be the graph obtained from $C_n$, $(n \geq 3)$, by subdividing the edges exactly once. Then $cdim_c(G) = 2n - 1$.

**Proof:** Let $V(C_n) = \{v_1, v_2, \dots, v_n\}$ and $\{u_1, u_2, \dots, u_n\}$ be the subdivided vertices of $C_n$. Then $G$ is a cycle contains $2n$ vertices. By Theorem 2.6, $cdim_c(G) = 2n - 1$.

**Theorem2.12.** For the crown graph $G = H_{n,n}, n \geq 3,\ cdim_c(G) = n$.

**Proof.** Let $V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$ and $E(G) = \{(u_i, v_j); 1 \leq i, j \leq n; i \neq j\}$,

Let $W = \{u_1, u_3, u_4, \dots, u_n, , v_2\}$. Then the circular representations of n tuples are as follows

$$cr(u_1/W) = (0, n, n, n, \dots, n, n, n, n)$$
$$cr(u_2/W) = (n, n, n, n, \dots, n, n, n, n + 2)$$
$$cr(u_3/W) = (n, 0, n, n, \dots, n, n, n, n)$$
$$cr(u_4/W) = (n, n, 0, n, \dots, n, n, n, n)$$

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$$cr(u_{n-1}/W) = (n, n, n, n, \dots, n, 0, n, n)$$
$$cr(u_n/W) = (n, n, n, n, \dots, n, n, 0, n)$$
$$cr(v_1/W) = (n + 2, n, n, n, \dots, n, n, n, n\ )$$
$$cr(v_2/W) = (n, n, n, n, \dots, n, n, n, 0)$$
$$cr(v_3/W) = (n, n + 2, n, n, \dots, n, n, n, n)$$
$$cr(v_4/W) = (n, n, n + 2, n, \dots, n, n, n, n)$$

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$$cr(v_{n-1}/W) = (n, n, n, n, \dots, n, n + 2, n, n)$$
$$cr(v_n/W) = (n, n, n, n, \dots, n, n, n + 2, n)$$

Since $cr(v/W)$ are distinct for all $v \in V(H_{n,n})$, it follows that $W$ is a circular resolving set of $G$. Since $G[W]$ is connected, $W$ is a connected circular resolving set of $G$. Therefore $cdim_c(G) \leq n$. We substantiate that $cdim_c(G) = n$. Consider, however, that $cdim_c(G) \leq n - 1$. Then, a set

$W'$exists such that $|W'| \leq n-1$. As a result, there are at least two vertices $u, v$ that satisfy the contradiction

$$cr(u/S') = cr(v/S') = (n, n, n, n, \dots, n, n).$$

Consequently, $cdim_c(G) = n$.

## 3.Some results on connected circular metric dimension of a graph

**Theorem 3.1.** For connected graph of order $n \geq 2$, $1 \leq cdim(G) \leq cdim_c(G) \leq n-1$.

**Proof:** Any circular resolving set of $G$ needs atleast one vertex and so $cdim(G) \geq 1$. Since any connected circular resolving set is also a circular resolving set of $G$, we have $cdim(G) \leq cdim_c(G)$. Also since $V(G) - x$ is a connected resolving set of $G$, where $x \in V(G)$ is not a cut vertex of $G$, we have $cdim_c(G) \leq n-1$. Thus $1 \leq cdim(G) \leq cdim_c(G) \leq n-1$.

**Remark 3.2.** The bounding bound in Theorem 3.1 bounds are sharp.

For $G = P_n, n \geq 2$, by theorem 2.5 $cdim_c(G) = 1$.

For the cycle $G = C_4$, by theorem 2.6 $cdim_c(G) = 3$ and for $G = K_n, n \geq 3$, $cdim_c(G) = n-1$.

**Remark 3.3.** Also, the bounds in Theorem 3.1 can be strict. For the star $G = K_{1,4}, cdim(G) = 3$ $cdim_c(G) = 4$ and $n = 5$. Thus $1 < cdim(G) < cdim_c(G) < n-1$.

**Theorem 3.4.** Let $G$ be a connected graph of order $n \geq 3$ has connected circular metric dimension 1 if and only if $G = P_n$.

**Proof.** Let $G = P_n$. Then the result follows from Theorem 2.5. Conversely, assume that $cdim_c(G) = 1$. Let $W = \{v\}$ be a minimum connected circular resolving set of $G$. Then $cr(u/W) = D^c(u, v)$ is a non-negative integer less than $2(n-1)$ for each $u \in V(G)$. There exists a vertex $u \in V(G)$ such that $d(u, v) = n-1$. This is because the representation of $V(G)$ with regard to $W$ are distinct. As a result, the circular diameter of $G$ is $2(n-1)$, implies that $G = P_n$.

**Theorem 3.5**. Let G be a connected graph of order $n \geq 3$. If every pair of vertices of G is a circular diametral path of G. Then $cdim_c(G) = n-1$

**Proof:** Assume that every pair of vertices of $G$ is circular diametral path of $G$. Therefore $D^c(u,v) = n$ for all $u,v \in V(G)$. Hence it follows that every circular resolving set of $G$ contains at least n-1 elements. Also, $G[W]$ is connected. Hence $cdim_c(G) = n-1$.

**Remark 3.6.** The converse of the Theorem 3.5. need not be true. For the graph $G = K_{1,n-1}$, $cdim_c(G) = n-1$. But there are at least two vertices say $x$ and $y$ in $G$ such that $x-y$ is not a circular diametral path of $G$.

**Theorem 3.7.** For any pair of integers $a$ and $b$ with $2 \leq a \leq b$, there exists a connected graph $G$ such that $cdim(G) = a$ and $cdim_c(G) = b$.

Proof: For $a = b$, let $G = K_{a+1}$ then by Theorems 1.1 and 2.7, $cdim(G) = cdim_c(G) = a$. So let $2 \leq a < b$. Let $P_{b-a}$ be a path of order b-a+1 and let $V(P_{b-a+1})$= $\{v_1, v_2, \dots, v_{b-a+1}\}$. Let $G$ be the graph obtained from by adding the new vertices $u_1, u_2, \dots, u_a$ and introducing the edge $v_{b-a+1}u_i$ $(1 \leq i \leq a)$. The graph is shown in Figure 3.1.

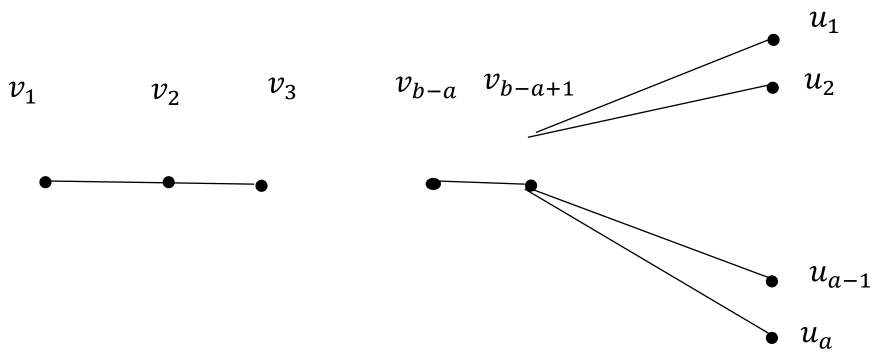


Figure 3.1

First, we prove that $cdim(G) = a$. Let $Z = \{u_1, u_2, \dots, u_a\}$. Then every circular resolving set of $G$ contains atleast $a-1$ vertices from $Z$ and the vertex $v_1$ and so $cdim(G) \geq a-1+1 = a$. Let $S = Z \cup \{v_1\}$. Then $S$ is a circular resolving set of $G$ so that $cdim(G) = a$. Next, we prove that $cdim_c(G) = b$. By Observation 2.4(ii), $Z_1 = \{v_2, v_3, \dots, v_{b-a+1}\}$ is a subset of every connected circular resolving set of $G$. Also it is easily seen that every connected circular resolving set of $G$ contains atleast a-1 vertices from $Z$ and the vertex $v_1$ and so $cdim_c(G) = b-a+1+a-1 = b$. Let $S_1 = S \cup Z_1$. Then $S_1$ is a connected circular resolving set of $G$ so that $cdim_c(G) = b$.

## Conclusion

This article established a novel circular distance metric called the connected circular metric dimension in graphs. We will develop this concept to incorporate more distance considerations in a subsequent investigation.

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