SOME OPERATORS ON P*GB CLOSED SETS

Aruna Glory Sudha[1]*, S. Zion Chella Ruth[2]

Abstract
T. Selvi and A. PunithaDharani [3] introduced pre*-closed sets and investigated some of their properties.This paperis devoted to p*gb Border, p*gb Frontier and p*gbExterior of a subset of a topological space. We investigate the fundamental properties of the above speculations and explore the inner relationship between them.

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[1]*Research Scholar, Department of Mathematics, Nazareth Margoschis College at Pillaiyanmanai, Nazareth. Registration Number: 21212152092002 (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India.)
[2]Assistant Professor, Department of Mathematics, Nazareth Margoschis College at Pillaiyanmanai, Nazareth (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India)

***Corresponding author:** Aruna Glory Sudha
*Research Scholar, Department of Mathematics, Nazareth Margoschis College at Pillaiyanmanai, Nazareth. Registration Number: 21212152092002 (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India.)

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1. INTRODUCTION

The concept of generalized Closed sets introduced by Levine [2] plays a significant role in General Topology. In 2012, T. Selvi and A. Punitha Dharani [3] introduced pre*-closed sets and investigated some of their properties. The characterizations of pre*-generalized b-closed sets and pre*-generalized b-open sets are given in [4]. This paperis devoted to p*gb Border, p*gb Frontier and p*gb Exterior of a subset of a topological space. We investigate the fundamental properties of the above speculations and explore the inner relationship between them.

2.PRELIMINARIES

Throughout this paper (X, τ) represent a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no changes of confusion. We recall the following definitions and results.

Definition 2.1.[1] Let (X, τ) be a topological space. A subset A of the space X is said to be b-open if A⊆int(cl(A))∪cl(int(A)) and b-closed if int(cl(A))∩cl(int(A)) ⊆A.

Definition 2.2.[1] Let (X, τ) be a topological space and A⊆X. The b-closure of A, denoted by bcl(A) and is defined by the intersection of all b-closed sets containing A.

Definition 2.3.[2] Let (X, τ) be a topological space. A subset A of X is said to be generalized closed (briefly g-closed) if cl(A)⊆U whenever A ⊆U and U is open in (X, τ). The complements of the above mentioned g closed set is g open set.

Definition 2.4. Let A be a subset of a topological space (X, τ). Then the union of all g-open sets contained in A is called the g-interior of A and it is denoted by int*(A). That is, int*(A)=∪{V:V⊆A and V∈g-O(X)}.

Definition 2.5. Let A be a subset of a topological space (X, τ). Then the intersection of all g-closed sets in X containing A is called the g-closure of A and it is denoted by cl*(A). That is, cl*(A)=∩{F: A⊆F and F∈g-C(X)}.

Definition 2.6. [3] Let (X, τ) be a topological space. A subset A of the space X is said to be pre*-open if A⊆ int*(cl(A)) and pre*-closed if cl*(int(A))⊆A.

Definition 2.7.[4] A subset A of a topological space (X, τ) is called a pre* generalized b-closed set (briefly, p*gb-closed) if bcl(A)⊆U whenever A⊆U and U is pre*-open in (X, τ).

Lemma 2.8.[4]For a topological space (X, τ), Every open set is p*gb-open.

Lemma 2.9. [4]

(a) Arbitrary intersection of p*gb-closed sets is p*gb-closed.
(b) Arbitrary union of p*gb-open sets is p*gb-open.

Remark 2.10.[4]

(a) The union of p*gb-closed sets need not be a p*gb-closed set.
(b) The intersection of p*gb-open sets is p*gb-open.

Definition 2.11.[5] Let X be a topological space and let x∈X. A subset N of X is said to be a p*gb-neighbourhood (shortly, p*gb-nbhd) of x if there exists a p*gb-open set U such that x∈U⊆N.

Theorem 2.12.[5] Every nbhd N of x∈X is a p*gb-nbhd of x.

Definition 2.13.[5] Let A be a subset of a topological space (X, τ). Then the union of all p*gb-open sets contained in A is called the p*gb-interior of A and it is denoted by p*gbint(A). That is, p*gbint(A)=∪{V:V⊆A and V∈p*gb-O(X)}.

Theorem 2.14.[5] Let A be a subset of a topological space (X, τ). Then

(a) p*gbint(A) is the largest p*gb-open set contained in A.
(b) A is p*gb-open if and only if p*gbint(A)=A.
(c) p*gbint(ϕ)= ϕ and p*gbint(X)=X.
(d) If A⊆B, then p*gbint(A)⊆p*gbint(B).
(e) p*gbint(p*gbint(A))=p*gbint(A).

Definition 2.15. [5] Let A be a subset of a topological space (X, τ). Then the intersection of all p*gb-closed sets in X containing A is called the p*gb-closure of A and it is denoted by p*gbcl(A). That is, p*gbcl(A)= ∩ {F:A⊆F and F∈p*gb-C(X)}. The intersection of p*gb-closed set is p*gb-closed, then p*gbcl(A) is p*gb-closed.

Theorem 2.16.[5] Let A be a subset of a topological space (X, τ). Then

(a) p*gbcl(A) is the smallest p*gb-closed set containing A.
(b) A is p*gb-closed if and only if p*gbcl(A)=A.

(c) p*gbcl(ϕ)= $\phi$and p*gbcl(X)=X.
(d) If A⊆B, then p*gbcl(A)⊆p*gbcl(B).
(e) p*gbcl(p*gbcl(A))=p*gbcl(A).

3.p*gb-border and p*gb-frontier

Definition 3.1. Let A be a subset of X. Then the set B_{p*gb}(A)=A\p*gbint(A) is called the p*gb-border of A. The set Fr_{p*gb}(A)=p*gbcl(A) \p*gbint(A) is called the p*gb-frontier of A.

Example 3.2. Let X= {a,b,c} with τ = {X, ϕ, {a},{a,b}}. Here, the p*gb closed sets are ={X, ϕ, {b},{c},{b,c}}.
Let A={a,c}. Then B_{p*gb}(A)=A\p*gbint(A) ={a,c}\{a,c}={ ϕ }and Fr_{p*gb}(A)=p*gbcl(A) \p*gbint(A) = X \{a,c}={b}.

Theorem 3.3. If a subset A of X is p*gb-closed, then B_{p*gb}(A)=Fr_{p*gb}(A).
Proof: Let A be a p*gb-closed subset of X. Then by Theorem 2.16, p*gbcl(A)=A. Now, Fr_{p*gb}(A)=p*gbcl(A)\p*gbint(A)=A\p*gbint(A)=B_{p*gb}(A).

Theorem 3.4. For a subset A of X, A=p*gbint(A)∪B_{p*gb}(A)
Proof: Let x∈A. If x∈p*gbint(A), then the result is obvious. If x∉p*gbint(A), then by the definition of B_{p*gb}(A), x ∈ B_{p*gb}(A). Hence x ∈ p*gbint (A)∪B_{p*gb}(A) and so A⊆p*gbint(A)∪B_{p*gb}(A). On the other hand, since p*gbint(A)⊆A and B_{p*gb}(A)⊆A, then we have p*gbint(A) ∪ B_{p*gb} (A)⊆A. This proves (i).

Theorem 3.5. For a subset A of X, p*gbint(A)∩B_{p*gb}(A)= ϕ
Proof. Suppose p*gbint(A) ∩ B_{p*gb}(A) ≠ ϕ. Let x∈p*gbint(A)∩B_{p*gb}(A). Then x∈p*gbint(A) and x∈B_{p*gb}(A). Since B_{p*gb}(A)=A\p*gbint(A), then x∈A. But x∈p*gbint(A), x∈A. A contradiction exists. Hence p*gbint(A)∩B_{p*gb}(A)= ϕ

Theorem 3.6 For a subset A of X, A is a p*gb-open set if and only if B_{p*gb}(A)= ϕ
Proof. Necessity: Let A be p*gb-open. Then by Theorem 3.4, p*gbint(A)=A. Now, B_{p*gb}(A)=A\p*gbint(A)=A\A= ϕ . Sufficiency: Suppose B_{p*gb} (A)= ϕ. This implies, A\p*gbint(A)= ϕ. Therefore A=p*gbint(A) and hence A is p*gb-open.

Theorem 3.7. For a subset A of X, B_{p*gb}(p*gbint(A))= ϕ.
By the definition of p*gb-border, B_{p*gb}(p*gbint (A))=p*gbint(A)\p*gbint(p*gbint(A)). By Theorem 3.4, p*gbint(p*gbint(A))=p*gbint(A) and hence B_{p*gb}(p*gbint(A))= ϕ.

Theorem 3.8. For a subset A of X, p*gbint(B_{p*gb}(A))= ϕ.
Proof. Let x ∈ p*gbint(B_{p*gb}(A)). Since B_{p*gb}(A)⊆A, by Theorem 3.5, p*gbint (B_{p*gb}(A))⊆p*gbint(A). Hence x ∈ p*gbint(A). Since p*gbint(B_{p*gb}(A))⊆B_{p*gb}(A), then x∈B_{p*gb} (A). Therefore x∈p*gbint(A)∩B_{p*gb}(A). By part (ii), x= ϕ.

Theorem 3.9. For a subset A of X, B_{p*gb} (B_{p*gb}(A))=B_{p*gb}(A)
Proof. By the definition of p*gb-border, B_{p*gb}(B_{p*gb}(A)) =B_{p*gb}(A)\p*gbint(B_{p*gb}(A)). By part (v), p*gbint(B_{p*gb}(A))= ϕ and hence B_{p*gb}(B_{p*gb}(A))=B_{p*gb}(A).

Corollary 3.10. For a topological space, B_{p*gb}(ϕ)= ϕ and B_{p*gb}(X)= ϕ.
Proof: As ϕ and X are p*gb-open, the aforementioned theorem B_{p*gb}(ϕ)= ϕ and B_{p*gb}(X)= ϕ.

Theorem 3.11. For a subset A of a space and X, the following statements are equivalent
(a) A is p*gb-open
(b) A=p*gbint(A)
(c) B_{p*gb}(A)=ϕ.

Proof: (a)→(b)Obvious from Theorem 2.14(b).
(b)→(c). Suppose that A=p*gbint(A). Then by Definition, B_{p*gb}(A)= p*gbint(A)\p*gbint(A)=ϕ
(c)→(a). Let B_{p*gb}(A)=ϕ. Then by Definition, A\p*gbint(A)=ϕ and hence A=p*gbint(A).

Theorem 3.12. Let A be a subset of X. Then, B_{p*gb}(A)=A∩p*gbcl(X\A)
Proof:Since B_{p*gb}(A)=A\p*gbint(A) and since p*gbint(A)=X\p*gbcl(X\A),
B_{p*gb}(A)=A\(X\p*gbcl(X\A))
=A∩(X\(X\p*gbcl(X\A))=A∩p*gbcl(X\A). This proves B_{p*gb}(A)=A∩p*gbcl(X\A).

Theorem 3.13. Let A be a subset of X. Then A is p*gb-closed if and only if Fr_{p*gb}(A)⊆A.
Proof: Necessity: Suppose A is p*gb-closed. Then by Theorem 2.16, p*gbcl(A)=A. Now, Fr_{p*gb}(A)=p*gbcl(A)\p*gbint(A)=A\p*gbint(A)⊆ A. Hence Fr_{p*gb}(A)⊆A. Sufficiency: Assume that, Fr_{p*gb}(A)⊆A. Then p*gbcl(A)\p*gbint(A)⊆A. Since p*gbint(A)⊆A, then we conclude that p*gbcl(A)⊆A. Also A⊆p*gbcl(A). Therefore p*gbcl(A)=A and hence A is p*gb-closed.

Theorem 3.14. For a subset A of X, p*gbcl(A)=p*gbint(A)∪Fr_{p*gb}(A).

Proof:

Since p*gbint(A)⊆p*gbcl(A) and Fr_{p*gb}(A) ⊆p*gbcl(A), then p*gbint(A) ∪ Fr_{p*gb} (A)⊆p*gbcl(A). Let x ∈ p*gbcl(A). Suppose x∉Fr_{p*gb}(A). Since, then x ∈ p*gbint(A). Hence x ∈ p*gbint(A) ∪ Fr_{p*gb}(A) and hence p*gbcl(A)⊆p*gbint(A)∪Fr_{p*gb}(A).

Theorem 3.15. For a subset A of X, p*gbint(A)∩Fr_{p*gb}(A)= ϕ.

Proof. Suppose p*gbint(A) ∩ Fr_{p*gb}(A)≠ ϕ. Let x∈p*gbint(A)∩Fr_{p*gb}(A). Then x∈p*gbint(A) and x∈Fr_{p*gb}(A), which is impossible to x belongs to both p*gbint(A) and Fr_{p*gb}(A), sinceFr_{p*gb} (A)=p*gbcl(A)\p*gbint(A). Hence p*gbint(A) ∩ Fr_{p*gb} (A)= ϕ.

Theorem 3.16. For a subset A of X, B_{p*gb}(A)⊆Fr_{p*gb}(A).

Proof. Since A⊆p*gbcl(A), then A\p*gbint(A) ⊆p*gbcl(A)\p*gbint(A). That implies, B_{p*gb}(A) ⊆Fr_{p*gb}(A).

4.p*gb-Exterior

Definition 4.1. Let A be a subset of a topological space (X, τ). The p*gb-interior of X\A is called the p*gb-exterior of A and it is denoted by Ext_{p*gb}(A). That is, Ext_{p*gb}(A)=p*gbint(X\A).

Example 4.2. Let X= {a,b,c} with τ = {X, ϕ, {a},{a,b}}. Here, the p*gb closed sets are ={X, ϕ, {b},{c},{b,c}}.

Let A={a,c}.Then Ext_{p*gb}(A)=p*gbint(X\A) = p*gbint({b}) = ϕ.

Theorem 4.3. For a subsets A and B of X, the followings are valid.

(i) Ext_{p*gb}(A)=X\p*gbcl(A).

(ii) Ext_{p*gb}(Ext_{p*gb} (A))=p*gbint(p*gbcl(A))⊇p*gbint(A).

(iii) If A⊆B, then Ext_{p*gb} (B)⊆ Ext_{p*gb} (A).

(iv) Ext_{p*gb} (A∪B)_Ext_{p*gb} (A)∩Ext_{p*gb} (B).

(v) Ext_{p*gb}(A∩B)_Ext_{p*gb} (A)∪Ext_{p*gb} (B).

(vi) Ext_{p*gb}(X)= $\phi$and Ext_{p*gb} (ϕ)=X.

(vii)Ext_{p*gb}(A)=Ext_{p*gb}(X\Ext_{p*gb}(A)).

Proof.

(i) We know that, X\p*gbcl(A)=p*gbint(X\A), then Ext_{p*gb}(A)=p*gbint(X\A)=X\p*gbcl(A).

(ii) Now, Ext_{p*gb}(Ext_{p*gb}(A))=Ext_{p*gb}(p*gbint(X\A)) =p*gbint(X\p*gbint(X\A)) =p*gbint(p*gbcl(A)) ⊇p*gbint(A).

(iii) Suppose A ⊆ B. Now, Ext_{p*gb}(B)=p*gbint(X\B)⊆p*gbint(X\A)=Ext $_{p*gb}$(A). This proves (iii).

(iv) Ext_{p*gb}(A∪B)=p*gbint(X\(A∪B))=p*gbint((X\A)∩(X\B))⊆p*gb(X\A)∩p*gbcl(X\B)=Ex t_{p*gb}(A)∩Ext_{p*gb} (B).

(v) Ext_{p*gb} (A∩B)=p*gbint(X\(A∩B))= p*gbint((X\A)∪(X\B))⊇p*gb(X\A)∪p*gbcl(X\B)=Ext_{p*gb}(A)∪Ext_{p*gb} (B).

(vi) Now, Ext_{p*gb}(X)=p*gbint(X\X)=p*gbint(ϕ) and Ext_{p*gb}(ϕ)= p*gbint(X\ ϕ)=p*gbint(X). Since ϕ and X are p*gb-open sets, then p*gbint(ϕ)= ϕ and p*gbint(X)=X. Hence Ext_{p*gb}(ϕ)=X and Ext_{p*gb}(X)= ϕ.

(vii) Now, Ext_{p*gb}(X\Ext_{p*gb}(A))=Ext_{p*gb}(X\p*gbint(X\A))=p*gbint(X\(X\p*gbint(X\A)))=p*gbint(p* gbint(X\A)) =p*gbint(X\A)=Ext_{p*gb}(A). This proves (vii).

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