ANTI SKOLEM MEAN LABELING OF TRIANGULAR SNAKE GRAPHS

B. Belsiya[1*], Dr. S. Alice Pappa[2]

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Abstract

A graph G = (V, E) with p vertices and q edges where $p < q + 1$ is said to be an Anti Skolem Mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from $\{1, 2, \ldots, q + 1\}$ in such a way that when each edge e = uv is labeled with $f(e = uv) = \frac{f(u)+ f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd then the resulting edges are distinct labels from the set $\{2, 3, \ldots, p\}$. In this case f is called an Anti Skolem Mean labeling of G. In this paper, we prove that Triangular Snake Graphs are Anti Skolem Mean graphs.

Keywords: Anti Skolem Mean Graph, Anti Skolem Mean labeling, Triangular Snake Graphs.

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[1]Research Scholar, Reg No. 19122142092004 Department of Mathematics, Nazareth Margoschis College, Pillayanmanai, (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627012, Tamil Nadu, INDIA)
[2]Department of Mathematics, Nazareth Margoschis College, Pillayanmanai, Nazareth – 628 617, Tamil Nadu, INDIA, (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627012, Tamil Nadu, INDIA.

*__Corresponding Author:__
B. Belsiya[1*]
[1]Research Scholar, Reg No. 19122142092004 Department of Mathematics, Nazareth Margoschis College, Pillayanmanai, (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627012, Tamil Nadu, INDIA)

Email: belsiyapersiah@gmail.com

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1. Introduction

Anti Skolem Mean graphs are finite, simple and undirected graph without loops or parallel edges. Detailed survey for all graph labeling we refer to Gallian [1]. For all other standard terminology and notations, we follow Harary [2]. The concept of Skolem Mean Labeling was introduced by A. Subramanian, D. S.T. Ramesh and V. Balaji [4]. We already investigate the new concept Anti Skolem Mean labeling of cycle related graphs. In this paper we investigate the Anti Skolem Mean labeling of Triangular Snake Graphs.

1.1 Definition

A graph $G=(V,E)$ with p vertices and q edges where $p<q+1$ is said to be anAnti Skolem Mean graph if it is possible to label the vertices $x\in V$ with distinct labels $f(x)$ from $\{1,2,...,q+1\}$ in such a way that when each edge $e=uv$ is labeled with $f(e=uv)=\frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd then the resulting edges get distinct labels from the set $\{2,3,...,p\}$.

1.2 Result

The only graph satisfies the condition $p<q+1$ are cycle related graphs.

1.3 Definition

A Triangular Snake graph T_n is attained from a path $a_1,a_2,...,a_n$ by joining a_i and a_{i+1} to a vertex b_i for $1\le i\le n-1$.

1.4 Definition

A Double triangular Snake graph $D(T_n)$ is attained from a path $a_1,a_2,...,a_n$ by joining a_i and a_{i+1} to a different vertices b_i, c_i, $1\le i\le n-1$.

1.5 Definition

A Triple Triangular Snake graph $T(T_n)$ is attained from a path $a_1,a_2,...,a_n$ by joining a_i and a_{i+1} to a different vertices b_i, c_i, d_i, $1\le i\le n-1$.

1.6 Definition

A Four Triangular Snake graph $F(T_n)$ is attained from a path $a_1,a_2,...,a_n$ by joining a_i and a_{i+1} to four different vertices b_i, c_i, d_i and e_i, $1\le i\le n-1$.

Main Results

Theorem

Triangular Snake graph T_n is an Anti Skolem Mean graph.

Proof:

Let the vertices of path P_n be $a_1,a_2,...,a_n$.

Let T_n be the Triangular Snake graph attained from the path P_n by joining a_i and a_{i+1} to different vertices v_i, $1\le i\le n-1$.

Define a function $f:V(T_n)\to\{1,2,...,q+1\}$ by

$f(a_i)=3i-2, 1\le i\le n,$

$f(b_i)=3i, 1\le i\le n-1.$

Then the edges are labeled with

$f(a_ia_{i+1})=3i, 1\le i\le n-1,$

$f(a_ib_i)=3i-1, 1\le i\le n-1,$

$f(a_{i+1}b_i)=3i+2, 1\le i\le n-1.$

Then the edges are labeled with distinct labeling. Hence, the Triangular Snake graph T_n is an Anti Skolem Mean graph.

Example Anti Skolem Mean labeling of Triangular Snake graph T_6 is given below.

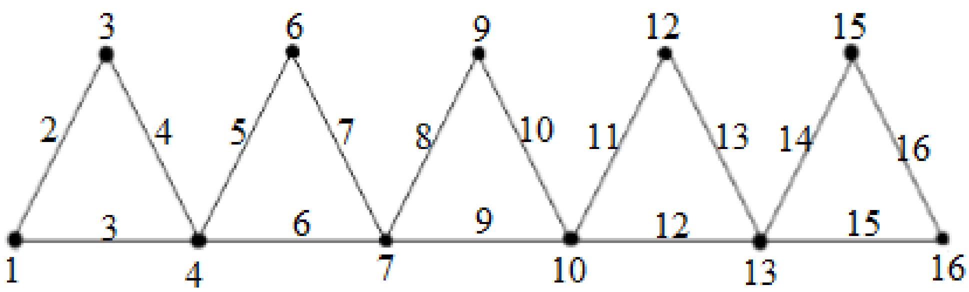

Fig. 1.Triangular Snake graph T_6

Theorem
The Double triangular Snake graph $D(T_n)$ is an Anti Skolem Mean graph.

Proof:
Let the vertices of path P_n be $a_1, a_2, ..., a_n$.
Let $D(T_n)$ be the Double Triangular Snake graph attained from the path P_n by joining a_i and a_{i+1} to two different vertices b_i, c_i, $1 \le i \le n-1$.
Define a function $f : V(D(T_n)) \to \{1,2,...,q+1\}$ by
$f(a_i) = 5i-4, 1 \le i \le n,$
$f(b_i) = 5i-2, 1 \le i \le n-1,$
$f(c_i) = 5i, 1 \le i \le n-1.$
Then the edges are labeled with
$f(a_i a_{i+1}) = 5i-1, 1 \le i \le n-1,$
$f(a_i b_i) = 5i-3, 1 \le i \le n-1,$
$f(a_{i+1} b_i) = 5i, 1 \le i \le n-1,$
$f(a_i c_i) = 5i-2, 1 \le i \le n-1,$
$f(a_{i+1} c_i) = 5i+1, 1 \le i \le n-1.$
Then the edges are labeled with distinct labeling.
Hence, the Double Triangular Snake $D(T_n)$ is an Anti Skolem Mean graph.
Example Anti Skolem Mean labeling of Double Triangular Snake $D(T_6)$ is given below.

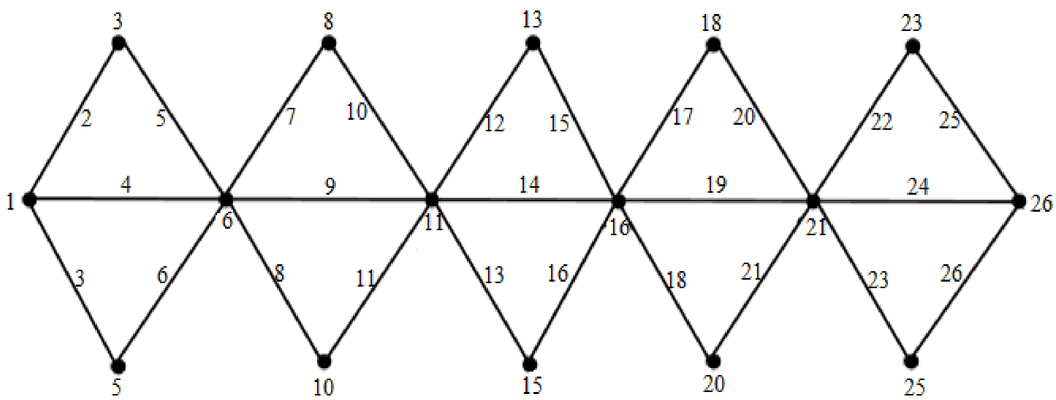

Fig. 2.Double Triangular Snake $D(T_6)$

Theorem
The Triple Triangular Snake $T(T_n)$ is an Anti Skolem Mean graph.
Proof:
Let the vertices of path P_n be $a_1, a_2, ..., a_n$.
Let $T(T_n)$ be the Triple Triangular Snake graph attained from the path P_n by joining a_i and a_{i+1} to three different vertices b_i, c_i, d_i, $1 \le i \le n-1$.
Define a function $f : V(T(T_n)) \to \{1,2,...,q+1\}$ by
$f(a_i) = 7i-6, 1 \le i \le n,$
$f(b_i) = 7i-2, 1 \le i \le n-1,$
$f(c_i) = 7i-4, 1 \le i \le n-1,$
$f(d_i) = 7i, 1 \le i \le n-1.$
Then the edges are labeled with
$f(a_i a_{i+1}) = 7i-2, 1 \le i \le n-1,$
$f(a_i b_i) = 7i-4, 1 \le i \le n-1,$
$f(a_{i+1} b_i) = 7i, 1 \le i \le n-1,$
$f(a_i c_i) = 7i-5, 1 \le i \le n-1,$
$f(a_{i+1} c_i) = 7i-1, 1 \le i \le n-1,$
$f(a_i d_i) = 7i-3, 1 \le i \le n-1,$
$f(a_{i+1} d_i) = 7i+1, 1 \le i \le n-1,$
Then the edges are labeled with distinct labeling.
Hence, the Triple Triangular Snake $T(T_n)$ is an Anti Skolem Mean graph.

Example Anti Skolem Mean labeling of Triple Triangular Snake $T(T_4)$ is given below.

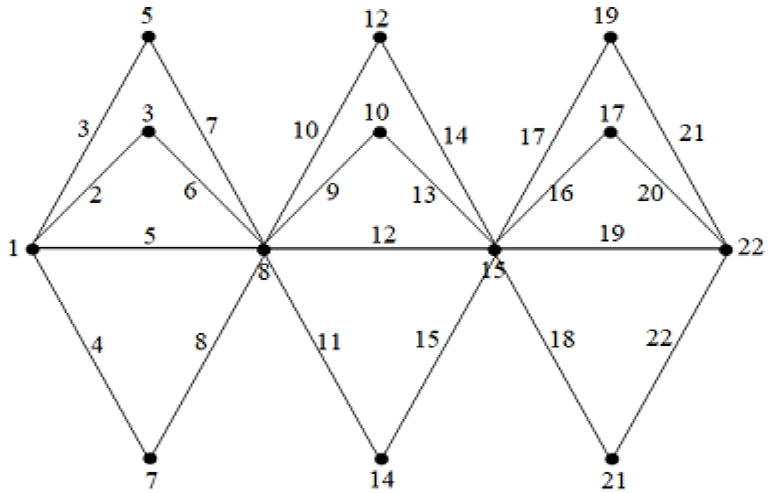

Fig. 3.Triple Triangular Snake $T(T_4)$

Theorem

Four Triangular Snake $F(T_n)$ is an Anti Skolem Mean graph.

Proof:

Let the vertices of path P_n be $a_1, a_2, ..., a_n$.

Let $F(T_n)$ be the Four Triangular Snake graph attained from the path P_n by joining a_i and a_{i+1} to three different vertices b_i, c_i, d_i and e_i, $1 \le i \le n-1$.

Define a function $f: V(F(T_n)) \to \{1,2,...,q+1\}$ by

$f(a_i) = 9i - 8, 1 \le i \le n,$

$f(b_i) = 9i - 4, 1 \le i \le n-1,$

$f(c_i) = 9i - 6, 1 \le i \le n-1,$

$f(d_i) = 9i - 2, 1 \le i \le n-1,$

$f(e_i) = 9i, 1 \le i \le n-1.$

Then the edges are labeled with

$f(a_i a_{i+1}) = 9i - 3, 1 \le i \le n-1,$

$f(a_i b_i) = 9i - 6, 1 \le i \le n-1,$

$f(a_{i+1} b_i) = 9i - 1, 1 \le i \le n-1,$

$f(a_i c_i) = 9i - 7, 1 \le i \le n-1,$

$f(a_{i+1} c_i) = 9i - 2, 1 \le i \le n-1,$

$f(a_i d_i) = 9i - 5, 1 \le i \le n-1,$

$f(a_{i+1} d_i) = 9i, 1 \le i \le n-1,$

$f(a_i e_i) = 9i - 4, 1 \le i \le n-1,$

$f(a_{i+1} e_i) = 9i + 1, 1 \le i \le n-1.$

Then the edges are labeled with distinct labeling.

Hence, the Four Triangular Snake $F(T_n)$ is an Anti Skolem Mean graph.

Example Anti Skolem Mean labeling of Four Triangular Snake $F(T_4)$ is given below.

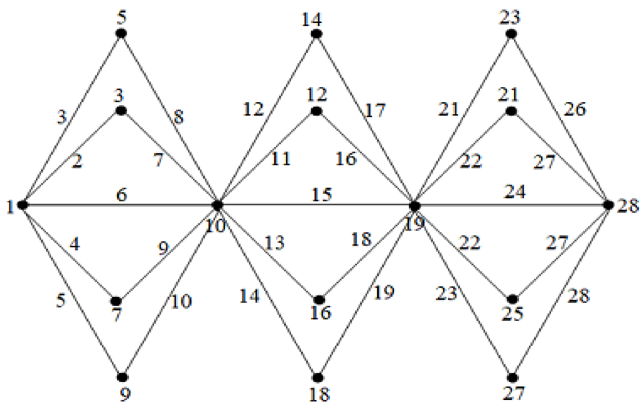

Fig. 4.Four Triangular Snake $F(T_4)$

2. Conclusion

Anti Skolem Mean graphs are not satisfies all graphs and the investigation of graphs are very interesting which satisfiesAnti Skolem Mean labeling. Anti Skolem Mean Labeling of Triangular Snake graphs areproved in this paper. We have already investigated cyclerelated graphs are Anti Skolem Mean Graphs and we planned to investigate that Quadrilateral Snake graphs are Anti Skolem Mean graphs.

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