TOPOLOGICAL SPACE GENERATED BY EDGES NEIGHBORHOODS OF DISCRETE TOPOLOGICAL GRAPHS

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Abstract

The present work aims is to convert the topological graph to a topology by neighborhoods of the graph edges. The resulting topology is proved as a discrete topology. A new definition for the subbase is derived and denoted by NS_{G_τ}. It contains all sets of the edges neighborhoods. The base NB_{G_τ} is extracted from the intersection of all elements of NS_{G_τ}. Then, the neighborhood topology $N\tau_{G_\tau}$ is extracted from the union of all elements of NB_{G_τ} with some examples.

Keywords: Topological graph, discrete topology, neighborhood topology.
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1. INTRODUCTION

Graph Theory is the well-recognized area of discrete mathematics that deals with the study of graphs. The graphs considered here are finite, simple, and undirected. A graph $G = (V, E)$ with vertex set $V(G)$ and edge set $E(G)$. For each vertex $v \in V(G)$, the set $N_G(V) = \{u \in V \backslash uv \in E\}$ refers to the open neighborhood of v in G. See [1-19, 21, 32-37] for details of graph theoretic terminology and its applications. The discrete topology is denoted by (X, τ), where X is a non-empty set and τ is a family of all subsets of X, where $\tau = P(X)$. The sets X and $\emptyset$ belong to τ, and both are open sets. The set $\mathcal{B} \subseteq \tau$ is called a base for τ if every open set in τ is a union of members of $\mathcal{B}$ [30]. A set $\sigma \subseteq \tau$ is called a subbase of τ if every open set in $\mathcal{B}$ is a finite intersection of elements of σ. Let $\{M_i;\ i \in I\}$ be a family of the subset of X where if $I = \emptyset$, then $\bigcup_{i\in \mathrm{I}} M_i = \emptyset$ and $\bigcap_{i\in \mathrm{I}} M_i = X$ [35]. Many papers joined graph theory and topology, see [20, 23-29]. In this work, converting the topological graph to a discrete topology by adjacent edges are studied. A new definition of subbase is introduced, containing all sets of the edges neighborhoods. The base is extracted from the intersection of all elements of the subbase. After that, the neighborhood (discrete) topology is extracted from the base elements with some examples.

2. Definition and Properties of Topological Graph

In this section, many properties that proved by authors in [22] for the discrete topological graph G_τ are given.

Definition 2.1 Let X be a non-empty set and τ be a discrete topological space. A discrete topological graph $G_\tau = (V, E)$ is a graph of the vertex set $V(G_\tau) = \tau - \{\emptyset, X\}$ and the edge set by $E(G_\tau) = \{A\,B;\ A \subset B\}$.

Proposition 2.2: Let G_τ be a discrete topological graph on X, where $|X| = 2$, then $G_\tau \cong N_2$.

Corollary 2.3: Let G_τ be a discrete topological graph on X, where $|X| = 3$. Then, $G_\tau \cong K_{3,3}$.

Proposition 2.4: Let G_τ be a discrete topological graph on X, where $|X| = 4$, then $G_\tau \cong K_{4,6,4}$.

Example 2.5: let $|X| = 5$, then $\tau =$
$$\left\{\begin{matrix} \emptyset, X, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\} \\ \{3,5\}, \{4,5\}, \{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,3,4\}, \{1,3,5\}, \{1,4,5\}, \{2,3,4\}, \{2,3,5\} \\ \{2,4,5\}, \{3,4,5\}, \{1,2,3,4\}, \{1,2,3,5\}, \{1,2,4,5\}, \{1,3,4,5\}, \{2,3,4,5\} \end{matrix}\right\}$$, so $V =$
$$\left\{\begin{matrix} \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\}, \{3,5\}, \\ \{4,5\}, \{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,3,4\}, \{1,3,5\}, \{1,4,5\}, \{2,3,4\}, \{2,3,5\}, \{2,4,5\} \\ \{3,4,5\}, \{1,2,3,4\}, \{1,2,3,5\}, \{1,2,4,5\}, \{1,3,4,5\}, \{2,3,4,5\} \end{matrix}\right\}$$

Proposition 2.6: Let G_τ be a discrete topological graph on a non-empty set X, where $|X| = n$. Then $|G_\tau| = 2^n - 2$.

3. Topological Space Generated by Topological Graph

In this section, converting the topological graph to a discrete topology are studied.

Definition 3.1. Let G_τ be a discrete topological graph. The neighborhoods of an edge e_i for any $e_i \in E(G_\tau)$ defined as:
$N(e_i) = \{e_j \in E(G_\tau)\colon\ e_j$ adjacent with e_i where $i \neq j\}$. Let NS_{G_τ} be a collection of all neighborhoods of E whose union equals E, such that $NS_{G_\tau}(E) = \{N(e_i)\}_{e_i \in E(G_\tau)}$.

Definition 3.2. Let NB_{G_τ} be a basis generated by finite intersection of members of $NS_{G_\tau}(E)$. Where it is defined as follows:
$NB_{G_\tau}(E) = \{A; A \subseteq E, A$ is a finite intersection of members of $NS_{G_\tau}\}$.

Definition 3.3. The topology $N\tau_{G_\tau}$ on a set E which is generated by NB_{G_τ} is called neighborhood topology of a graph G_τ.

Example 3.4. Let G_τ be a topological graph for $|X| = 2$. We find the neighborhood topology $N\tau_{G_\tau}$ of G_τ. Since the graph is null, there is no edge neighborhood. Hence, $N\tau_{G_\tau}$ is not topology on E. See Figure 1.

Figure 1: Discrete topological graph N_2 .

Example 3.5. Let G_τ be a topological graph for $|X| = 3$. Thus, we extract the neighborhood topology $N\tau_{\,G_\tau}$ of topological graph G_τ.
Let $E(\,G_\tau\,) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.
$N(e_1) = \{e_2, e_6\}$, $N(e_2) = \{e_1, e_3\}$, $N(e_3) = \{e_2, e_4\}$,
$N(e_4) = \{e_3, e_5\}$, $N(e_5) = \{e_4, e_6\}$, $N(e_6) = \{e_1, e_5\}$,
$NS_{G_\tau}(E) = \{\{e_2, e_6\}, \{e_1, e_3\}, \{e_2, e_4\}, \{e_3, e_5\}, \{e_4, e_6\}, \{e_1, e_5\}$,

By taking the intersection of sets of $NS_{G_\tau}(E)$ we get the base as:
$NB_{G_\tau}(E) = \{\emptyset, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_6\}, \{e_2, e_6\}, \{e_1, e_3\}, \{e_2, e_4\}$,
$\{e_3, e_5\}, \{e_4, e_6\}, \{e_1, e_5\}\}$,

By taking all unions. The neighborhood topology can be written as follows:
$N\tau_{G_\tau} = \{\emptyset, E\} \cup \{\{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_6\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_4\}$,
$\{e_1, e_5\}, \{e_1, e_6\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_2, e_5\}, \{e_2, e_6\}, \{e_3, e_4\}, \{e_3, e_5\}, \{e_3, e_6\}$
$, \{e_4, e_5\}, \{e_4, e_6\}, \{e_5, e_6\}, \{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_1, e_2, e_5\}, \{e_1, e_2, e_6\}$
$, \{e_1, e_3, e_4\}, \{e_1, e_3, e_5\}, \{e_1, e_3, e_6\}, \{e_1, e_4, e_5\}, \{e_1, e_4, e_6\}, \{e_1, e_5, e_6\}$
$, \{e_2, e_3, e_4\}, \{e_2, e_3, e_5\}, \{e_2, e_3, e_6\}, \{e_2, e_4, e_5\}, \{e_2, e_4, e_6\}, \{e_2, e_5, e_6\}$,
$\{e_3, e_4, e_5\}, \{e_3, e_4, e_6\}, \{e_3, e_5, e_6\}, \{e_4, e_5, e_6\}, \{e_1, e_2, e_3, e_4\}, \{e_1, e_2, e_3, e_5\}$,
$\{e_1, e_2, e_3, e_6\}, \{e_1, e_2, e_4, e_5\}, \{e_1, e_2, e_4, e_6\}, \{e_1, e_2, e_5, e_6\}, \{e_1, e_3, e_4, e_5\}$,
$\{e_1, e_3, e_4, e_6\}, \{e_1, e_3, e_5, e_6\}, \{e_1, e_4, e_5, e_6\}, \{e_2, e_3, e_4, e_5\}, \{e_2, e_3, e_4 e_6\}$
$, \{e_2, e_3, e_5, e_6\}, \{e_2, e_4, e_5, e_6\}, \{e_3, e_4, e_5, e_6\}, \{e_1, e_2, e_3, e_4, e_5\}$
$, \{e_1, e_2, e_3, e_4, e_6\}, \{e_1, e_2, e_3, e_5, e_6\}, \{e_1, e_2, e_4, e_5, e_6\}, \{e_1, e_3, e_4, e_5, e_6\}$,
$\{e_2, e_3, e_4, e_5, e_6\}\}$.

Hence, $N\tau_{G_\tau}$ is a discrete topology on $E(G_\tau)$. See Figure 2.

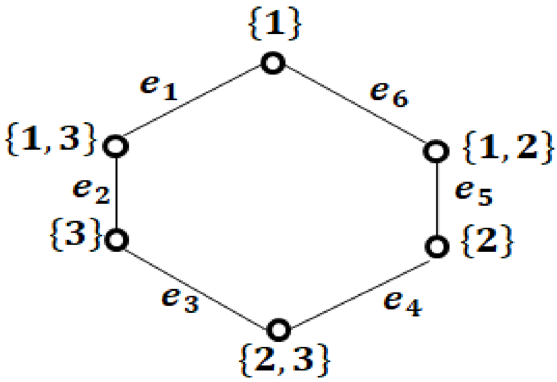

Figure 2: Discrete topological graph C_6 .

Example 3.6. Let G_τ be a topological graph for $|X| = 4$. We find the neighborhood topology $N\tau_{G_\tau}$ of the graph G_τ.

Let $E(G_\tau) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}, e_{21}, e_{22}, e_{23}, e_{24}, e_{25}, e_{26}, e_{27}, e_{28}, e_{29}, e_{30}, e_{31}, e_{32}, e_{33}, e_{34}, e_{35}, e_{36}\}$.

$N(e_1) = \{e_2, e_3, e_4, e_5, e_6, e_7, e_{25}, e_{26}\}$,
$N(e_2) = \{e_1, e_3, e_4, e_5, e_6, e_8, e_{14}, e_{25}, e_{27}, e_{31}\}$,
$N(e_3) = \{e_1, e_2, e_4, e_5, e_6, e_9, e_{20}, e_{26}, e_{29}, e_{33}\}$,
$N(e_4) = \{e_1, e_2, e_3, e_5, e_6, e_{13}, e_{27}, e_{28}\}$,
$N(e_5) = \{e_1, e_2, e_3, e_4, e_6, e_{15}, e_{21}, e_{28}, e_{30}, e_{35}\}$,
$N(e_6) = \{e_1, e_2, e_3, e_4, e_5, e_{19}, e_{29}, e_{30}\}$,
$N(e_7) = \{e_1, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{25}, e_{26}\}$,
$N(e_8) = \{e_2, e_7, e_9, e_{10}, e_{11}, e_{12}, e_{14}, e_{25}, e_{27}, e_{31}\}$,
$N(e_9) = \{e_3, e_7, e_8, e_{10}, e_{11}, e_{12}, e_{20}, e_{26}, e_{29}, e_{33}\}$,
$N(e_{10}) = \{e_7, e_8, e_9, e_{11}, e_{12}, e_{16}, e_{22}, e_{32}, e_{34}, e_{36}\}$,
$N(e_{11}) = \{e_7, e_8, e_9, e_{10}, e_{12}, e_{17}, e_{31}, e_{32}\}$,
$N(e_{12}) = \{e_7, e_8, e_9, e_{10}, e_{11}, e_{23}, e_{33}, e_{34}\}$,
$N(e_{13}) = \{e_1, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{27}, e_{28}\}$,
$N(e_{14}) = \{e_2, e_8, e_{13}, e_{15}, e_{16}, e_{17}, e_{18}, e_{25}, e_{27}, e_{31}\}$,
$N(e_{15}) = \{e_1, e_{13}, e_{14}, e_{16}, e_{17}, e_{18}, e_{21}, e_{28}, e_{30}, e_{35}\}$,
$N(e_{16}) = \{e_{10}, e_{13}, e_{14}, e_{15}, e_{17}, e_{18}, e_{22}, e_{32}, e_{34}, e_{36}\}$,
$N(e_{17}) = \{e_{11}, e_{13}, e_{14}, e_{15}, e_{16}, e_{18}, e_{31}, e_{32}\}$,
$N(e_{18}) = \{e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{24}, e_{35}, e_{36}\}$,
$N(e_{19}) = \{e_6, e_{20}, e_{21}, e_{22}, e_{23}, e_{24}, e_{29}, e_{30}\}$,
$N(e_{20}) = \{e_1, e_9, e_{19}, e_{21}, e_{22}, e_{23}, e_{24}, e_{26}, e_{29}, e_{33}\}$,
$N(e_{21}) = \{e_1, e_{15}, e_{19}, e_{20}, e_{22}, e_{23}, e_{24}, e_{28}, e_{30}, e_{35}\}$,
$N(e_{22}) = \{e_{10}, e_{16}, e_{19}, e_{20}, e_{21}, e_{23}, e_{24}, e_{32}, e_{34}, e_{36}\}$,
$N(e_{23}) = \{e_{12}, e_{19}, e_{20}, e_{21}, e_{22}, e_{23}, e_{24}, e_{33}, e_{34}\}$,
$N(e_{24}) = \{e_{18}, e_{19}, e_{20}, e_{21}, e_{22}, e_{23}, e_{35}, e_{36}\}$,
$N(e_{25}) = \{e_1, e_2, e_7, e_8, e_{14}, e_{26}, e_{27}, e_{31}\}$,
$N(e_{26}) = \{e_1, e_3, e_7, e_9, e_{20}, e_{25}, e_{29}, e_{33}\}$,
$N(e_{27}) = \{e_2, e_4, e_8, e_{13}, e_{14}, e_{25}, e_{28}, e_{31}\}$,
$N(e_{28}) = \{e_4, e_5, e_{13}, e_{15}, e_{21}, e_{27}, e_{30}, e_{35}\}$,
$N(e_{29}) = \{e_3, e_6, e_9, e_{19}, e_{20}, e_{26}, e_{30}, e_{33}\}$,
$N(e_{30}) = \{e_5, e_6, e_{15}, e_{19}, e_{21}, e_{28}, e_{29}, e_{35}\}$,

$N(e_{31}) = \{e_2, e_8, e_{11}, e_{17}, e_{25}, e_{27}, e_{28}, e_{32}\}$,
$N(e_{32}) = \{e_{10}, e_{11}, e_{17}, e_{16}, e_{22}, e_{31}, e_{34}, e_{36}\}$,
$N(e_{33}) = \{e_3, e_9, e_{12}, e_{20}, e_{23}, e_{26}, e_{29}, e_{34}\}$,
$N(e_{34}) = \{e_{10}, e_{12}, e_{16}, e_{22}, e_{23}, e_{32}, e_{33}, e_{36}\}$,
$N(e_{35}) = \{e_5, e_{15}, e_{18}, e_{21}, e_{24}, e_{28}, e_{30}, e_{36}\}$,
$N(e_{36}) = \{e_{10}, e_{16}, e_{18}, e_{22}, e_{24}, e_{32}, e_{34}, e_{35}\}$,
$NS_{G_\tau}(E) = \{\{e_2, e_3, e_4, e_5, e_6, e_7, e_{25}, e_{26}\}$,
$\{e_1, e_3, e_4, e_5, e_6, e_8, e_{14}, e_{25}, e_{27}, e_{31}\}$,
$\{e_1, e_2, e_4, e_5, e_6, e_9, e_{20}, e_{26}, e_{29}, e_{33}\}, \{e_1, e_2, e_3, e_5, e_6, e_{13}, e_{27}, e_{28}\}$,
$\{e_1, e_2, e_3, e_4, e_6, e_{15}, e_{21}, e_{28}, e_{30}, e_{35}\}, \{e_1, e_2, e_3, e_4, e_5, e_{19}, e_{29}, e_{30}\}$,
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$\{e_3, e_7, e_8, e_{10}, e_{11}, e_{12}, e_{20}, e_{26}, e_{29}, e_{33}\}$,
$\{e_7, e_8, e_9, e_{11}, e_{12}, e_{16}, e_{22}, e_{32}, e_{34}, e_{36}\}$,
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$\{e_1, e_{13}, e_{14}, e_{16}, e_{17}, e_{18}, e_{21}, e_{28}, e_{30}, e_{35}\}$,
$\{e_{10}, e_{13}, e_{14}, e_{15}, e_{17}, e_{18}, e_{22}, e_{32}, e_{34}, e_{36}\}$,
$\{e_{11}, e_{13}, e_{14}, e_{15}, e_{16}, e_{18}, e_{31}, e_{32}\}, \{e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{24}, e_{35}, e_{36}\}$,
$\{e_6, e_{20}, e_{21}, e_{22}, e_{23}, e_{24}, e_{29}, e_{30}\}, \{e_1, e_9, e_{19}, e_{21}, e_{22}, e_{23}, e_{24}, e_{26}, e_{29}, e_{33}\}$,
$\{e_1, e_{15}, e_{19}, e_{20}, e_{22}, e_{23}, e_{24}, e_{28}, e_{30}, e_{35}\}$,
$\{e_{10}, e_{16}, e_{19}, e_{20}, e_{21}, e_{23}, e_{24}, e_{32}, e_{34}, e_{36}\}$,
$\{e_{12}, e_{19}, e_{20}, e_{21}, e_{22}, e_{23}, e_{24}, e_{33}, e_{34}\}, \{e_{18}, e_{19}, e_{20}, e_{21}, e_{22}, e_{23}, e_{35}, e_{36}\}$,
$\{e_1, e_2, e_7, e_8, e_{14}, e_{26}, e_{27}, e_{31}\}, \{e_1, e_3, e_7, e_9, e_{20}, e_{25}, e_{29}, e_{33}\}$,
$\{e_2, e_4, e_8, e_{13}, e_{14}, e_{25}, e_{28}, e_{31}\}, \{e_4, e_5, e_{13}, e_{15}, e_{21}, e_{27}, e_{30}, e_{35}\}$,
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$\{e_2, e_8, e_{11}, e_{17}, e_{25}, e_{27}, e_{28}, e_{32}\}, \{e_{10}, e_{11}, e_{17}, e_{16}, e_{22}, e_{31}, e_{34}, e_{36}\}$,
$\{e_3, e_9, e_{12}, e_{20}, e_{23}, e_{26}, e_{29}, e_{34}\}, \{e_{10}, e_{12}, e_{16}, e_{22}, e_{23}, e_{32}, e_{33}, e_{36}\}$,
$\{e_5, e_{15}, e_{18}, e_{21}, e_{24}, e_{28}, e_{30}, e_{36}\}, \{e_{10}, e_{16}, e_{18}, e_{22}, e_{24}, e_{32}, e_{34}, e_{35}\}\}$.

we find NB_{G_τ}, and $N\tau_{G_\tau}$ by the same technique of example 3.5. So we get NB_{G_τ} which has all sets of singleton e_i where $\{e_i\} \in NB_{G_\tau}$, for all $i = 1,2,3, \dots ,36$. Since $N\tau_{G_\tau}$ is the union of all sets of NB_{G_τ}. Then, the number of all sets of $N\tau_{G_\tau}$ is 2^{36} and it is a discrete topology. See Figure 4.

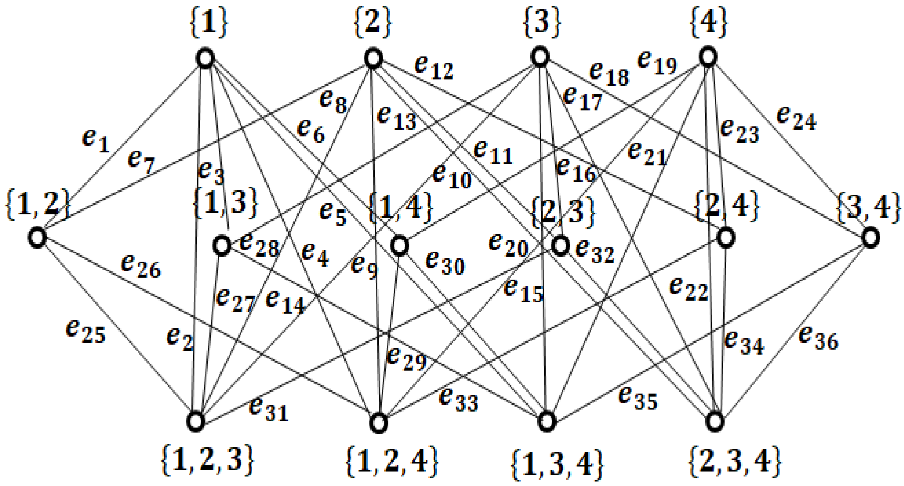

Figure 4: Discrete topology graph $K_{4,6,4}$.

Open problems

1- Converting the topological graph to the discrete topology by other ways, by the adjacent or non-adjacent edges or vertices.

2- Apply many types of domination parameters on the topological graph such as: Pitchfork domination, co-even domination, and co-independent domination.

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