# MATRIX REPRESENTATION OF FELICITOUS FUZZY GRAPHS 

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#### Abstract

In this paper, a new concept of Felicitous Fuzzy Matrix and Intuitionistic Felicitous Fuzzy matrix are introduced. And few new operations of Felicitous Fuzzy matrix are introduced. In addition, some findings about both the new operators and the current operators are presented.


Keywords: Felicitous Fuzzy Graphs (FFGs), Felicitous Fuzzy Matrix(FFG), Intuitionistic Felicitous Fuzzy Matrix(IFFM).
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## 1. Introduction

Atanassov,[1] introduced the concept of intuitionistic fuzzy sets, which is a generalization of fuzzy subsets. Since then, much research has been done with this concept by Atanasov etal. The term fuzzy matrix has an important role in fuzzy algebra. The definition of fuzzy matrix has been taken from Dubois and Prade [2]. This type of fuzzy matrices consist of applicable matrices which are modeled to represents uncertain aspects. Thomson [3] defined convergence of a square fuzzy matrix.
Pal and Shyamal [6] introduced two new operators on fuzzy matrices and proved several properties of them. From the concept of intuitionistic fuzzy set, Pal [5] introduced intuitionistic fuzzy determinant. Later on, Pal and Shyamal [6] introduced intuitionistic fuzzy matrices (IFMs) and distance between intuitionistic fuzzy
matrices. Bhowmik and Pal [7] presented some results on intuitionistic fuzzy matrices, intuitionistic circulant fuzzy matrices and intuitionistic fuzzy matrices.
The general rectangular or square array of the numbers is known as matrix and if the elements of it are intuitionistic fuzzy, then the matrix is called intuitionistic fuzzy matrix. If we delete some rows or some columns or both, then the intuitionistic fuzzy matrix is called intuitionistic fuzzy submatrix. The concept of non-empty subset in set theory and the principle of combination are applied for the construction and the calculation of the number of intuitionistic fuzzy submatrices of a given intuitionistic fuzzy matrix.
The stucture of this paper is organized as follows. In section 2, preliminaries and definitions are given and in section 3, different kinds of felicitous fuzzy matrix are given.

## 1. Preliminaries

## Definition 1.

A felicitous fuzzy matrix of order $n \times m$ is defined as $P=\left[p_{i j 1}\right]$, where $p_{i j n}$ is the membership value of the $\mathrm{ij}^{\text {th }}$ element in P satisfying the following conditions, given that $(\mathrm{V}, \mathrm{E})$ - graph G has a proper labeling $\mathrm{f}: \mathrm{V}(\mathrm{G}) \rightarrow\{\mathrm{E}+1, \mathrm{E}+2, \ldots, \mathrm{~V}+\mathrm{E}\}$, where V and E represent the number of nodes and the number of arcs respectively, then the arc label $\beta(u v)$ of each arc uv $\in E(G)$ is defined as $\mathrm{p}_{\mathrm{ij} \delta}=\left(\mathrm{p}_{\mathrm{ij} \eta}+\mathrm{p}_{\mathrm{ij} \eta}\right)(\bmod (\mathrm{P} * \mathrm{~h})$ ), where $\mathrm{h}=0.01$ if $3 \leq \mathrm{n} \leq 49$ and $\mathrm{h}=0.001$ if $50 \leq \mathrm{n} \leq$ 99 and so on. G is said to be felicitous fuzzy matrix such that $\left\{\mathrm{p}_{\mathrm{ij} \delta}\right.$ : uv $\left.\in \mathrm{E}(\mathrm{G})\right\}=[1, \mathrm{t}]$.

## Definition 2.

An intuitionistic felicitous fuzzy matrix of order $\mathrm{n} \times \mathrm{m}$ is defined as $\mathrm{P}=\left[\left\langle\mathrm{p}_{\mathrm{ij},}, \mathrm{q}_{\mathrm{ij} \delta}\right\rangle\right]$, where $\mathrm{p}_{\mathrm{ijn}}$ and $p_{i j \delta}$ are the membership and the non-membership values of the $\mathrm{ij}^{\text {th }}$ element in P satisfying the following conditions,
(i) An IFFG is of the form $G=(P, Q)$ where $P=V_{1}, V_{2}, \ldots, V_{n}$ such that $\eta_{P}: V \rightarrow[0,1]$ and $\delta_{P}: V \rightarrow$ $[0,1]$ denote the membership and the non-membership values of any element $v_{i} \in V$ respectively and $0 \leq \mathrm{p}_{\mathrm{ij} \eta}+\mathrm{p}_{\mathrm{ij} \delta} \leq 1$, if $\mathrm{v}_{\mathrm{i}} \in \mathrm{V}, \mathrm{i}=1,2, \ldots, \mathrm{n}$.
(ii) $\mathrm{E} \subseteq \mathrm{V} \times \mathrm{V}$ where $\mathrm{p}_{\mathrm{ikn}}: \mathrm{V} \times \mathrm{V} \rightarrow[0,1]$ for $\mathrm{p}_{\mathrm{ik} \delta}: \mathrm{V} \times \mathrm{V} \rightarrow[0,1]$ such that
$\mathrm{p}_{\mathrm{ij} \delta} \leq \min \left\{\mathrm{p}_{\mathrm{ik} n}, \mathrm{p}_{\mathrm{kj}}\right\}$
$\mathrm{P}_{\mathrm{ik} \delta} \leq \max \left\{\mathrm{p}_{\mathrm{ik} n}, \mathrm{p}_{\mathrm{k} \eta}\right\}$
$0 \leq p_{\mathrm{ijn}}+\mathrm{p}_{\mathrm{ij} \delta} \leq 1$, if $\left(\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{j}}\right) \in \mathrm{E}$ and $1 \leq \mathrm{i}, \mathrm{j}, \mathrm{k} \leq \mathrm{n}$.
(iii) G has a proper labeling $\mathrm{f}: \mathrm{V}(\mathrm{G}) \rightarrow\{\mathrm{Q}+1, \mathrm{Q}+2, \ldots, \mathrm{P}+\mathrm{Q}\}$ where P and Q represent the number of nodes and the number of arcs $u v \in E(G)$ defined as $p_{\mathrm{ij} \delta}=$
$\left(p_{\text {ikn }}+p_{k j \eta}\right)(\operatorname{modP} * h)$ and
$\mathrm{p}_{\mathrm{ik} \delta}=\left(\mathrm{p}_{\mathrm{ikn}}+\mathrm{p}_{\mathrm{kjn}}\right)(\operatorname{modP} * \mathrm{~h})$, where $\mathrm{h}=0.01$ if $3 \leq \mathrm{n} \leq 49$ and $\mathrm{h}=0.001$ if $50 \leq \mathrm{n} \leq 99$ and so on, then G is said to be intuitionistic felicitous fuzzy matrix, such that
$\left\{\mathrm{p}_{\mathrm{ij},}, \mathrm{p}_{\mathrm{ij} \delta}: \mathrm{uv} \in \mathrm{E}(\mathrm{G})\right\}=[1, \mathrm{P}]$

## Definition 3.

Let P and Q be two intuitionistic felicitous fuzzy matrices, such that $\mathrm{P}=\left[\left\langle\mathrm{p}_{\mathrm{ij},}, \mathrm{p}_{\mathrm{ij} \delta}\right\rangle\right]$ and $\left.\mathrm{Q}=\left[\left\langle\mathrm{q}_{\mathrm{ij},}, \mathrm{q}_{\mathrm{ij}}\right\rangle\right\rangle\right] \in \mathrm{Gn} \times \mathrm{m}$.
(i) Matrix addition and subtraction are given by

$$
\begin{aligned}
& \quad \mathrm{P}+\mathrm{Q}=\left[\left\langle\max \left\{\mathrm{p}_{\mathrm{ij},}, \mathrm{q}_{\mathrm{ij} \mathrm{\eta}}\right\}, \min \left\{\mathrm{p}_{\mathrm{ij} \delta}, \mathrm{q}_{\mathrm{ij} \delta}\right\}\right\rangle\right] \text { and } \\
& \mathrm{P}-\mathrm{Q}=\left[\left\langle\left\{\mathrm{p}_{\mathrm{ij} \mathrm{\eta}}-\mathrm{q}_{\mathrm{ij} j}\right\},\left\{\mathrm{p}_{\mathrm{ij} \delta}-\mathrm{q}_{\mathrm{ij} \delta}\right\}\right\rangle\right] \\
& \text { Where }\left\{\mathrm{p}_{\mathrm{ij} \eta}-\mathrm{q}_{\mathrm{ij} \mathrm{\eta}}\right\}=\left\{\begin{array}{c}
\mathrm{p}_{\mathrm{ij} \mathrm{\eta}}, \mathrm{p}_{\mathrm{ij} \eta} \geq \mathrm{q}_{\mathrm{ij}} \\
0, \text { otherwise }
\end{array}\right. \\
& \text { and }\left\{\mathrm{p}_{\mathrm{ij} \delta}-\mathrm{q}_{\mathrm{ij} \delta}\right\}=\left\{\begin{array}{c}
\mathrm{p}_{\mathrm{ij} \delta} \delta \mathrm{p}_{\mathrm{ij} \delta} \geq \mathrm{q}_{\mathrm{ij} \delta} \\
0, \text { otherwise }
\end{array}\right.
\end{aligned}
$$

(ii) Componentwise matrix multiplication is given by

$$
\mathrm{P} * \mathrm{Q}=\left[\left\langle\min \left\{\mathrm{p}_{\mathrm{ij},}, \mathrm{q}_{\mathrm{ij} \mathrm{\eta}}\right\}, \max \left\{\mathrm{p}_{\mathrm{ij} \delta}, \mathrm{q}_{\mathrm{ij} \delta}\right\}\right\rangle\right]
$$

(iii) Let $P$ and $Q$ be two intuitionistic felicitous fuzzy matrices of order $n \times m$ and $m \times t$ respectively. Then the matrix product PQ is given by
$P Q=\left[\left\langle\sum_{k} \min \left\{p_{i k \eta}, q_{k j \delta}\right\}, \prod_{k} \max \left\{p_{i k \eta}, q_{k j \delta}\right\}\right\rangle\right] \in G_{n \times t}$.

## Definition 4.

Let $P$ be an $m \times m$ intuitionistic felicitous fuzzy matrix over distributive lattice $\left(G_{n}(L), \leq\right.$ $,+, *)$, Then P satisfies idempotent laws, as follows:
(i) $\mathrm{P}+\mathrm{P}=\mathrm{P}$
(ii) $\mathrm{P} * \mathrm{P}=\mathrm{P}$

## 2. Main Results

### 3.1 Partial ordered set (Poset) of intuitionistic felicitous fuzzy matrices

Theorem 1. Let $\mathrm{G}_{\mathrm{n}}$ be the set of all $\mathrm{n} \times \mathrm{n}$ intuitionistic felicitous fuzzy matrices and ' $\leq$ ' be comparable fuzzy matrix relation. Then $\left(\mathrm{G}_{\mathrm{n}}, \leq\right)$ is a poset.
Proof. Let $\mathrm{P}, \mathrm{Q}, \mathrm{R} \in \mathrm{G}_{\mathrm{n}}$. Then
(i) $\mathrm{P} \leq \mathrm{P}$ is true, since $\mathrm{p}_{\mathrm{ij} \mathrm{\eta}} \leq \mathrm{q}_{\mathrm{ij} \mathrm{\eta}}$ and $\mathrm{p}_{\mathrm{ij} \delta} \geq \mathrm{q}_{\mathrm{ij} \delta}$. Hence the relation ' $\leq$ ' is reflexive.
(ii) $P \leq Q$ and $Q \geq P$ is possible only when $P=Q$. Since $P \leq Q$ when $p_{i j \eta} \leq q_{i j \eta}$ and $p_{i j \delta} \geq q_{i j}$ and $Q$ $\leq \mathrm{P}$ when $\mathrm{q}_{\mathrm{ijj}} \leq \mathrm{p}_{\mathrm{ijn}}$ and $\mathrm{q}_{\mathrm{ij} \delta} \geq \mathrm{p}_{\mathrm{ij} \delta} . \mathrm{P}=\mathrm{Q}$.

Therefore the relation ' $\leq$ ' is anti-symmetric.
(iii) $\mathrm{P} \leq \mathrm{Q} \rightarrow \mathrm{p}_{\mathrm{ij}} \leq \mathrm{q}_{\mathrm{ij} \mathrm{\eta}}$ and $\mathrm{p}_{\mathrm{ij} \delta} \geq \mathrm{q}_{\mathrm{ij} \delta}$ and $\mathrm{Q} \leq \mathrm{R}$ when $\mathrm{q}_{\mathrm{ij} \mathrm{\eta}} \leq \mathrm{r}_{\mathrm{ij} \mathrm{\eta}}$ and $\mathrm{q}_{\mathrm{ij} \delta} \geq \mathrm{r}_{\mathrm{ij} \delta}$. This implies $\mathrm{P} \leq \mathrm{R}$ since $\mathrm{p}_{\mathrm{ij} \mathrm{\eta}} \leq \mathrm{r}_{\mathrm{ij} \mathrm{\eta}}$ and $\mathrm{p}_{\mathrm{ij} \delta} \geq \mathrm{r}_{\mathrm{ij} \delta}$. Hence the relation ' $\leq$ ' is transitive. Therefore, a non-empty set of intuitionistic felicitous fuzzy matrices, $\mathrm{G}_{\mathrm{n}}$ satisfies the partial order relations. Hence $G_{n}$ is a poset.

Definition 5. Let $P, Q, R \in G_{n}$. Then the lattice of intuitionistic felicitous fuzzy matrices $\left(\mathrm{G}_{\mathrm{n}}(\mathrm{L}), \leq,+, *\right)$ is said to be distributive lattice of intuitionistic felicitous fuzzy matrices if
(i) $\mathrm{P} *(\mathrm{Q}+\mathrm{R})=(\mathrm{P} * \mathrm{Q})+(\mathrm{P} * \mathrm{R})$
(ii) $\mathrm{P}+(\mathrm{Q} * \mathrm{R})=(\mathrm{P}+\mathrm{Q}) *(\mathrm{P}+\mathrm{R})$.

## Example 1.

(i) The example for the distributive property of intuitionistic felicitous fuzzy matrices isgiven below.

Consider $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ be three $4 \times 4$ intuitionistic felicitous fuzzy matrices, where

$$
\begin{aligned}
& P=\left(\begin{array}{cccc}
(0.05,0.08) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.06,0.07) & (0.01,0.01) & (0.00,0.02) \\
(0.00,0.02) & (0.01,0.01) & (0.06,0.07) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.08,0.05)
\end{array}\right) \\
& Q=\left(\begin{array}{cccc}
(0.08,0.05) & (0.03,0.03) & (0.02,0.00) & (0.01,0.01) \\
(0.03,0.03) & (0.07,0.06) & (0.01,0.01) & (0.02,0.00) \\
(0.02,0.00) & (0.01,0.01) & (0.06,0.07) & (0.03,0.03) \\
(0.01,0.01) & (0.00,0.02) & (0.03,0.03) & (0.05,0.08)
\end{array}\right) \\
& R=\left(\begin{array}{cccc}
(0.11,0.14) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.12,0.13) & (0.01,0.01) & (0.02,0.00) \\
(0.00,0.02) & (0.01,0.01) & (0.13,0.12) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.14,0.11)
\end{array}\right) \\
& \text { Now } P+R=\left(\begin{array}{cccc}
(0.11,0.14) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.12,0.06) & (0.01,0.01) & (0.02,0.00) \\
(0.02,0.00) & (0.01,0.01) & (0.13,0.07) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.14,0.08)
\end{array}\right) \\
& P *(Q+R)=\left(\begin{array}{cccc}
(0.05,0.08) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.06,0.07) & (0.01,0.01) & (0.00,0.02) \\
(0.00,0.02) & (0.01,0.01) & (0.06,0.07) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.08,0.08)
\end{array}\right) \\
& P * Q=\left(\begin{array}{cccc}
(0.05,0.08) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.06,0.07) & (0.01,0.01) & (0.00,0.02) \\
(0.00,0.02) & (0.01,0.01) & (0.06,0.07) & (0.03,0.03) \\
(0.01,0.01) & (0.00,0.02) & (0.03,0.03) & (0.08,0.08)
\end{array}\right) \\
& P * R=\left(\begin{array}{cccc}
(0.05,0.14) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.06,0.13) & (0.01,0.01) & (0.00,0.02) \\
(0.00,0.02) & (0.01,0.01) & (0.06,0.12) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.08,0.11)
\end{array}\right) \\
& (P * Q)+(P * R)=\left(\begin{array}{cccc}
(0.05,0.08) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.06,0.07) & (0.01,0.01) & (0.00,0.02) \\
(0.00,0.02) & (0.01,0.01) & (0.06,0.07) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.08,0.08)
\end{array}\right)
\end{aligned}
$$

Therefore, $P *(Q+R)=(P * Q)+(P * R)$.
(ii) The example for the distributive property of intuitionistic felicitous fuzzy matrices isgiven below.
Consider P,Q,R be three $4 \times 4$ intuitionistic felicitous fuzzy matrices, where

$$
(P+Q) *(P+R)=\left(\begin{array}{cccc}
(0.08,0.08) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.07,0.07) & (0.01,0.01) & (0.02,0.00) \\
(0.00,0.02) & (0.01,0.01) & (0.06,0.07) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.08,0.05)
\end{array}\right)
$$

$$
\begin{aligned}
& P=\left(\begin{array}{cccc}
(0.05,0.08) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.06,0.07) & (0.01,0.01) & (0.00,0.02) \\
(0.00,0.02) & (0.01,0.01) & (0.06,0.07) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.08,0.05)
\end{array}\right) \\
& Q=\left(\begin{array}{cccc}
(0.08,0.05) & (0.03,0.03) & (0.02,0.00) & (0.01,0.01) \\
(0.03,0.03) & (0.07,0.06) & (0.01,0.01) & (0.02,0.00) \\
(0.02,0.00) & (0.01,0.01) & (0.06,0.07) & (0.03,0.03) \\
(0.01,0.01) & (0.00,0.02) & (0.03,0.03) & (0.05,0.08)
\end{array}\right) \\
& R=\left(\begin{array}{cccc}
(0.11,0.14) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.12,0.13) & (0.01,0.01) & (0.02,0.00) \\
(0.00,0.02) & (0.01,0.01) & (0.13,0.12) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.14,0.11)
\end{array}\right) \\
& \text { Now } Q * R=\left(\begin{array}{cccc}
(0.08,0.14) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.07,0.13) & (0.01,0.01) & (0.02,0.00) \\
(0.02,0.00) & (0.01,0.01) & (0.06,0.17) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.05,0.11)
\end{array}\right) \\
& P+(Q * R)=\left(\begin{array}{cccc}
(0.08,0.08) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.07,0.07) & (0.01,0.01) & (0.02,0.00) \\
(0.00,0.02) & (0.01,0.01) & (0.06,0.07) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.08,0.05)
\end{array}\right) \\
& P+Q=\left(\begin{array}{cccc}
(0.08,0.05) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.07,0.06) & (0.01,0.01) & (0.02,0.00) \\
(0.02,0.00) & (0.01,0.01) & (0.06,0.07) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.08,0.05)
\end{array}\right) \\
& P+R=\left(\begin{array}{cccc}
(0.11,0.08) & (0.03,0.03) & (0.00,0.02) & (0.01,0.01) \\
(0.03,0.03) & (0.12,0.07) & (0.01,0.01) & (0.02,0.00) \\
(0.00,0.02) & (0.01,0.01) & (0.13,0.07) & (0.03,0.03) \\
(0.01,0.01) & (0.02,0.00) & (0.03,0.03) & (0.14,0.05)
\end{array}\right)
\end{aligned}
$$

Therefore, $\mathrm{P}+(\mathrm{Q} * \mathrm{R})=(\mathrm{P}+\mathrm{Q}) *(\mathrm{P}+\mathrm{R})$.
Theorem 2. In a distributive lattice of intuitionistic felicitous fuzzy matrices $\left(\mathrm{G}_{\mathrm{n}}(\mathrm{L}), \leq\right.$ $,+, *)$ for $\mathrm{P}, \mathrm{Q}, \mathrm{R} \in \mathrm{G}_{\mathrm{n}}(\mathrm{L}), \mathrm{P}+\mathrm{Q}=\mathrm{P}+\mathrm{R}$ and $\mathrm{P} * \mathrm{Q}=\mathrm{P} * \mathrm{R}$, then $\mathrm{Q}=\mathrm{R}$.
Proof. Let $\mathrm{P}, \mathrm{Q}, \mathrm{R} \in\left(\mathrm{G}_{\mathrm{n}}(\mathrm{L}), \leq,+, *\right)$, then
$\mathrm{Q}=\left[\min \left\{\mathrm{q}_{\mathrm{ijn}}, \max \left\{\mathrm{p}_{\mathrm{ij} \eta}, \mathrm{q}_{\mathrm{ij} j}\right\}\right\}, \max \left\{\mathrm{q}_{\mathrm{ij} \delta}, \min \left\{\mathrm{p}_{\mathrm{ij} \delta}, \mathrm{q}_{\mathrm{ij} \delta}\right\}\right\}\right][$ By absorption property $]$
$=\quad \mathrm{Q} *\left[\max \left\{\mathrm{p}_{\mathrm{ij},}, \mathrm{r}_{\mathrm{i} j \mathrm{j}}\right\}, \min \left\{\mathrm{p}_{\mathrm{ij} \delta}, \mathrm{r}_{\mathrm{ij} \delta}\right\}\right][$ SinceP $+\mathrm{Q}=\mathrm{P}+\mathrm{R}]$
$=\quad\left[\min \left\{\mathrm{q}_{\mathrm{ij}}, \mathrm{p}_{\mathrm{ij} \eta}\right\}, \max \left\{\mathrm{q}_{\mathrm{ij}}, \mathrm{p}_{\mathrm{ij} \delta}\right\}\right]+\left[\min \left\{\mathrm{q}_{\mathrm{ij}}, \mathrm{r}_{\mathrm{ij} j}\right\}, \max \left\{\mathrm{q}_{\mathrm{ij} \delta}, \mathrm{r}_{\mathrm{ij} \delta}\right\}\right]$
$=\quad\left[\min \left\{\mathrm{q}_{\mathrm{ij}}, \mathrm{p}_{\mathrm{ijn}}\right\}, \max \left\{\mathrm{q}_{\mathrm{ij}}, \mathrm{p}_{\mathrm{ij} \delta}\right\}\right]+\left[\min \left\{\mathrm{q}_{\mathrm{ij}}, \mathrm{r}_{\mathrm{ij}}\right\}, \max \left\{\mathrm{q}_{\mathrm{ij} \delta}, \mathrm{r}_{\mathrm{ij} \delta}\right\}\right]$
$=\quad\left[\min \left\{\mathrm{q}_{\mathrm{ijn}}, \mathrm{p}_{\mathrm{ij} \eta}\right\}, \max \left\{\mathrm{q}_{\mathrm{ij}}, \mathrm{p}_{\mathrm{ij} \delta}\right\}\right]+\left[\min \left\{\mathrm{q}_{\mathrm{ij}}, \mathrm{r}_{\mathrm{ij}}\right\}, \max \left\{\mathrm{q}_{\mathrm{ij} \delta}, \mathrm{r}_{\mathrm{ij} \delta}\right\}\right]$
$=\quad \mathrm{R} *\left[\max \left\{\mathrm{p}_{\mathrm{ij} \eta}, \mathrm{q}_{\mathrm{ij} j}\right\}, \min \left\{\mathrm{p}_{\mathrm{ij} \delta}, \mathrm{q}_{\mathrm{ij} \delta}\right\}\right]$ [ By distributive law]
$=\quad \mathrm{R} *(\mathrm{P}+\mathrm{R})=\mathrm{R}[$ By absorption property $]$
Therefore, $\mathrm{Q}=\mathrm{R}$.

## Example 2.

The example for the distributive property of intuitionistic felicitous fuzzy matrices is given below.
Let $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ be three $4 \times 4$ intuitionistic felicitous fuzzy matrices, where

$$
\left.\begin{array}{c}
P=\left(\begin{array}{ccccc}
(0.031,0.034) & (0.003,0.003) & (0.000,0.002) & (0.001,0.001) \\
(0.003,0.003) & (0.032,0.033) & (0.001,0.001) & (0.002,00.00) \\
(0.000,0.002) & (0.001,0.001) & (0.033,0.032) & (0.003,0.003) \\
(0.001,0.001) & (0.002,0.000) & (0.003,0.003) & (0.034,0.031)
\end{array}\right) \\
Q \\
\\
R
\end{array} \begin{array}{lllll}
(0.021,0.024) & (0.003,0.003) & (0.000,0.002) & (0.001,0.001) \\
(0.003,0.003) & (0.022,0.023) & (0.001,0.001) & (0.002,00.00) \\
(0.000,0.002) & (0.001,0.001) & (0.023,0.022) & (0.003,0.003) \\
(0.001,0.001) & (0.002,0.000) & (0.003,0.003) & (0.024,0.021)
\end{array}\right)
$$

$$
\begin{gathered}
Q+R=\left(\begin{array}{cccc}
(0.031,0.034) & (0.003,0.003) & (0.000,0.002) & (0.001,0.001) \\
(0.003,0.003) & (0.032,0.013) & (0.001,0.001) & (0.002,00.00) \\
(0.000,0.002) & (0.001,0.001) & (0.033,0.012) & (0.003,0.003) \\
(0.001,0.001) & (0.002,0.000) & (0.003,0.003) & (0.034,0.011)
\end{array}\right) \\
P-(Q+R)=\left(\begin{array}{llll}
(0.000,0.011) & (0.000,0.000) & (0.000,0.000) & (0.000,0.000) \\
(0.000,0.000) & (0.000,0.013) & (0.000,0.000) & (0.000,00.00) \\
(0.000,0.000) & (0.000,0.000) & (0.000,0.012) & (0.000,0.000) \\
(0.000,0.000) & (0.000,0.000) & (0.000,0.000) & (0.000,0.011)
\end{array}\right)
\end{gathered}
$$

Therefore, $(\mathrm{P}-\mathrm{Q})-\mathrm{R} \geq \mathrm{P}-(\mathrm{Q}+\mathrm{R})$

## Example 3.

Let P,Q,R be four $4 \times 4$ intuitionistic felicitous fuzzy matrices, where

$$
\begin{gathered}
P=\left(\begin{array}{cccc}
(0.021,0.024) & (0.003,0.003) & (0.000,0.002) & (0.001,0.001) \\
(0.003,0.003) & (0.022,0.023) & (0.001,0.001) & (0.002,00.00) \\
(0.000,0.002) & (0.001,0.001) & (0.023,0.022) & (0.003,0.003) \\
(0.001,0.001) & (0.002,0.000) & (0.003,0.003) & (0.024,0.021)
\end{array}\right) \\
Q=\left(\begin{array}{llll}
(0.031,0.034) & (0.003,0.003) & (0.000,0.002) & (0.001,0.001) \\
(0.003,0.003) & (0.032,0.033) & (0.001,0.001) & (0.002,00.00) \\
(0.000,0.002) & (0.001,0.001) & (0.033,0.032) & (0.003,0.003) \\
(0.001,0.001) & (0.002,0.000) & (0.003,0.003) & (0.034,0.031)
\end{array}\right) \\
R=\left(\begin{array}{llll}
(0.011,0.014) & (0.003,0.003) & (0.000,0.002) & (0.001,0.001) \\
(0.003,0.003) & (0.012,0.013) & (0.001,0.001) & (0.002,00.00) \\
(0.000,0.002) & (0.001,0.001) & (0.013,0.012) & (0.003,0.003) \\
(0.001,0.001) & (0.002,0.000) & (0.003,0.003) & (0.014,0.011)
\end{array}\right) \\
\text { Now, P-Q}=\left(\begin{array}{llll}
(0.021,0.014) & (0.000,0.000) & (0.000,0.000) & (0.000,0.000) \\
(0.000,0.000) & (0.022,0.013) & (0.000,0.000) & (0.000,00.00) \\
(0.000,0.000) & (0.000,0.000) & (0.023,0.012) & (0.000,0.000) \\
(0.000,0.000) & (0.000,0.000) & (0.000,0.000) & (0.024,0.011)
\end{array}\right) \\
P
\end{gathered}\left(\begin{array}{llll}
(0.031,0.014) & (0.000,0.000) & (0.000,0.000) & (0.000,0.000) \\
(0.000,0.000) & (0.032,0.013) & (0.000,0.000) & (0.000,00.00) \\
(0.000,0.000) & (0.000,0.000) & (0.033,0.012) & (0.000,0.000) \\
(0.000,0.000) & (0.000,0.000) & (0.000,0.000) & (0.034,0.011)
\end{array}\right)
$$

Therefore, if $\mathrm{P} \leq \mathrm{Q}$, then $\mathrm{P}-\mathrm{R} \leq \mathrm{Q}-\mathrm{R}$.

## 3. Conclusion

Matrix theory is a very essential tool to model a large number of problems that occur in science, engineering, medical science and even in social science. In this paper, we have introduced a new matrix representation of felicitous fuzzy matrix over distributive lattice and studied subdivision of intuitionistic felicitous fuzzy graphs. Some algebraic operations of intuitionistic felicitous fuzzy matrices are represented over distributive lattice.

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