Section A-Research paper
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Basic Operations on Pythagorean Neutrosophic Hypersoft Matrices

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Abstract: The main goal of this work is to extend the idea of Pythagorean Neutrosophic Hypersoft sets (PNHSS) to the Pythagorean Neutrosophic Hypersoft matrix (PNHSM) through the fundamental study of matrices and appropriate instances.

Keywords: Uncertainty (UC), Soft set (SS), Hypersoft set (HS), PNHSS, PNHSMs.

1. Introduction

Numerous theories are utilized in the literature to address the ambiguity and UC of numerous problems that arise in Engineering, Economics, Social Science, etc., All ideas, however have their own drawbacks. Multiple qualities and UC are present in multi-criteria Decision-making (D-M) situations, Since NSs fully address indeterminacy, whereas NSSs address vagueness and UC, they are utilized to deal with such kinds. The concept of NSS cannot be applied to such problems when qualities are multiple and further divided.

To get around these issues, Smarandache [3] demonstrated a different method of handling UC by extending the SS to the HS and its hybrids, such as the FHS, IFHS, and NHSS, by converting the function into a multi-argument function. The matrix representation and aggregate operators of this idea were presented by Delhi and Broumi in [5].

Section 3 extensively covers the concept of PNHSM, providing definitions and appropriate examples for better comprehension. Different potential types of PNHSM and their fundamental operators, along with their corresponding properties, are suggested in sections 4 and 5.

2. Preliminaries

Definition 2.1. [4] Let $\widetilde{\Delta}$ be the universal set (US) and $P(\widetilde{\Delta})$ be a power set (PS) of $\widetilde{\Delta}$. Consider $\tilde{A}^{\cdots}{}_{1}, \tilde{A}^{\cdots}{}_{2}, \ldots, \tilde{A}^{\cdots}{}_{\overset{\frown}{\kappa}}$ for $\overset{\frown}{\kappa} \geq 1$ be $\overset{\frown}{\kappa}$ well - defined attributes, whose corresponding attributive values are $\widetilde{G}^{\cdots}{}_{1}, \widetilde{G}^{\cdots}{}_{2}, \ldots, \widetilde{G}^{\cdots}{}_{\overset{\frown}{\kappa}}$ with $\widetilde{G}^{\cdots}{}_{\overset{\frown}{\bar{\imath}}} \cap \widetilde{G}^{\cdots}{}_{\overset{\frown}{\bar{\jmath}}} = \emptyset$, for $\overset{\frown}{\bar{\imath}} \neq \overset{\frown}{\bar{\jmath}}$, $\overset{\frown}{\bar{\imath}}, \overset{\frown}{\bar{\jmath}} \dot{\in} \{1, 2, \ldots, \overset{\frown}{\kappa}\}$ and their relation $\widetilde{G}^{\cdots}{}_{1} \times \widetilde{G}^{\cdots}{}_{2}, \ldots \times \widetilde{G}^{\cdots}{}_{\overset{\frown}{\kappa}} = \dot{\eta}$, $(\ddot{\gamma}, \widetilde{G}^{\cdots}{}_{1} \times \widetilde{G}^{\cdots}{}_{2} \times \ldots \times \widetilde{G}^{\cdots}{}_{\overset{\frown}{\kappa}})$ is said to be PNHSS over $\widetilde{\Delta}$ where $\ddot{\gamma}: \widetilde{G}^{\cdots}{}_{1} \times \widetilde{G}^{\cdots}{}_{2} \times \ldots \times \widetilde{G}^{\cdots}{}_{\overset{\frown}{\kappa}} \rightarrow P(\widetilde{\Delta})$ and $\ddot{\gamma}(\widetilde{G}^{\cdots}{}_{1} \times \widetilde{G}^{\cdots}{}_{2} \times \ldots \times \widetilde{G}^{\cdots}{}_{\overset{\frown}{\kappa}}) = \left\{\left(\dot{\eta}, < \check{x}, T_{\ddot{\gamma}(\dot{\eta})}(\underbrace{\check{x}}), I_{\ddot{\gamma}(\dot{\eta})}(\underbrace{\check{x}}), F_{\ddot{\gamma}(\dot{\eta})}(\underbrace{\check{x}}) >\right) : \underbrace{\check{x}} \dot{\in} \widetilde{\Delta}, \dot{\eta} \dot{\in} \widetilde{G}^{\cdots}{}_{1} \times \widetilde{G}^{\cdots}{}_{2} \times \ldots \times \widetilde{G}^{\cdots}{}_{\overset{\frown}{\kappa}}\right\}$ where T is the membership value of truthiness, I is the membership value of indeterminacy and F is the membership value of falsity such that $T_{\ddot{\gamma}(\dot{\eta})}(\check{x}), I_{\ddot{\gamma}(\dot{\eta})}(\check{x}), F_{\ddot{\gamma}(\dot{\eta})}(\check{x}) \dot{\in} [0,1]$ also $\left(T_{\ddot{\gamma}(\dot{\eta})}(\underbrace{\check{x}})\right)^2 + \left(I_{\ddot{\gamma}(\dot{\eta})}(\underbrace{\check{x}})\right)^2 + \left(F_{\ddot{\gamma}(\dot{\eta})}(\underbrace{\check{x}})\right)^2 \leq 2$.

3. Pythagorean Neutrosophic Hypersoft Matrix (PNHSM)

3.1. PNHSM. Let $\tilde{\Delta} = \{\tilde{\Delta}^1, \tilde{\Delta}^2, \ldots \tilde{\Delta}^{r}\}$ be the US and $\dot{P}(\tilde{\Delta})$ be the PS of $\tilde{\Delta}$. Consider $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_c$, $for\ c \geq 1$, where c is well-defined attributes, whose corresponding attributive values are $\tilde{A}_1^{a}, \tilde{A}_2^{b}, \ldots, \tilde{A}_c^{z}$ and their relation $\tilde{A}_1^{a} \times \tilde{A}_2^{b} \times \ldots \times \tilde{A}_c^{z}$, where $a, b, \ldots, z = 1,2,\ldots,n$; then, the pair $\left(\ddot{\gamma}, \tilde{A}_1^{a} \times \tilde{A}_2^{b}, \ldots \times \tilde{A}_c^{z}\right)$ is said to be PNHSS over $\tilde{\Delta}$, where $\ddot{\gamma}: \left(\tilde{A}_1^{a} \times \tilde{A}_2^{b} \times \ldots \times \tilde{A}_c^{z}\right) \rightarrow \dot{P}(\tilde{\Delta})\ and$ $\ddot{\gamma}\left(\tilde{A}_1^{a} \times \tilde{A}_2^{b} \times \ldots \times \tilde{A}_c^{z}\right) = \left\{\left(\delta, < x, T_{\ddot{\gamma}(\delta)}(x), I_{\ddot{\gamma}(\delta)}(x), F_{\ddot{\gamma}(\delta)}(x) > \right): x \in \tilde{\Delta}, \delta \in \tilde{A}_1^{a} \times \tilde{A}_2^{b} \times \ldots \times \tilde{A}_c^{z}\right\}$.

If $ך_{ij} = X_{R_{\ddot{\gamma}(\delta)}}(\tilde{\Delta}^{i}, \tilde{A}_j^{k})$; $i = 1,2,3\ldots r$, $j = 1,2,3\ldots c$, $k = a, b, \ldots, z$;
$[ך_{ij}]_{r \times c} =$
$(ך_{11}\ ך_{12} \ldots ך_{1c}\ ך_{21}\ ך_{22} \ldots ך_{2c} \ldots\ldots\ldots\ ך_{r1}\ ך_{r2} \ldots ך_{rc})$, where $ך_{ij} =$ $(T_{\tilde{A}_j^{k}}(\tilde{\Delta}_i), I_{\tilde{A}_j^{k}}(\tilde{\Delta}_i), F_{\tilde{A}_j^{k}}(\tilde{\Delta}_i))$, $(\tilde{\Delta}_i) \in \tilde{\Delta}, \tilde{A}_j^{k} \in \left(\tilde{A}_1^{a} \times \tilde{A}_2^{b} \times \ldots \times \tilde{A}_c^{z}\right) = (T^{ך}_{ijk}, I^{ך}_{ijk}, F^{ך}_{ijk})$. The collection of all PNHSMs over $\tilde{\Delta}$ is denoted by $\text{PNHSM}(\tilde{\Delta})_{r \times c}$.

Example 1. Consider a problem on requirement of teachers for Government postings (GPs) based on their CVs received. Let $\tilde{\Delta} = \{T_1, T_2, T_3, T_4, T_5\}$ be the set of candidates. Attributes: $\tilde{A}_1$= Qualification(Q), $\tilde{A}_2$= Reservation (R), $\tilde{A}_3$ = Gender (G), $\tilde{A}_4$ = Grade (GR). Parameters: $\tilde{A}_1^{a} = \{M.phil, B.Sc, M.Sc, Ph.D.\}$; $\tilde{A}_2^{b}$= {MBC, BC, SC, Others}; $\tilde{A}_3^{c}$ = {Male, Female}; $\tilde{A}_4^{d}$= {Very High, High, Average}. Let $\ddot{\gamma}$ (($\tilde{A}_1^{a} \times \tilde{A}_2^{b} \times \tilde{A}_3^{c} \times \tilde{A}_4^{d}$)) = $\ddot{\gamma}$ (M. Phil, BC, Female, High) = $\{T_1, T_2, T_4, T_5\}$. Pythagorean Neutrosophic values (PNVs) assigned by distinct D-Ms are

Table 1: PNVs to each T_i against (Q).

$\tilde{A}_1^{a}$ **(Q)**	T_1	T_2	T_3	T_4	T_5
$M.phil.$	(.6, .4, .5)	(.5, .4, .3)	(.7, .5, .1)	(.7, .2, .5)	(.6, .7, .3)
$B.Sc.$	(.6, .5, .3)	(.6, .5, .4)	(.5, .3, .1)	(.6, .5, .2)	(.5, .6, .3)
$M.Sc.$	(.5, .4, .3)	(.5, .3, .2)	(.6, .4, .3)	(.5, .6, .2)	(.6, .5, .1)
$Ph.D.$	(.6, .5, .2)	(.6, .7, .1)	(.5, .6, .3)	(.6, .4, .2)	(.5, .3, .1)

Table 2: PNVs to each T_i against (R).

$\tilde{A}_2^{b}$ **(R)**	T_1	T_2	T_3	T_4	T_5
BC	(.6, .5, .3)	(.7, .6, .3)	(.5, .4, .1)	(.8, .7, .1)	(.6, .4, .3)
MBC	(.5, .6, .2)	(.5, .4, .1)	(.8, .7, .1)	(.6, .5, .1)	(.5, .6, .1)
SC	(.8, .7, .2)	(.7, .6, .1)	(.6, .5, .1)	(.5, .4, .2)	(.6, .4, .1)
Others	(.6, .7, .1)	(.5, .2, .1)	(.6, .4, .2)	(.5, .4, .1)	(.5, .4, .1)

Table 3: PNVs to each T_i against (G).

$\tilde{\ddot{A}}_3^{\grave{c}}$ **(G)**	T^{1}	T^{2}	T^{3}	T^{4}	T^{5}
M	(.5, .4, .7)	(.6, .5, .1)	(.7, .5, .2)	(.6, .4, .2)	(.5, .4, .1)
F	(.4, .8, .7)	(.6, .4, .7)	(.6, .5, .1)	(.5, .4, .6)	(.8, .7, .5)

Table 4: PNVs to each T_i against (GR).

$\tilde{\ddot{A}}_4^{d}$ **(GR)**	T^{1}	T^{2}	T^{3}	T^{4}	T^{5}
Very high	(.5, .4, .1)	(.5, .6, .3)	(.7, .5, .1)	(.6, .4, .1)	(.5, .4, .2)
High	(.8, .3, .4)	(.6, .2, .5)	(.7, .6, .1)	(.7, .3, .5)	(.6, .5, .7)
Average	(.7, .6, .1)	(.6, .4, .3)	(.5, .4, .1)	(.6, .7, .1)	(.6, .4, .2)

$\ddot{\gamma}$ (($\tilde{\ddot{A}}_1^{\acute{a}} \times \tilde{\ddot{A}}_2^{\acute{b}} \times \tilde{\ddot{A}}_3^{\grave{c}} \times \tilde{\ddot{A}}_4^{d}$)) = $\ddot{\gamma}$ (M. Phil, BC, F, High)

= {<< T^{1}, (M. Phil {.6, .4, .5}, BC {.6, .5, .3}, F {.4, .8, .7}, High {.8, .3, .4})>>,

<< T^{2}, (M. Phil {.5, .4, .3}, BC {.7, .6, .3}, F {.6, .4, .7}, High {.6, .2, .5})>>,

<< T^{4}, (M. Phil {.7, .2, .5}, BC {.8, .7, .1}, F {.5, .4, .6}, High {.7, .3, .5})>>,

<< T^{5}, (M. Phil {.6, .7, .3}, BC {.6, .4, .3}, F {.8, .7, .5}, High {.6, .5, .7})>>}.

	$\tilde{\ddot{A}}_1^{\acute{a}}$	$\tilde{\ddot{A}}_2^{\acute{b}}$	$\tilde{\ddot{A}}_3^{\grave{c}}$	$\tilde{\ddot{A}}_4^{d}$
T^{1}	(M.Phil., (.6, .4, .5))	(BC, (.6, .5, .3))	(F, (.4, .8, .7))	(High, (.8, .3, .4))
T^{2}	(M.Phil., (.5, .4, .3))	(BC, (.7, .6, .3))	(F, (.6, .4, .7))	(High, (.6, .2, .5))
T^{4}	(M.Phil., (.7, .2, .5))	(BC, (.8, .7, .1))	(F, (.5, .4, .6))	(High, (.7, .3, .5))
T^{5}	(M.Phil., (.6, .7, .3))	(BC, (.6, .4, .3))	(F, (.8, .7, .5))	(High, (.6, .5, .7))

$[\daleth]_{4\times4}=$

$[((.6,.4,.5))\ \ ((.6,.5,.3))\ \ ((.4,.8,.7))\ \ ((.8,.3,.4))\ ((.5,.4,.3))\ \ ((.7,.6,.3))\ \ ((.6,.4,.7))\ \ ((.6,.2,.5))\ ((.$

4. TYPES OF PNHSMs

Let $\daleth = [\daleth_{ij}]$ and $\gimel = [\gimel_{ij}] \in$ PNHSM$(\tilde{\Delta})_{r\times c}$ where $\daleth_{ij} = \left(T^{\daleth}_{ijk}, I^{\daleth}_{ijk}, F^{\daleth}_{ijk}\right)$ and $\gimel_{ij} = \left(T^{\gimel}_{ijk}, I^{\gimel}_{ijk}, F^{\gimel}_{ijk}\right)$. Then,

- **Square PNHSM** if $r = c$.
- **Row PNHSM** if $c = 1$.
- **Column PNHSM** if $r = 1$.
- **Zero PNHSM if** $T^{\daleth}_{ijk} = 0$, $I^{\daleth}_{ijk} = 1$, $F^{\daleth}_{ijk} = 1$.
- **Universal PNHSM** if $T^{\daleth}_{ijk} = 1$, $I^{\daleth}_{ijk} = 0$, $F^{\daleth}_{ijk} = 0$.
- **Complement PNHSM** if $T^{\daleth}_{ijk} = F^{\daleth}_{ijk}$, $I^{\daleth}_{ijk} = 1 - I^{\daleth}_{ijk}$, $F^{\daleth}_{ijk} = T^{\daleth}_{ijk}$.
- **Sub Matrix of PNHSM** if $T^{\daleth}_{ijk} \le T^{\gimel}_{ijk}$, $I^{\daleth}_{ijk} \ge I^{\gimel}_{ijk}$, $F^{\daleth}_{ijk} \ge F^{\gimel}_{ijk}$.
- **Super Matrix of PNHSM** if $T^{\daleth}_{ijk} \ge T^{\gimel}_{ijk}$, $I^{\daleth}_{ijk} \le I^{\gimel}_{ijk}$, $F^{\daleth}_{ijk} \le F^{\gimel}_{ijk}$.
- **Equal Matrix of PNHSM** if $T^{\daleth}_{ijk} = T^{\gimel}_{ijk}$, $I^{\daleth}_{ijk} = I^{\gimel}_{ijk}$, $F^{\daleth}_{ijk} = F^{\gimel}_{ijk}$.
- **Transpose of square PNHSM** if $\left(T^{\daleth}_{ijk}, I^{\daleth}_{ijk}, F^{\daleth}_{ijk}\right)^t = \left(T^{\daleth}_{jki}, I^{\daleth}_{jki}, F^{\daleth}_{jki}\right)$.
- **Symmetric PNHSM** if $\daleth^t = \daleth$.

- **Scalar Multiplication of PNHSM** For any scalar '$s□$', $s□$ ˥ = $[s□\,˥_{ij□}]$; $0 \le s□ \le 1$.
- **Max-Min Product of PNHSM** Let ˥ = $[˥_{ij□}]$, Ⅎ $=[Ⅎ_{jm□}]$ ∈ ´PNHSM$(\widetilde{\Delta})_{ŗ□\times c□}$ where $˥_{ij□} = \left(T^{˥}_{ijk□}, I^{˥}_{ijk□}, F^{˥}_{ijk□}\right)$ and $Ⅎ_{jm□} = \left(T^{Ⅎ}_{jkm□}, I^{Ⅎ}_{jkm□}, F^{Ⅎ}_{jkm□}\right)$.
  If ˥ = $[˥_{ij□}]_{ŗ□\times c□}$ and Ⅎ = $[Ⅎ_{jm□}]_{c□\times □}$, then ˥ ⊗ Ⅎ = $[\delta_{im□}]_{ŗ□\times □}$, where $[\delta_{im□}]$ = $\left(max_{jk□} min\left(T^{˥}_{ijk□}, T^{Ⅎ}_{jkm□}\right), min_{jk□}\, max\left(I^{˥}_{ijk□}, I^{Ⅎ}_{jkm□}\right), min_{jk□}\, max\left(F^{˥}_{ijk□}, F^{Ⅎ}_{jkm□}\right)\right)$.

Theorem 1. Let ˥ = $[˥_{ij}]$, Ⅎ $=[Ⅎ_{ij}]$ ∈´ PNHSM$(\widetilde{\Delta})_{ŗ\times c}$ where $˥_{ij} = \left(T^{˥}_{ijk}, I^{˥}_{ijk}, F^{˥}_{ijk}\right)$ and $Ⅎ_{ij} = \left(T^{Ⅎ}_{ijk}, I^{Ⅎ}_{ijk}, F^{Ⅎ}_{ijk}\right)$. For two scalars s, $ẃ$ ∈´ [0,1], then

(i) $s□\,(ẃ\,˥) = (s□ẃ)\,˥$
(ii) If $s□ < ẃ$, then $s□\,˥ < ẃ\,˥$
(iii) If ˥ ⊆ Ⅎ , then $s□\,˥ \subseteq s□Ⅎ$
(iv) $(s□˥)^{t} = s□˥^{t}$
(v) $\left(˥^{t}\right)^{t} = ˥$

Proof.

(i) $s\,(ẃ\,˥) = s\,[ẃ\,˥_{ij}] = s\,[(ẃT^{˥}_{ijk}, ẃ\,I^{˥}_{ijk}, ẃ\,F^{˥}_{ijk})]$
$= [(sẃT^{˥}_{ijk}, sẃ\,I^{˥}_{ijk}, sẃ\,F^{˥}_{ijk})]$
$= s□ẃ\,[(T^{˥}_{ijk□}, I^{˥}_{ijk□}, F^{˥}_{ijk□})]$
$= s□ẃ\,[˥_{ij□}] = (s□ẃ)\,˥$.

(ii) Since $T^{˥}_{ijk}, I^{˥}_{ijk}, F^{˥}_{ijk}$ ∈´ [0,1], so
$sT^{˥}_{ijk} \le ẃT^{˥}_{ijk}, sI^{˥}_{ijk} \le ẃI^{˥}_{ijk}, sF^{˥}_{ijk} \le ẃF^{˥}_{ijk}$.
$s˥ = [s˥_{ij}] = [(sT^{˥}_{ijk}, s\,I^{˥}_{ijk}, s\,F^{˥}_{ijk})] \le [(ẃT^{˥}_{ijk}, ẃ\,I^{˥}_{ijk}, ẃF^{˥}_{ijk})]$
$= [ẃ˥_{ij}] = ẃ˥$.

(iii) $˥ \subseteq Ⅎ \Rightarrow [˥_{ij}] \subseteq [Ⅎ_{ij}]$
$\Rightarrow T^{˥}_{ijk} \le T^{Ⅎ}_{ijk},\ I^{˥}_{ijk} \ge I^{Ⅎ}_{ijk},\ F^{˥}_{ijk} \ge F^{Ⅎ}_{ijk}$
$\Rightarrow s□T^{˥}_{ijk□} \le s□T^{Ⅎ}_{ijk□}, s□I^{˥}_{ijk□} \ge s□I^{Ⅎ}_{ijk□}, s□F^{˥}_{ijk□} \ge s□F^{Ⅎ}_{ijk□}$
$\Rightarrow s□[˥_{ij□}] \subseteq s□\,[Ⅎ_{ij□}] \Rightarrow s□˥ \subseteq s□Ⅎ$.

(iv) $(s\,˥)^{t} = \left[\left(sT^{˥}_{ijk}, s\,I^{˥}_{ijk}, s\,F^{˥}_{ijk}\right)\right]^{t} = [(sT^{˥}_{jki}, s\,I^{˥}_{jki}, s\,F^{˥}_{jki})]$
$= s□\,[(T^{˥}_{jki□}, I^{˥}_{jki□}, F^{˥}_{jki□})] = s□\left[\left(T^{˥}_{ijk□}, I^{˥}_{ijk□}, F^{˥}_{ijk□}\right)\right]^{t}$
$= s□˥^{t}$.

(v) $\left(˥^{t}\right)^{t} = \left(\left[\left(T^{˥}_{jki}, I^{˥}_{jki}, F^{˥}_{jki}\right)\right]\right)^{t} = \left[\left(T^{˥}_{ijk}, I^{˥}_{ijk}, F^{˥}_{ijk}\right)\right] = ˥$.

Theorem 2. Let ˥ = $[˥_{ij}]_{ŗ\times c}$, Ⅎ $= [Ⅎ_{jm}]_{c\times \acute{y}}$ ∈´ PNHSM$(\widetilde{\Delta})_{ŗ\times c}$; $˥_{ij} = \left(T^{˥}_{ijk}, I^{˥}_{ijk}, F^{˥}_{ijk}\right)$ *and* $Ⅎ_{jm} = \left(T^{Ⅎ}_{jkm}, I^{Ⅎ}_{jkm}, F^{Ⅎ}_{jkm}\right)$. Then, $(˥ \otimes Ⅎ)^{t} = Ⅎ^{t} \otimes ˥^{t}$.

Proof.

Let ˥ ⊗ Ⅎ = $[\delta_{im}]_{ŗ\times \acute{y}}$; then, $(˥ \otimes Ⅎ)^{t} = [\delta_{mi}]_{\acute{y}\times ŗ}$,
$˥^{t} = [˥_{ji}]_{c\times ŗ}$ and $Ⅎ^{t} = [Ⅎ_{mj}]_{\acute{y}\times c}$.

$(˥ \otimes Ⅎ)^{t} = \left(T^{Ⅎ}_{mjk}, I^{Ⅎ}_{mjk}, F^{Ⅎ}_{mjk}\right)_{\acute{y}\times c} \otimes \left(T^{˥}_{jki}, I^{˥}_{jki}, F^{˥}_{jki}\right)_{c\times ŗ}$

$= Ⅎ^t \otimes ℸ^t$.

5. OPERATORS OF PNHSMs

Let $ℸ = [ℸ_{i\bar{j}}]$ and $Ⅎ = [Ⅎ_{i\bar{j}}] \in$ PNHSM$(\widetilde{\Delta})_{r \times c}$ where $ℸ_{i\bar{j}} = \left(T^{ℸ}_{ij\bar{k}}, I^{ℸ}_{ij\bar{k}}, F^{ℸ}_{ij\bar{k}}\right)$ and $Ⅎ_{i\bar{j}} = \left(T^{Ⅎ}_{ij\bar{k}}, I^{Ⅎ}_{ij\bar{k}}, F^{Ⅎ}_{ij\bar{k}}\right)$.

- $ℸ \cup Ⅎ = \left[max(T^{ℸ}_{ij\bar{k}}, T^{Ⅎ}_{ij\bar{k}}), min\ (I^{ℸ}_{ij\bar{k}}, I^{Ⅎ}_{ij\bar{k}}), min\ (F^{ℸ}_{ij\bar{k}}, F^{Ⅎ}_{ij\bar{k}})\right]$
- $ℸ \cap Ⅎ = \left[min(T^{ℸ}_{ij\bar{k}}, T^{Ⅎ}_{ij\bar{k}}), max\ (I^{ℸ}_{ij\bar{k}}, I^{Ⅎ}_{ij\bar{k}}), max\ (F^{ℸ}_{ij\bar{k}}, F^{Ⅎ}_{ij\bar{k}})\right]$
- $ℸ \widetilde{@} Ⅎ = \left[\frac{(T^{ℸ}_{ij\bar{k}} + T^{Ⅎ}_{ij\bar{k}})}{2}, \frac{(I^{ℸ}_{ij\bar{k}} + I^{Ⅎ}_{ij\bar{k}})}{2}, \frac{(F^{ℸ}_{ij\bar{k}} + F^{Ⅎ}_{ij\bar{k}})}{2}\right]$
- $ℸ\ \widetilde{@}^{\widetilde{w}} Ⅎ = \left[\frac{(\widetilde{w}^1 T^{ℸ}_{ij\bar{k}} + \widetilde{w}^2 T^{Ⅎ}_{ij\bar{k}})}{\widetilde{w}^1 + \widetilde{w}^2}, \frac{(\widetilde{w}^1 I^{ℸ}_{ij\bar{k}} + \widetilde{w}^2 I^{Ⅎ}_{ij\bar{k}})}{\widetilde{w}^1 + \widetilde{w}^2}, \frac{(\widetilde{w}^1 F^{ℸ}_{ij\bar{k}} + \widetilde{w}^2 F^{Ⅎ}_{ij\bar{k}})}{\widetilde{w}^1 + \widetilde{w}^2}\right]; \widetilde{w}^1, \widetilde{w}^2 > 0.$
- $ℸ \$ Ⅎ = \left[\sqrt{T^{ℸ}_{ij\bar{k}}.T^{Ⅎ}_{ij\bar{k}}}, \sqrt{I^{ℸ}_{ij\bar{k}}.I^{Ⅎ}_{ij\bar{k}}}, \sqrt{F^{ℸ}_{ij\bar{k}}.F^{Ⅎ}_{ij\bar{k}}}\right]$
- $ℸ\ \$^{\widetilde{w}} Ⅎ =$

$$\sqrt[\widetilde{w}^1+\widetilde{w}^2]{\left(T^{ℸ}_{ij\bar{k}}\right)^{\widetilde{w}^1}.\left(T^{Ⅎ}_{ij\bar{k}}\right)^{\widetilde{w}^2}}, \sqrt[\widetilde{w}^1+\widetilde{w}^2]{\left(I^{ℸ}_{ij\bar{k}}\right)^{\widetilde{w}^1}.\left(I^{Ⅎ}_{ij\bar{k}}\right)^{\widetilde{w}^2}},$$
$$\sqrt[\widetilde{w}^1+\widetilde{w}^2]{\left(F^{ℸ}_{ij\bar{k}}\right)^{\widetilde{w}^1}.\left(F^{Ⅎ}_{ij\bar{k}}\right)^{\widetilde{w}^2}}; \widetilde{w}^1, \widetilde{w}^2 > 0$$

- $ℸ \oslash Ⅎ = \left[\frac{2T^{ℸ}_{ij\bar{k}}T^{Ⅎ}_{ij\bar{k}}}{T^{ℸ}_{ij\bar{k}} + T^{Ⅎ}_{ij\bar{k}}}, \frac{2I^{ℸ}_{ij\bar{k}}I^{Ⅎ}_{ij\bar{k}}}{I^{ℸ}_{ij\bar{k}} + I^{Ⅎ}_{ij\bar{k}}}, \frac{2F^{ℸ}_{ij\bar{k}}F^{Ⅎ}_{ij\bar{k}}}{F^{ℸ}_{ij\bar{k}} + F^{Ⅎ}_{ij\bar{k}}}\right]$
- $ℸ\ \oslash^{\widetilde{w}} Ⅎ = \frac{\widetilde{w}^1 + \widetilde{w}^2}{\left(\frac{\widetilde{w}^1}{T^{ℸ}_{ij\bar{k}}}\right) + \left(\frac{\widetilde{w}^2}{T^{Ⅎ}_{ij\bar{k}}}\right)}, \frac{\widetilde{w}^1 + \widetilde{w}^2}{\left(\frac{\widetilde{w}^1}{I^{ℸ}_{ij\bar{k}}}\right) + \left(\frac{\widetilde{w}^2}{I^{Ⅎ}_{ij\bar{k}}}\right)}, \frac{\widetilde{w}^1 + \widetilde{w}^2}{\left(\frac{\widetilde{w}^1}{F^{ℸ}_{ij\bar{k}}}\right) + \left(\frac{\widetilde{w}^2}{F^{Ⅎ}_{ij\bar{k}}}\right)}; \widetilde{w}^1, \widetilde{w}^2 > 0$

Theorem 3. $ℸ = [ℸ_{i\bar{j}}]$, $Ⅎ = [Ⅎ_{i\bar{j}}] \in$ PNHSM$(\widetilde{\Delta})_{r \times c}$, where $ℸ_{i\bar{j}} = \left(T^{ℸ}_{ij\bar{k}}, I^{ℸ}_{ij\bar{k}}, F^{ℸ}_{ij\bar{k}}\right)$ and $Ⅎ_{i\bar{j}} = \left(T^{Ⅎ}_{ij\bar{k}}, I^{Ⅎ}_{ij\bar{k}}, F^{Ⅎ}_{ij\bar{k}}\right)$. Then,

- ✔ $(ℸ \cup Ⅎ) = (Ⅎ \cup ℸ)$
- ✔ $(ℸ \cap Ⅎ) = (Ⅎ \cap ℸ)$
- ✔ $ℸ \cup ℸ = ℸ$
- ✔ $ℸ \cap ℸ = ℸ$
- ✔ $[ℸ_{i\bar{j}}] \subseteq [\tilde{1}]$
- ✔ $[\tilde{0}] \subseteq [ℸ_{i\bar{j}}]$
- ✔ $[ℸ_{i\bar{j}}] \subseteq [ℸ_{i\bar{j}}]$
- ✔ $[ℸ_{i\bar{j}}] \cup [\tilde{1}] = [\tilde{1}]$
- ✔ $[ℸ_{i\bar{j}}] \cup [\tilde{0}] = [ℸ_{i\bar{j}}]$
- ✔ $[ℸ_{i\bar{j}}] \cap [\tilde{0}] = [\tilde{0}]$
- ✔ $[ℸ_{i\bar{j}}] \cap [\tilde{1}] = [ℸ_{i\bar{j}}]$
- ✔ $(ℸ \cup Ⅎ)^C = ℸ^C \cap Ⅎ^C$
- ✔ $(ℸ \cap Ⅎ)^C = ℸ^C \cup Ⅎ^C$

- ✔ $(⅂^C \cap ꓷ^C)^C = ⅂ \cup ꓷ$
- ✔ $(⅂^C \cup ꓷ^C)^C = ⅂ \cap ꓷ$
- ✔ $(O^C)^C = O$
- ✔ $[0]^C = [1]$

Proof.

- ✔ $(⅂ \cup ꓷ) = [(T^{⅂}_{ijk□}, T^{ꓷ}_{ijk□}), min(I^{⅂}_{ijk□}, I^{ꓷ}_{ijk□}), min(F^{⅂}_{ijk□}, F^{ꓷ}_{ijk□})]$
  $= (T^{ꓷ}_{ijk}, I^{ꓷ}_{ijk}, F^{ꓷ}_{ijk}) \cup (T^{⅂}_{ijk}, I^{⅂}_{ijk}, F^{⅂}_{ijk}) = (ꓷ \cup ⅂).$

Remaining parts are proved in a similar way.

Theorem 4. Let $⅂ = [⅂_{ij}]$, $ꓷ = [ꓷ_{ij}] \in'$ PNHSM$(\widetilde{\Delta})_{r\times c}$, where $⅂_{ij} = (T^{⅂}_{ijk}, I^{⅂}_{ijk}, F^{⅂}_{ijk})$ and $ꓷ_{ij} = (T^{ꓷ}_{ijk}, I^{ꓷ}_{ijk}, F^{ꓷ}_{ijk})$. Then,

- ✔ $(⅂ \widetilde{@} ꓷ) = (ꓷ' \widetilde{@} ⅂)$
- ✔ $(⅂ \widetilde{@} ꓷ)^C = ⅂^C \widetilde{@} ꓷ^C$
- ✔ $(⅂ \widetilde{@}^{\widetilde{w}} ꓷ) = (ꓷ' \widetilde{@}^{\widetilde{w}} ⅂)$
- ✔ $(⅂' \widetilde{@}^{\widetilde{w}} ꓷ)^C = ⅂^C \widetilde{@}^{\widetilde{w}} ꓷ^C$
- ✔ $(⅂ \breve{\$} ꓷ) = (ꓷ' \breve{\$} ⅂)$
- ✔ $(⅂' \$^{\widetilde{w}} ꓷ) = (ꓷ' \$^{\widetilde{w}} ⅂)$
- ✔ $(⅂ \oslash □ꓷ) = (ꓷ \oslash □⅂)$
- ✔ $(⅂ \oslash □^{\widetilde{w}} ꓷ) = (ꓷ \oslash □^{\widetilde{w}} ⅂)$
- ✔ $(⅂ \cup ꓷ)^t = ⅂^t \cup ꓷ^t$
- ✔ $(⅂ \cap ꓷ)^t = ⅂^t \cap ꓷ^t$
- ✔ $(⅂' \widetilde{@} ꓷ)^t = ⅂^t \widetilde{@} ꓷ^t$
- ✔ $(⅂ \widetilde{@}^{\widetilde{w}} ꓷ)^t = ⅂^t \widetilde{@}^{\widetilde{w}} ꓷ^t$
- ✔ $(⅂' \breve{\$} ꓷ)^t = ⅂^t \$ ꓷ^t$
- ✔ $(⅂ \$^{\widetilde{w}} ꓷ)^t = ⅂^t \$^{\widetilde{w}} ꓷ^t$
- ✔ $(⅂ \oslash □ꓷ)^t = ⅂^t \oslash □ꓷ^t$
- ✔ $(⅂ \oslash □^{\widetilde{w}} ꓷ)^t = ⅂^t \oslash □^{\widetilde{w}} ꓷ^t$

Proof.

- ✔ $(⅂' \widetilde{@} ꓷ) = \left[\left(\frac{(T^{⅂}_{ijk□} + T^{ꓷ}_{ijk□})}{2}\right), \left(\frac{(I^{⅂}_{ijk□} + I^{ꓷ}_{ijk□})}{2}\right), \left(\frac{(F^{⅂}_{ijk□} + F^{ꓷ}_{ijk□})}{2}\right)\right]$
  $= (ꓷ' \widetilde{@} ⅂).$

Remaining parts are proved in a similar way.

Theorem 5. Let $⅂ = [⅂_{ij}]$, $ꓷ = [ꓷ_{ij}]$ and $í = [í_{ij}] \in'$ PNHSM$(\widetilde{\Delta})_{r\times c}$, where $⅂_{ij} = (T^{⅂}_{ijk}, I^{⅂}_{ijk}, F^{⅂}_{ijk})$ and $ꓷ_{ij} = (T^{ꓷ}_{ijk}, I^{ꓷ}_{ijk}, F^{ꓷ}_{ijk})$, $í_{ij} = (T^{í}_{ijk}, I^{í}_{ijk}, F^{í}_{ijk})$.

- ✔ $(⅂ \cup ꓷ) \cup í = ⅂ \cup (ꓷ \cup í)$
- ✔ $(⅂ \cap ꓷ) \cap í = ⅂ \cap (ꓷ \cap í)$
- ✔ $(⅂' \widetilde{@} ꓷ) \cap í = (⅂ \cap í) \widetilde{@} (ꓷ \cap í)$
- ✔ $(⅂ \widetilde{@} ꓷ) \cup í = (⅂ \cup í) \widetilde{@} (ꓷ \cup í)$

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- ✔ (ꓶ´@̃ ꓷ) @̃ί ≠ ꓶ @̃ (ꓷ @̃ ί)
- ✔ (ꓶ $ ꓷ) $˘ί ≠ ꓶ $ (ꓷ´$˘ί)
- ✔ (ꓶ ⊘ ▯ꓷ)⊘ ▯ί ≠ ꓶ ⊘ ▯ (ꓷ ⊘ ▯ί)
- ✔ ꓶ ∩ (ꓷ ∪ ί) = (ꓶ ∩ ꓷ) ∪ (ꓶ ∩ ί)
- ✔ (ꓶ ∩ ꓷ) ∪ ί = (ꓶ ∪ ί) ∩ (ꓷ ∪ ί)
- ✔ ꓶ ∪ (ꓷ ∩ ί) = (ꓶ ∪ ꓷ) ∩ (ꓶ ∪ ί)
- ✔ (ꓶ ∪ ꓷ) ∩ ί = (ꓶ ∩ ί) ∪ (ꓷ ∩ ί)
- ✔ (ꓶ ∩ ꓷ)@̃ ί = (ꓶ´@̃ ί) ∩ (ꓷ´@̃ ί)
- ✔ (ꓶ ∩ ꓷ) ⊘ ▯ ί = (ꓶ ⊘ ▯ί) ∩ (ꓷ ⊘ ▯ ί)
- ✔ (ꓶ ∪ ꓷ) ⊘ ▯ ί = (ꓶ ⊘ ▯ί) ∪ (ꓷ ⊘ ▯ ί)
- ✔ ꓶ @̃ (ꓷ ∪ ί) = (ꓶ´@̃ ꓷ) ∪ (ꓶ´@̃ ί)
- ✔ ꓶ´@̃ (ꓷ ∩ ί) = (ꓶ´@̃ ꓷ) ∩ (ꓶ´@̃ ί)
- ✔ ꓶ $ (ꓷ ∪ ί) = (ꓶ´$˘ꓷ) ∪ (ꓶ´$˘ί)
- ✔ (ꓶ ∪ ꓷ)$ ί = (ꓶ´$˘ί) ∪ (ꓷ´$ ί)
- ✔ ꓶ ⊘ ▯ (ꓷ ∪ ί) = (ꓶ ⊘ ▯ꓷ) ∪ (ꓶ ⊘ ▯ί)
- ✔ ꓶ $ (ꓷ ∩ ί) = (ꓶ´$ ꓷ) ∩ (ꓷ´$˘ί)
- ✔ (ꓶ ∪ ꓷ)$ ί = (ꓶ´$˘ί) ∪ (ꓷ´$ ί)

Proof.

(i) (ꓶ ∪ ꓷ) ∪ ί

$$= \left[\left(T^{ꓶ}_{ijK}, T^{ꓷ}_{ijK}\right), min\left(I^{ꓶ}_{ijK}, I^{ꓷ}_{ijK}\right), min(F^{ꓶ}_{ijK}, F^{ꓷ}_{ijK})\right] \cup \left[\left(T^{ί}_{ijK}, I^{ί}_{ijK}, F^{ί}_{ijK}\right)\right]$$

$$= \left[\left(T^{ꓶ}_{ijK}, T^{ꓷ}_{ijK}, T^{ί}_{ijK}\right), min\left(I^{ꓶ}_{ijK}, I^{ꓷ}_{ijK}, I^{ί}_{ijK}\right), min(F^{ꓶ}_{ijK}, F^{ꓷ}_{ijK}, F^{ί}_{ijK})\right]$$

$$= \left(T^{ꓶ}_{ijK}, I^{ꓶ}_{ijK}, F^{ꓶ}_{ijK}\right) \cup \left[\left(T^{ꓷ}_{ijK}, T^{ί}_{ijK}\right), min\left(I^{ꓷ}_{ijK}, I^{ί}_{ijK}\right), min(F^{ꓷ}_{ijK}, F^{ί}_{ijK})\right]$$

$$= \left(T^{ꓶ}_{ijK}, I^{ꓶ}_{ijK}, F^{ꓶ}_{ijK}\right) \cup \left[\left(T^{ꓷ}_{ijK}, I^{ꓷ}_{ijK}, F^{ꓷ}_{ijK}\right) \cup \left(T^{ί}_{ijK}, I^{ί}_{ijK}, F^{ί}_{ijK}\right)\right]$$

= ꓶ ∪ (ꓷ ∪ ί)

Remaining parts are proved in a similar way.

Conclusion

The various types and properties of the PNHSM have been well established. Substantial verifications for the proposed properties over the frameworks have additionally been given.

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