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THE USE OF EMPIRICAL MATHEMATICAL EQUATIONS FOR RELATIVE PERMEABILITY IN EVALUATING OIL AND GAS RESERVOIRS

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Abstract

Relative permeability for a fluid is the relative flow of that fluid when its saturation in the reservoir rock is less than a hundred percent. It is also the ratio of effective to absolute permeability at a given saturation condition of the rock pore spaces and the wetting characteristics of the fluid/rock surfaces. Direct relative permeability data is very difficult to determine for the Niger Delta reservoirs because of.

- Problems connected with obtaining sufficient and representative data for the area;
- No comprehensive work on this determination has so far been done in the area;
- Existing correlations from literature gives inaccurate values for the area.

In this investigation, empirical equations for two phase relative permeability data (oil/water, gas/oil systems), are developed by processing a large set of core data samples from special core analysis of various wells of the Niger Delta reservoirs.
The generalized empirical equation:

$Krf = RWe\lambda s$, serves as a:

- First estimate of the relative permeability values for Niger Delta reservoirs which have little or no available data information.
- Means of cross-checking relative permeability values obtained through other methods.
- Means of obtaining the relative permeability data when good estimate of fluid saturations are possible.
- Good and economical starting set of relative permeability data during history matching phase of reservoir simulation for the Niger Delta reservoirs.

Keywords; Relative permeability, reservoirs, Rocks, empirical equations, and fluids

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INTRODUCTION

Permeability is a measure of the flow of saturating fluid through a porous rock medium. When the rock medium is one hundred percent saturated by the fluid, the permeability so obtained is absolute whereas if the fluid saturating the rock is less than one hundred percent, the fluid permeability is regarded as effective. The ratio of effective permeability of a fluid to absolute permeability is the relative permeability of that fluid. Relative permeability is a function of the rock pore spaces and the wetting characteristics of both fluid/fluid and the fluid/rock surfaces. Relative permeability data can be obtained through laboratory methods involving steady and unsteady state displacement processes, capillary pressure data, field data, and published correlations.

Laboratory methods for relative permeability determination involve the use of cores. In the Niger Delta of Nigeria, coring is sometimes very difficult especially in the unconsolidated formations. In general, coring and/or special core analyses are not very common in the Niger Delta because:

(a) It is very expensive to do.
(b) It is not very easy to obtain core samples at reservoir conditions of temperature and pressure on surface.
(c) Some operators play down on the importance of coring especially at times of production cuts and low crude prices when the prospects of future field development are not very clear. Normally operators core the first wells of a field to enable decisions concerning field development to be made.

However, in the absence of core analysis, correlations are used for estimative relative permeability. Some of the global correlations used in literature 1, 2, and 3, for drainage and inhibition processes give a first approximation estimate of relative permeability. It has been observed that while it is convenient to employ any of these correlations in our calculations, it is important to determine how well each of them actually represents the formation under consideration. This is particularly important during the history matching process of reservoir simulation where the shapes of the relative permeability functions are usually adjusted by changing the parameters of the fitting equations to match the actual production data. In this regard selection of a relative permeability correlation that will give very close values to the actual production data results in a quick and inexpensive solution that converges fast to the desired value.

In this study we obtained empirical equations for estimating relative permeability values that match closely actual data from the Niger Delta reservoirs. In this paper, we shall regard a good laboratory determined data as actual data when the core samples emanate from fresh, well preserved whole core samples taken using native crude oil as drilling fluid and selected pressure core techniques. The main objectives of our investigation are to obtain correlating equations that will:

(1) Give closer data values to the actual values than any of the global correlations;
(2) Give quicker and inexpensive convergence during history matching phase of any reservoir simulation;
(3) In cases where there are inadequate data or laboratory measurements or no data at all, our correlation will come handy as a check or good estimate for the desired relative permeability values.

DEVELOPMENT AND GENERATION OF EQUATION:

One of the objectives of this investigation is to obtain a set of relative permeability functions which will estimate closely actual relative permeability values to be used in reservoir simulation. More than 250 core data from some oil/gas fields representative of the Niger Delta were processed and used in the determination of two- phase relative permeability values. Linear regression analysis was employed to develop relevant logarithmic form of equations that approximated laboratory measured relative permeability values. The coefficient of correlation for each equation was determined and was found to be very close to unity. A statistical test was also carried out to remove “Out of place” data from the set. The proposed equation is an exponential curve of the form:

Krf = $We^{\lambda s}$ ………………… (1)
where Krf = relative permeability to fluid
W, λ = constants
s = saturation.

Irreducible/Critical Saturations: Considering that the oil/water imbibitions and gas/oil drainage data were measured in the presence of irreducible water and critical gas saturations, which were not taken care of in the above equation, Equation 1 was modified by “R” to:

$$Krf = WRe^{\lambda s} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\text{where } R = Rw = \frac{1 - Swi - Sor}{Sw - Swi} \text{ for Krw} \ldots\ldots (3)$$

$$R = Row = \frac{1 - Swi - Sor}{1 - Sw} \text{ for krow} \ldots\ldots (4)$$

$$R = Rg = \frac{1 - Sgi - Sor}{Sg - Sgi} \text{ for krg} \ldots\ldots (5)$$

$$R = Rog = \frac{1 - Sgi - Sor}{1 - Sg - Sor} \text{ for krog} \ldots\ldots (6)$$

Usir

letermined. Thus, for oil/water imbibitions

syst

Krw = 0.00007135 Rwe l0.65Sw ………… (7)
Krow = o.00009586 Row e 1 2.85Sow ……..(8)

For gas/oil Drainage System:

Krg = 0.0001068 Rge 15.O2Sg………… (9)

Krog = 0.000003778 Rog e 12.87Sog…….(10)

APPICATION/COMPARISM WITH EXPERIMENTAL DATA

Values for relative permeability for 3 different reservoirs in the Niger Delta were determined using our proposed gas/oil drainage and oil/water imbibitions equations. The values obtained are tabulated in Tables 1 to 3. Also calculated and tabulated along with our values are experimental values, values from Wyllie [2] Pirson[3] and modified .Wyllie[4] equations. Table 1 shows the variation in relative permeability with respect to saturation for a formation with irreducible water and residual oil saturations of 0.207 and 0.263 respectively. The values from the five different methods were compared. Tables 2 and 3 also show the values obtained when irreducible saturations are 0.194 and 0.113 respectively and residual oil saturations are 0.286 and 0.218 respectively (See Appendix - 1 for the equations). Tables 4 and 5 are, for as Gas-oil system.

RESULTS / DISCUSSIONS

Values of oil relative permeability displayed in Table 1 —3 were plotted against their corresponding saturations. Figures 1 - 3 show how the various correlations 2 -4 compare with the experimental data for the three different formations (“A”, “B and C”) in the Niger Delta. It was observed that our proposed equations consistently corresponded more closely with the experimental data than any of the other correlations 2-4 (The modified Wyllie[4] presented here is still under investigation by the author.) In Figure 4 we plotted relative permeabilities (oil and water) against saturations (oil and water) on a semi-log paper. The curve shows that we can use the proposed equations to

Table 1: Calculated and Experimental values of relative permeability for a formation "A" in the xyz Reservoir Niger Delta, Nigeria.

Formation "A"

θ = 17.1%
Swi = 20.7%
Kair = 58.2md
Ko (Swi) = 48.8md
Kw (so r) = 14.4md
ROS = 26.3%

Sw	Krw (EXPT)	Krw (MODEL)	Kro (EXPT)	Kro (MODEL)	Kro WYLLIE)	Kro (PIRSON)	Kro * (MODIFIED)
24.00	0.001	0.0148	0.700		0.9 16	0.60	0.607
33.00	0.003	0.0103	0.400	0.416	0.697	0.43l	0310
39.40	0.005	0.0134	0.210	0404	0.551	0.356	0.202
45.00	0.010	0.0188	0.100	0.1084	0.436	0303	0.138
52.10	027	0.03094	0.039	0.050	0308	0247	0.083
62.70	0.106	0.07152	0.008	0.0164	0.159	0.176	0,035
70.0	0.205	0.1326	0.002	0.008	0.088	0.13	0.017

*MODIFIED WYLLIE Eq. - by the author.

Table 2: Calculated and experimental values of relative permeability for a formation B" in the xyz Reservoir Niger Delta, Nigeria.

Formation "B"
θ = 14.8%
Swi = 19.4%
Kair = 23.6%
Ko (Swi) = 20.8 md
Kw (So r) = 2.5 md
ros
ROS = 28.6%

Sw	Krw (EXPT)	Krw (MODEL)	Kro (EXPT)	Kro (MODEL)	Kro WYLLIE)	Kro (PIRSON)	Kro * (MODIFIED)
22.00	0.003	0.0148	0.80	1.00	0.936	0.64	0.65
29.00	0.012	0.0085	0.58 1	0.644	0.765	0.484	0.377
40.00	0.017	0.0127	0.235	0.185	0.725	035	0.182
49.50	0.023	0.0239	0.033	0.065	0337	0.269	0.095
60.70	0.059	0.0577	0.004	0.012	0.1753	0.192	0.039
69.00	0.097	0.1162	0.001	0.005148	0.092	0.143	0.018

* MODIFIED WYLLIE Eq. = by the author**.**

Table 3: Calculated and experimental values of relative Permeability for a formation “C” in xyz Reservoir the Niger Delta, Nigeria.

Formation “C”

θ = 19%
Swi = 11.3%
Kair = 726.Omd
Ko(Swi) = 604.OOmd
Kw(or) = 229.OOmd
ROS = 21.8%

Sw	Krw (EXPT)	Krw (MODEL)	Kro (EXPT)	Kro (MODEL)	Kro WYLLIE)	Kro (PIRSON)	Kro * (MODIFIED)
15.00	0.001	0.006	0.700		0.906	0.698	0.606
22.00	0.003	0.005	0.465		0.762	0.560	0375
28.90	0.006	0.006	0.271	0.837	0.617	0.482	0.247
41.90	0.038	0.014	0.055	0.366	0.378	0.361	0.112
57.00	0.137	0.045	0.010	0.037	0.173	0.248	0.039
66.00	0.215	0.100	0.006	0.015	0.091	0.189	0.017
75.00	0.29	0.22	0.001	0.006	0.039	0.064	0.006

*MODIFIED WYLLIE Equation - by the author.

Estimate closely actual relative permeability values for the Niger Delta especially in areas with no core analysis or no data information.

Conclusions: A set of empirical equations has been developed to estimate accurately the relative permeability of reservoirs in the Niger Delta especially in areas with no data information. The adoption of these equations will result in quicker and less expensive match of production history during reservoir simulation. The equations are also particularly suitable for monitoring gas movement in solution gas drive systems.

Nomenclature:

Kair = relative permeability to air, md

Krg = gas relative permeability, fraction
KrOg = Oil relative permeability in G/0 system, fraction
KrOw = Oil relative permeability in 0/W system, fraction
Krw = water relative permeability fraction
Ko (Swi) = Oil permeability at irreducible water sat., md
Kw (S or) = Water permeability at residual oil sat., md
Row, Rw, Rg = Correlation parameters (see equations3-5)
Ros = residual Oil Saturation
w, = Empirical constants
Sg = Gas saturation, fraction
Sgi = critical gas saturation, fraction
Sw = Water saturation, fraction
Swi = Irreducible water saturation, fraction

Table 4
Calculated and experimental values of two phase relative penne ability for a formation “D” in xyz Reservoir the Niger Delta.

FORMATION “D” Gas - oil system
θ = 17.2%
Swi = 13.4%
Kair = 323.2md
Ko (Swi) = 278.7md
Sgc = 2%
Sor = 14%

Sg	Krg (EXPT)	Krg (Model)	Kro (EXPT)	Kro (Model
4.1	0.001	0.008	0318	0.89
7.0	0.009	0.005	0.624	0.888
15.80	0.039	0.007	0388	0.6342
25.00	0.081	0.017	0.204	0.230
40.90	0.182	0.110	0.440	0.081
55.80	0.369	0.72	0.006	0.014
68.90	0.570	xx	0.001	0.003
72.6	0.634	xx		

xx Values greater than 1.0

Table 5:
Calculated and experimental values of two phase relative permeability for formation “E.. in the xyz Reservoir Niger Delta.

FORMATION "E" Gas-Oil System

θ = 12.5%
Swi = 14.5%
Kair = 45md
Ko (Swi) = 35.9md
Sgc = 5.5%
Sor = 22%

Sg	Krg (EXPT)	Krg (Model)	Kro (EXPT)	Kro (Model
8.6	0.001	0.009	0.511	0.520
11.80	0.004	0.0072	0.411	0352
19.60	0.024	0.010	0.212	0.146
35.10	0.110	0.051	0.056	0.027
41.70	0.207	0.1123	0.029	0.014
51.90	0.434	0.41	0.008	0.005
60.50	0.625	xx	0.001	0.0025
63.50	0.696	xx	-	-

xx Values greater than l.0

θ = Porosity
ww = Constant for Krw eq.
Wow = Constant in Oil / water system
Wog = Constant in gas / oil system
Wg = Constant in Krg eq

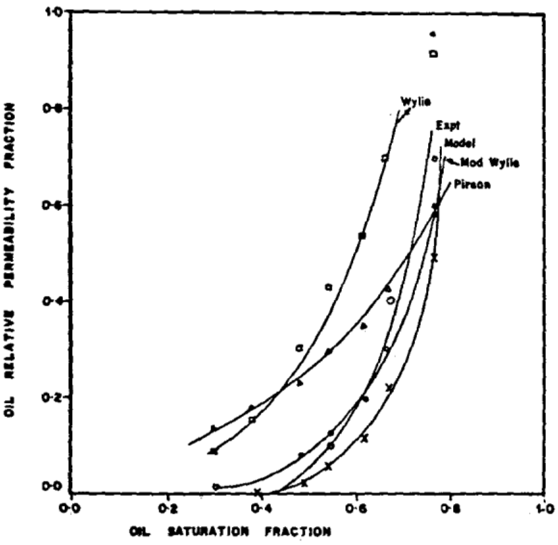

Fig 1: PLOT OF Kro versus So Formation A

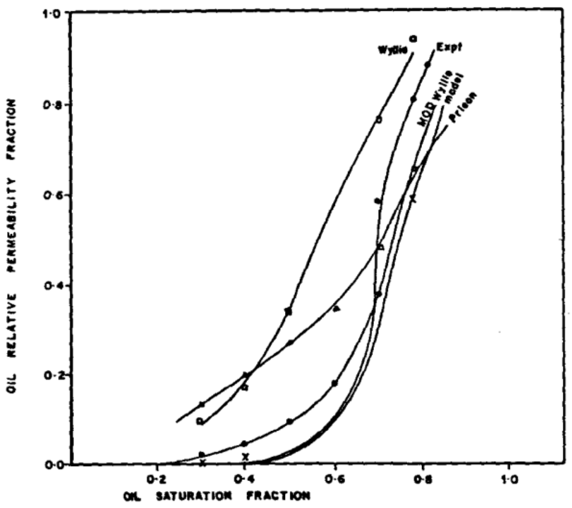

Fig 2: PLOT OF Kro versus So Formation B

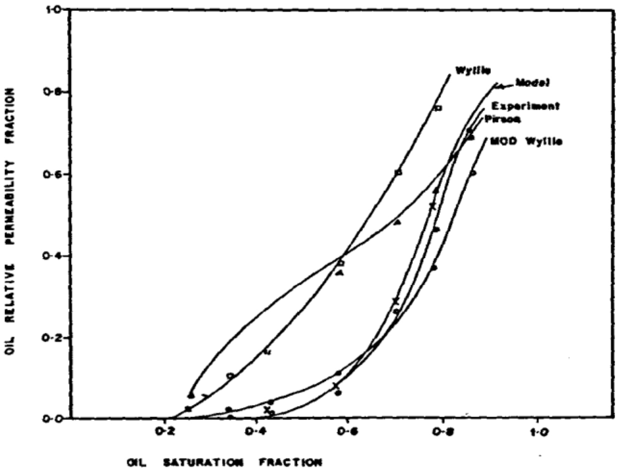

Fig 3: PLOT OF Kro versus So Formation C

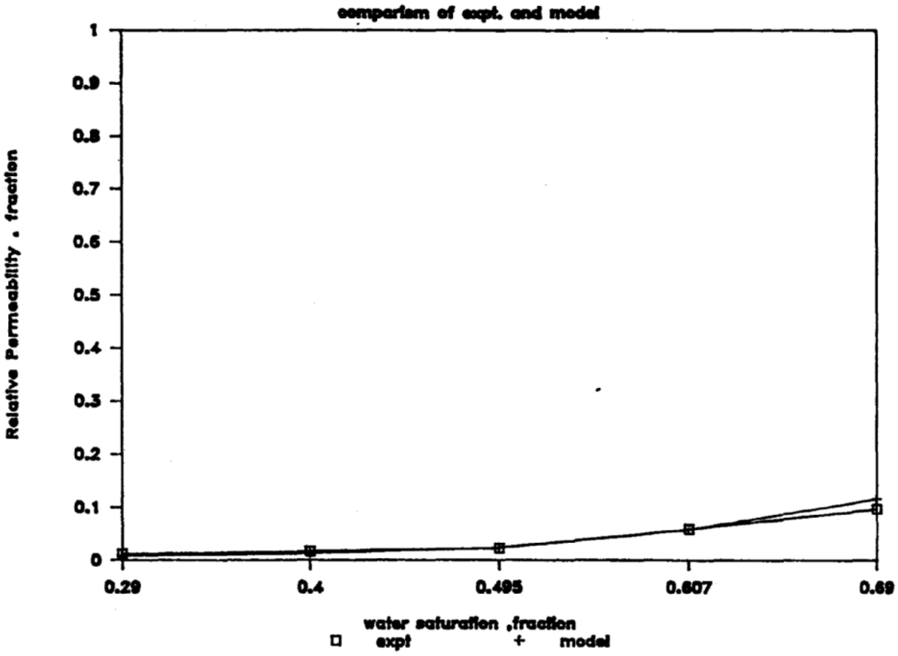

Fig 4: RELATIVE PERMEABILITY TO WATER. 0/W s**yst**

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APPENDIX - 1
Wyllie, Pirson and Modified Wylie Equations;

Wylie equation:

$Kro = (1\_S^*)^2\ [1(5^*)^2]$ **…………….** (11)

Where
$S^* = (Sw - Swi) / (1 - Swi)$ **………..** (12)
Pirson equation:
$Kro = (1-S^*)\ [-(S^*)\ ¼\ Sw½]^2$ **………** (13)

Modified Wyllie equation:
$Kro = [(1 - Sm)(1+Sm)\}^2\ [(1-Sm^2)\ ‘(l+Sm^2)]$ **…** (14)

where: $Sm = S^*$ **…………………………….** (15)