DIFFERENT KINDS OF CORDIAL LABELING ON IDENTITY GRAPHS

[1]N. Mohamed Rilwan, [2]A. Syed Hussain

[1]Assistant Professor, Department of Mathematics,
Sadakathullah Appa College, Tirunelveli-627 011, Tamil Nadu, India.
Corresponding Email ID: rilwan2020@gmail.com

[2]Research Scholar (Full time), Register No 18131282091017,
Department of Mathematics, Sadakathullah Appa College, Tirunelveli-627011,
Tamil Nadu, India. Email ID: hussainmphil@gmail.com

Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012,
Tamil Nadu, India.

Abstract

A graph $G = (V, E)$ is said to be *identity graph* if the vertex set $V(G) = \Gamma$ and the edge set $E(G) = \{(x, y) \cup (x, e) | x * y = e, x \neq y \text{ and } x, y \in \Gamma\}$ where Γ is a group and e as an identity element. In this paper, we substantiate some of the results about E-cordial, *k*-cordial labeling on identity graph associated with some groups.

Keywords: Identity graph, E-cordial, *k*-cordial, cyclic group, dihedral group.
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1 Introduction:

In this paper, we prefer to work related on graph from algebraic structures which means that there is an inter-relation between groups and graphs. In particular, we focus on identity graph which was first introduced by W. B. Vasantha Kandasamy [5]. In view of [5], "Every finite group can be view in form of graph". Rosa [4] initiated the concept on certain valuations of the vertices of a graph in 1967. Graph labeling investigation started in middle of 1990's. Graph labeling was first introduced by Rosa [4] in 1967. A *graph labeling* is an assignment of integers to the vertices (or) edges (or) both with respect to certain defined conditions. For the past five decades, the researchers and the findings on graph labeling has brought forth a new revelation and its broad range of applications like communication network, coding theory, circuit design etc. In particular, k-cordial labeling for certain classes of graphs were studied in 2009 by M.Z.Youssef [7].

I. Cahit [1] introduced a variation of both graceful and harmonious labeling. In [3], Hovey defined a function g is said to be a *cordial* of G if the number of vertices (edges) labeled with 0 and number of vertices (edges) labeled with 1 differ by at most 1. In 1991, Hovey [3] proposed the idea generalizations of harmonious and cordial labeling. In [7], a graph $G = (V, E)$ is said to be k-*cordial* if there is a labeling function g on the vertex set of G to $\{\overline{0}, \overline{1}, \ldots, \overline{k-1}\}$, it induces an edge labeling such that $xy = \big(g(x) + g(y)\big)(\text{mod}\, k)$ for all x, y in $V(G)$, then $|i - j| \leq 1$ where i and j be the number of vertices (edges) of labels of G. In this research script, we venture to explicate the existence of k-cordial and E-cordial on identity graph associated to some special groups.

2 Preliminaries:

Definition 2.1. [2] Let G be a graph. A labeling $g: V(G) \rightarrow \{0,1\}$ is called a *binary labeling* of G. A binary labeling g of G induces an edge labeling $g*$ of G as follows: $g*(u,v) = |g(u) - g(v)|$ for every edge $e = uv \in E(G)$. Let $v_g(0)$ and $v_g(1)$ be the number of vertices of G labeled with 0 and 1 under g and $e_{g*}(0)$ and $e_{g*}(1)$ be the number of edges labeled with 0 and 1 under $g*$ respectively. The binary labeling of G is said to be *cordial* if $|v_g(0) - v_g(1)| \leq 1$ and $|e_{g*}(0) - e_{g*}(1)| \leq 1$.

Definition 2.2. [6] Let G be a graph with vertex set $V(G)$ and edge set $E(G)$ and let g from $E(G)$ to $\{0,1\}$. Define $g*$ on $V(G)$ by $g*(v) = \sum\{g(uv) | uv \in E(G)\}$ (mod 2). The function g is called an E-*cordial* labeling of G if $|v_{g*}(0) - v_{g*}(1)| \leq 1$ and $|e_g(0) - e_g(1)| \leq 1$. A graph is called E-*cordial* if it admits E-cordial labeling.

Definition 2.3. [7] Let $G(V,E)$ be a graph. A vertex labeling g from $V(G)$ to $\mathbb{Z}_k$ induces an edge labeling g^+ from $E(G)$ to $\mathbb{Z}_k$, defined by $g^+(xy) = (g(x) + g(y))(\text{mod}\, k)$, for all edges which belongs to the edge set $E(G)$. For $i \in \mathbb{Z}_k$, let $n_i(g) = |\{v \in V(G) | g(v) = i\}|$ and $m_i(g) = |\{e \in E(G) | g^+(e) = i\}|$. A labeling g of a graph G is called k-*cordial* if $|n_i(g) - n_j(g)| \leq 1$ and $|m_i(g) - m_j(g)| \leq 1$ for all $i, j \in \mathbb{Z}_k$.

Theorem 2.4. [6] Necessary condition for a graph G, to admit an E-cordial labeling is $n \not\equiv 2(\text{mod}\, 4)$, where n denotes the number of vertices of G.

Definition 2.5. [5] A graph G is said to be $identity\ graph$ if the vertex set $V(G)$ contains the elements of group Γ and the realtion between two vertices x, y is defined on the edge set $E(G) = \{(x,y) \cup (x,e) | x * y = e, x \neq y$ and $x, y \in \Gamma\}$ where is e an identity element. That is if we say two elements x, y in the group are adjacent or can be joined by edge if $x * y = e$.

Definition 2.6. The dihedral group of order $2n$ is the group formed by the symmetries of a regular n-gon and it is denoted by D_{2n}. Let r be the rotation of the n-gon by $\frac{2\pi}{n}$ radians and let s be the reflection across the line connecting to the center of the object.

(1) $e, r, r^2, \ldots, r^{n-1}$ are all distinct and $r^n = e$
(2) $o(s) = 2$
(3) $s \neq r^i$ for any i
(4) $r^i s \neq r^j s$ for all $0 \leq i, j \leq n-1$ with $i \neq j$.

From this, we can conclude that $D_{2n} = \{e, r, r^2, \ldots, r^{n-1}, s, rs, r^2 s, \ldots, r^{n-1}s\}$.

It is interesting to note that when we construct the identity graph under the group $\mathbb{Z}_n$, we grasp that if n is odd, the identity graph $Id(\mathbb{Z}_n)$ contains only $\left\lfloor \frac{n}{2} \right\rfloor$ triangles and if n is even, the identity graph $Id(\mathbb{Z}_n)$ contains $\frac{n-2}{2}$ triangles and a line incident with apex vertex.

Let us construct the identity graph under the group D_{2n}, identity graph $Id(D_{2n})$ contains $s-1$ traingles when $2n \equiv 0 \pmod 4$ and s traiangles when $2n \not\equiv 0 \pmod 4$ where s is a quotient and it contains $2k+3$ lines when $n \equiv 0, 2 \pmod 4$ and $2k+1$ lines when $n \equiv 1, 3 \pmod 4$ where k denotes number of triangles occur in the identity graph $Id(D_{2n})$ graph.

3 E- cordial Labeling on Identity graph:

Theorem 3.1. *Let Γ be a finite cyclic group of order n where $n \geq 3$. Then the Identity graph $Id(\Gamma)$ admits E-cordial labeling.*

Proof. Let $\Gamma = \langle a | a^n = 1 \rangle$ be a finite cyclic group of order n.
Case 1: n is odd and $n \equiv 1,3 \pmod 4$. Let $e_1, e_2, \ldots, e_{3\left\lfloor \frac{n}{2} \right\rfloor}$ be the edges of $Id(\Gamma)$. Let us consider the edges $e_1, e_2, \ldots, e_k$ be the matchings M_k where k be the number of triangles in $Id(\Gamma)$ and the remaining edges $e_{k+1}, e_{k+2}, \ldots, e_{3\left\lfloor \frac{n}{2} \right\rfloor}$ be star S_{n-k}. Define a labeling $l: E(G) \rightarrow \{0,1\}$ as follows: For $1 \leq i \leq k$ and for $k+1 \leq i \leq 3\left\lfloor \frac{n}{2} \right\rfloor$,

$$l(e_i) = \begin{cases} 0 & if \quad i \; is \; odd \\ 1 & if \quad i \; is \; even \end{cases}$$

Case 2: n is even and $n \equiv 0 \pmod 4$. Let $e_1, e_2, \ldots, e_{3\left(\frac{n-2}{2}\right)}, e_{3\left(\frac{n-2}{2}\right)+1}$ be the edges of $Id(\Gamma)$. Let us consider the edges $e_1, e_2, \ldots, e_k$ be the matchings $M_k$where k be the number of triangles in $Id(\Gamma)$ and $e_{k+1}, e_{k+2}, \ldots, e_{3\left(\frac{n-2}{2}\right)}$ be the edges of star S_{n-k}. For $1 \leq i \leq 3\left(\frac{n-2}{2}\right)$,

$$l(e_i) = \begin{cases} 0 & \text{if} \quad i \text{ is odd} \\ 1 & \text{if} \quad i \text{ is even} \end{cases}$$

and $l(e_p) = 1$ where e_p denote an edge incident with pendent vertex. From the above said labeling, we infer that vertex and edge conditions of E-cordial labeling established.

Theorem 3.2. *Let Γ be a group. Then the identity graph $Id(\Gamma)$ where $\Gamma = D_{2n}, n \geq 3$ and $n \not\equiv 1\ (mod\ 2)$ admits E-cordial labeling.*

Proof. Since $n \equiv 0$ (mod 2). Let $e_1, e_2, \dots, e_k$ be the matchings M_k where k be the number of triangles in $Id(\Gamma)$ and $e_{k+1}, e_{k+2}, \dots, e_{3k}$ be the edges of star S_{2k} and $e_{3k+1}, e_{3k+2}, \dots, e_p$ where e_p denotes the number of edges incident with pendent vertices. Define a labeling $l: E(G) \to \{0,1\}$ as follows:
For $1 \leq i \leq k$, $k+1 \leq i \leq 3k$ and $3k+1 \leq i \leq p$,

$$l(e_i) = \begin{cases} 0 & \text{if} \quad i \text{ is odd} \\ 1 & if \quad i \text{ is even} \end{cases}$$

From the above said labeling, it is evident that E-cordial labeling exists.

Theorem 3.3. *Let Γ be a group. Then the identity graph $Id(\Gamma)$ where $\Gamma = \mathbb{Z}_m \times \mathbb{Z}_n, 2 \leq p \leq q$ admits E-cordial labeling.*

Proof. Since $2 \leq$ m $\leq$ n and $\eta = mn$. The identity graph $Id(\Gamma)$ where $\Gamma = \mathbb{Z}_m \times \mathbb{Z}_n$ contains η vertices.
Case 1: $m \equiv 2$ (mod 4) and $\eta \not\equiv 2$ (mod 4). Let $e_1, e_2, \dots, e_{3\left(\frac{\eta-2}{2}\right)}$ be the edges of $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$. Let us consider the edges $e_1, e_2, \dots, e_{3\left(\frac{\eta-4}{2}\right)}$ be the matchings in $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$ and the remaining edges $e_{\left(\frac{\eta-4}{2}\right)+1}, e_{\left(\frac{\eta-4}{2}\right)+2}, \dots, e_{3\left(\frac{\eta-2}{2}\right)}$ be a star $S_{n-\left(\frac{\eta-4}{2}\right)}$. Define a labeling $l: E(G) \to \{0,1\}$ as follows:
For $1 \leq i \leq \left(\frac{\eta-4}{2}\right)$, $\left(\frac{\eta-4}{2}\right) + 1 \leq i \leq 3\left(\frac{\eta-2}{2}\right)$

$$l(e_i) = \begin{cases} 0 & \text{if} \quad i \text{ is odd} \\ 1 & \text{if} \quad i \text{ is even} \end{cases}$$

Case 2: $p \equiv 0$ (mod 4) and $\eta \equiv 0$ (mod 4).

Subcase 2.1. When n is even. Let $e_1, e_2, \dots, e_{3\left(\frac{\eta-2}{2}\right)}$ be the edges of $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$. Let us consider the edges $e_1, e_2, \dots, e_{\left(\frac{\eta-4}{2}\right)}$ be the matchings in $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$ and the remaining edges $e_{\left(\frac{\eta-4}{2}\right)+1}, e_{\left(\frac{\eta-4}{2}\right)+2}, \dots, e_{3\left(\frac{\eta-2}{2}\right)}$ be a star $S_{n-\left(\frac{\eta-4}{2}\right)}$. Define a labeling $l: E(G) \to \{0,1\}$ as follows:
For $1 \leq i \leq \left(\frac{\eta-4}{2}\right)$ and for $\left(\frac{\eta-4}{2}\right) + 1 \leq i \leq 3\left(\frac{\eta-2}{2}\right)$,

$$l(e_i) = \begin{cases} 0 & \text{if} \quad i \text{ is odd} \\ 1 & \text{if} \quad i \text{ is even} \end{cases}$$

Subcase 2.2. When n is odd.

The identity graph $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$ contains $\left(\frac{\eta-2}{2}\right)$ triangles and one line. Let $e_1, e_2, \ldots, e_{\left(\frac{3\eta-4}{2}\right)}$ be the edges of $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$. Let us consider $e_1, e_2, \ldots, e_{\left(\frac{\eta-2}{2}\right)}$ be the matchings in $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$ and remaining edges $e_{\left(\frac{\eta-2}{2}\right)+1}, e_{\left(\frac{\eta-2}{2}\right)+2}, \ldots, e_{\left(\frac{3\eta-4}{2}\right)}$ be a star $S_{n-\left(\frac{\eta-2}{2}\right)}$.

Define a labeling $l: E(G) \rightarrow \{0,1\}$ as follows:

For $1 \leq i \leq \left(\frac{\eta-2}{2}\right)$ and for $\left(\frac{\eta-2}{2}\right) + 1 \leq i \leq \left(\frac{3\eta-4}{2}\right)$,

$$l(e_i) = \begin{cases} 0 & \text{if} \quad i \text{ is odd} \\ 1 & \text{if} \quad i \text{ is even} \end{cases}$$

Case 3: $m \equiv 1,3 \pmod 4$. If n is even, the identity graph $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$ contains $\left(\frac{\eta-2}{2}\right)$ triangles and one line. Let $e_1, e_2, \ldots, e_{\left(\frac{3\eta-4}{2}\right)}$ be the edges of $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$. Let us consider $e_1, e_2, \ldots, e_{\left(\frac{\eta-2}{2}\right)}$ be the matchings in $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$ and remaining edges $e_{\left(\frac{\eta-2}{2}\right)+1}, e_{\left(\frac{\eta-2}{2}\right)+2}, \ldots, e_{\left(\frac{3\eta-4}{2}\right)}$ be a star $S_{n-\left(\frac{\eta-2}{2}\right)}$. Define a labeling $l: E(G) \rightarrow \{0,1\}$ as follows:

For $1 \leq i \leq \left(\frac{\eta-2}{2}\right)$ and for $\left(\frac{\eta-2}{2}\right) + 1 \leq i \leq \left(\frac{3\eta-4}{2}\right)$

$$l(e_i) = \begin{cases} 0 & \text{if} \quad i \text{ is odd} \\ 1 & \text{if} \quad i \text{ is even} \end{cases}$$

Subcase 3.2. If n is odd, the identity graph $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$ contains $\left(\frac{\eta-1}{2}\right)$ triangles. Let $e_1, e_2, \ldots, e_{3\left(\frac{\eta-1}{2}\right)}$ be the edges of $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$. Let us consider these edges $e_1, e_2, \ldots, e_{\left(\frac{\eta-1}{2}\right)}$ be the matchings in $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$ and remaining edges $e_{\left(\frac{\eta-1}{2}\right)+1}, e_{\left(\frac{\eta-1}{2}\right)+2}, \ldots, e_{3\left(\frac{\eta-1}{2}\right)}$ be a star $S_{n-\left(\frac{\eta-2}{2}\right)}$. Define a labeling $l: E(G) \rightarrow \{0,1\}$ as follows:

For $1 \leq i \leq \left(\frac{\eta-1}{2}\right)$ and for $\left(\frac{\eta-1}{2}\right) + 1 \leq i \leq 3\left(\frac{\eta-1}{2}\right)$,

$$l(e_i) = \begin{cases} 0 & \text{if} \quad i \text{ is odd} \\ 1 & \text{if} \quad i \text{ is even} \end{cases}$$

This labeling proves that, $Id(\Gamma)$ admits E-cordial labeling.

The following results are immediate from the above theorem.

Proposition 3.4. Let $Id(\mathbb{Z}_m)$, $Id(\mathbb{Z}_n)$ be two E-cordial graphs then $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$ is also E-cordial.

Proposition 3.5. Let $Id(\mathbb{Z}_m)$, $Id(\mathbb{Z}_n)$ be two non E-cordial graphs then $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$ is E-cordial.

Proposition 3.6. Let $Id(\mathbb{Z}_m)$ be E-cordial and $Id(\mathbb{Z}_n)$ be non E-cordial graphs then $Id(\mathbb{Z}_m \times \mathbb{Z}_n)$ is E-cordial.

4 k-cordial labeling on Identity graph:

In this section, we demonstrate the k-cordial labeling on Identity graph associated with the groups $\mathbb{Z}_n$ and $D_{2\mathrm{n}}$.

Definition 4.1. [7] Let $G(V,E)$ be a graph. A vertex labeling f from $V(G)$ to $\mathbb{Z}_k$ induces an edge labeling f^+ from $E(G)$ to $\mathbb{Z}_k$, defined by $f^+(xy) = (g(x) + g(y))(mod\, k)$, for all edges $xy \in E(G)$. For $i \in \mathbb{Z}_k$, let $n_i(f) = |\{v \in V(G)|f(v) = i\}|$ and $m_i(f) = |\{e \in E(G)|\, f^+(e) = i\}|$. A labeling f of a graph G is called *k-cordial* if $|n_i(f) - n_j(f)| \leq 1$ and $|m_i(f) - m_j(f)| \leq 1$ for all $i,j \in \mathbb{Z}_k$.

Theorem 4.2. *Let Γ be a finite cyclic group of order n where n is odd and $n \geq 3$. Then the Identity graph $Id(\Gamma)$ admits k-cordial labeling.*

Proof. Let $\Gamma = \langle a | a^n = 1\rangle$ be a finite cyclic group of order n. It is clear that the identity graph $Id(\Gamma)\backslash\{v_0\}$ contains $\left\lfloor\frac{n}{2}\right\rfloor$ matchings namely $G_1, G_2, \dots, G_{\left\lfloor\frac{n}{2}\right\rfloor}$ where v_0 is an apex vertex. Let $X = \left\{v_1, v_2, \dots, v_{\left\lfloor\frac{n}{2}\right\rfloor}\right\}$ and $Y = \left\{{v_1}^*, {v_2}^*, \dots, {v_{\left\lfloor\frac{n}{2}\right\rfloor}}^*\right\}$ be the partitions of the bipartite graph $Id(\Gamma)\backslash\{v_0\}$. Note that v_i is adjacent only to ${v_i}^*$. Define a labeling $l\colon V(G) \to \{0,1,\dots,n-1\}$ as follows:

$$l(x) = \begin{cases} 0 & \text{if} \quad x = v_0 \\ i & \text{if} \quad x = v_i,\ 1 \leq i \leq \left\lfloor\dfrac{n}{2}\right\rfloor \\ \left(n - \left\lfloor\dfrac{n}{2}\right\rfloor\right) + i & \text{if} \quad x = v_i \end{cases}$$

It is clear that, for all $a^i, a^j \in \Gamma, ||l(v_i)| - |l(v_j)|| \leq 1$ because all the vertex labels are assigned only once, and considering the edge labels it is clear that the maximum number of occurrence of an edge label is twice. Therefore we conclude that $||l(e_i)| - |l(e_j)|| \leq 1$.

Theorem 4.3. *Let Γ be a finite cyclic group of order n where n is even and $n \geq 4$. Then the Identity graph $Id(\Gamma)$ admits k-cordial labeling.*

Proof. Let $\Gamma = \langle a | a^n = 1\rangle$ be a finite cyclic group of order n. In identity graph $Id(\Gamma)$ we denote the apex vertex by v_0 and let $l(v_0) = 0$ and $l(v_p) = \frac{n}{2}$. From the construction of identity graph $Id(\Gamma)$ it is clear that the identity graph $Id(\mathbb{Z}_n)\backslash\{v_0, v_p\}$ contains $\frac{n}{2} - 1$ line graph, namely $G_1, G_2, \dots, G_{\frac{n}{2}-2}, G_{\frac{n}{2}-1}$.

Let $X = \left\{v_1, v_2, \dots, v_{\frac{n}{2}-1}\right\}$ and $Y = \left\{{v_1}^*, {v_2}^*, \dots, {v_{\frac{n}{2}-1}}^*\right\}$ be the partitions of the bipartite graph $Id(\Gamma)\setminus\{v_0, v_p\}$. Define a labeling $l: V(G) \to \{0,1, \dots, n-1\}$ as follows:

$$l(x) = \begin{cases} i & \text{if} \quad x = v_i \quad \text{where } 1 \le i \le \dfrac{n}{2} - 2 \\ \dfrac{n}{2} + i + 1 & \text{if} \quad x = {v_i}^* \text{ where } 1 \le i \le \dfrac{n}{2} - 2 \\ \dfrac{n}{2} - 1 & \text{if} \quad x = v_{\frac{n}{2}-1} \\ \dfrac{n}{2} + 1 & \text{if} \quad x = {v_{\frac{n}{2}}}^* - 1 \end{cases}$$

In view of above defined labeling, one can obtain the k-cordial labeling.

Remark 4.4. In view of Theorems 4.2 and 4.3, all the edge labels of k-cordial labeling on identity graphs are (i) odd integers when $n \equiv 0 \pmod 4$ and (ii) even integers when $n \not\equiv 0 \pmod 4$.

Theorem 4.5. *Let Γ be a group. Then the identity graph $Id(\Gamma)$ where $\Gamma = D_{2n}$ admits k-cordial labeling where $n \ge 3$ and $2n \not\equiv 0 \ (mod\ 4)$.*

Proof. Let us consider an identity graph $Id(\Gamma)$ where Γ be a dihedral group. By the construction of identity graph $Id(\Gamma)$contains s traingles when $2n \not\equiv 0 \pmod 4$ where s is the quotient. Whenever n is prime, there are n lines contained in $Id(\Gamma)$ and if n is non-prime then $Id(\Gamma)$ contains $2k + 3$ lines when $n \equiv 0, 2 \pmod 4$, $2k + 1$ lines when $n \equiv 1, 3 \pmod 4$ where k is the number of triangles in the graph. In this context all the lines and triangles are joined to a simple apex vertex and assign the label for the apex vertex as 0. That is $l(v_0) = 0$. Consider the vertex set $V(Id(\Gamma)) = T_1 \cup T_2 \cup L$ where $T_1 = \{u_1, u_2, \dots, u_k\}$, $T_2 = \{v_1, v_2, \dots, v_k\}$ and $L = \{w_1, w_2, \dots, w_k\}$ where $i = 2k + 1$ (or) $2k + 3$. Hence u_i is adjacent to v_i where $1 \le i \le k$. Now assign the label to u_i, v_i $(1 \le i \le k)$ as $l(u_i) = i$ and $l(v_i) = 2k + 3 + 2(k - i)$ and also assign the label for w_i as the remaining numbers in as ascending order $\{0,1, \dots, k - 1\}$. Now by using the above labeling, it is clear that vertex and edge labeling exists.

Theorem 4.6. *Let Γ be a group. Then the identity graph $Id(\Gamma)$ where $\Gamma = D_{2n}$ admits k-cordial labeling where $n \ge 3$ and $2n \equiv 0 \ (mod\ 4)$.*

Proof. When n is prime, there are n lines contained in $Id(\Gamma)$ and if n is non-prime then $Id(\Gamma)$ contains $2k + 3$ lines when $n \equiv 0, 2 \pmod 4$, $2k + 1$ lines when $n \equiv 1, 3 \pmod 4$ where k is the number of triangles in the graph. In this context all the lines and triangles are joined to a single apex vertex and assign the label for the apex vertex as 0. That is $l(v_0) = 0$. Consider the vertex set $V(Id(\Gamma)) = T_1 \cup T_2 \cup L$ where $T_1 = \{u_1, u_2, \dots, u_k\}$, $T_2 = \{v_1, v_2, \dots, v_k\}$ and $L =$

$\{w_1, w_2, \dots, w_t\}$ where $t = 2k + 1$ (or) $2k + 3$. Hence u_i is adjacent to v_i where $1 \leq i \leq k$. Now assign the label to u_i, v_i $(1 \leq i \leq k)$ as $l(u_i) = i$ and $l(v_i) = n - 2k + 1$ and also assign the label for w_i as the remaining numbers in as ascending order $\{0,1, \dots, k-1\}$. In view of the above labeling, it is clear that vertex and edge labeling exists.

Conclusion:

In this paper we initiating the E-cordial and k-cordial labeling concepts for the identity graphs for certain groups.

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References :

[1] I. Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, *Ars Combin*., **23** (1987), 201-208.

[2] I. Cahit, On Cordial and 3-eqiutable labeling of graphs, *Util. Math.*, **37** (1990) 189-198.

[3] M. Hovey, A-cordial graphs, *Discrete Math* **93** (1991) 183-194.

[4] A. Rosa, On certain valuations of the vertices of a graph, Theory of graphs *Internet Symposium, Rome,* July 1996, Gordon and Breach, N.Y and Dunod Paris, (1967) 349-355.

[5] Smarandache, Florentin and W.B. Vasantha Kandasamy, "*Groups as graphs*" (2009). https://digitalrepository.unm.edu/math_fsp/219

[6] R.Yilmaz and I. Cahit, E-cordial graphs, *Ars Combin.*, **46** (1997) 251-266.

[7] M.Z. Youssef, On k-cordial labeling, *Australian Journal of Combinatorics,* **43** (2009) 31-37.