Section A-Research paper
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K – Minus and K – Regular inverse ordering in generalized regular Neutrosophic Fuzzy Matrices

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Abstract

This study is concerned with the k - minus and k - regular ordering of Neutrosophic fuzzy matrices. We also explore the extensions of intuitionistic to k - minus and k - regular neutrosophic fuzzy matrices. Finally, we explain some examples of k - minus and k – regular ordering. Then we determine the right and left g- inverses theorem outcomes.

Keywords: Neutrosophic Fuzzy Matrices, k – minus and k – regular, Right and Left g-inverses.

1.Introduction :

Matrices serves an important part in many fields, including mathematics, statistics, physics, engineering, data processing and so on. However, Mathematics is used in everyday life to solve problems in economics, engineering, social science, medicine and other fields. As a result of the numerous sorts of uncertainties found in real-world problems, we cannot apply typical classical matrices properly. Fuzzy sets are today's probability. Rough sets are used as a mathematical tool for copying with uncertainties if sets are ambiguous.

Ben Israel and Grevile [2] discussed the concept of generalized inverses. In [1], Atanassov introduce intuitionistic fuzzy sets. Meenakshi and Gandhimathi [9], investigated the regular intuitionistic fuzzy matrices. Pal and Khan work on intuitionistic fuzzy matrices is a generalization of the work on fuzzy matrices in [9]. Kavitha, et.el [8], suggested minus ordering on fuzzy neutrosophic soft matrices. In [3], Cen explained T - ordering in fuzzy matrices and investigated the relationship between T - ordering and generalized inverse. Cho's fuzzy matrices equations are reported in [4] along with their consistency. Recently, Meenakshi and Jenita [11] introduced the idea of k - regular fuzzy matrices as a generalization of regular fuzzy matrices. Jenita, et.el, presented by Generalized Regular Intuitionistic fuzzy matrices [6].

Inverse generalized intuitionistic fuzzy matrices were introduced by Pradhan and Paul [13].Generalized inverses of the block of the original matrix described by Pradha and Pal [12], a technique for calculating the inverse of an intuitionistic fuzzy matrix. Jenita and Karuppasammy [6] presentation of the k -regularity of fuzzy block matrices. Jenita and Karuppasammy [5] discussed fuzzy generalized regular block intuitionistic fuzzy matrices. Zadeh introduced a Fuzzy sets [17]. Intuitionistic fuzzy sets and system was introduced by Atanassov [1]. Convergence of powers of a fuzzy matrix introduced by Thomson [16]. Smarandach discusses about Neutrosophic set as a generalization of fuzzy sets [15]. Punithavalli and Karthika discuss some characterization on Moore – Penrose inverse of symmetric Neutrosophic Fuzzy Matrices [14].

2.Premilinaries :

Definition 2.1 [14]

Let $\mathcal{S}$ be a fixed set that is not empty. A Neutrosophic set X is an object that substantially derives from the sets $\{\mathcal{T},\mathcal{I},\mathcal{F}\}$ where $\mathcal{T}$, $\mathcal{I}$ and $\mathcal{F}$ stands for the degree of membership, degree of Indeterminacy and degree of non – membership which can be defined as $0 \leq \mathcal{T}+\mathcal{I}+\mathcal{F} \leq 3$ for every $l \times z$ matrix.

Definition 2.2 [14]

A Neutrosophic Fuzzy Matrices $\mathcal{S} = \mathcal{S}_{ij}^{T}, \mathcal{S}_{ij}^{I}, \mathcal{S}_{ij}^{F} \in NFMs_{l\times z}$ is know as to be regular that there exist a matrix $\mathcal{U} = \mathcal{U}_{ij}^{T}, \mathcal{U}_{ij}^{I}, \mathcal{U}_{ij}^{F} \in NFMs_{l\times z}$ to such that $\mathcal{SUS} = \mathcal{U}$.Then $\mathcal{U}$ is called g – inverse of $\mathcal{S}$. Let $\mathcal{S}(1)$ = $\left\{\frac{\mathcal{U}}{\mathcal{SUS}=\mathcal{U}}\right\}$ the arrangement of all fuzzy matrices of order $l \times z$.

Definition 2.3 [15]

Let $\mathcal{S}$ and $\mathcal{U}$ be a two Neutrosophic Fuzzy Matrices .Where $\mathcal{S} = \{\mathcal{S}^{T}, \mathcal{S}^{I}, \mathcal{S}^{F}\}$ and $\mathcal{U} = \{\mathcal{U}^{T}, \mathcal{U}^{I}, \mathcal{U}^{F}\}$. Then we defined as,

$$\mathcal{S} \oplus \mathcal{U} = max(\mathcal{S}^{T},\mathcal{U}^{T}), min(\mathcal{S}^{I},\mathcal{U}^{I}), min(\mathcal{S}^{F},\mathcal{U}^{F})$$

$$\mathcal{S} \otimes \mathcal{U} = min(\mathcal{S}^{T},\mathcal{U}^{T}), max(\mathcal{S}^{I},\mathcal{U}^{I}), max(\mathcal{S}^{F},\mathcal{U}^{F}).$$

Definition 2.4 [14]

Let $\mathcal{S} = \mathcal{S}_{ij}^{T}, \mathcal{S}_{ij}^{I}, \mathcal{S}_{ij}^{F}$ and $\mathcal{U} = \mathcal{U}_{ij}^{T}, \mathcal{U}_{ij}^{I}, \mathcal{U}_{ij}^{F}$ be two Neutrosophic Fuzzy Matrices product $\mathcal{SU}$ is defined by,

$\mathcal{SU} = \{\max(min(\mathcal{S}^{T},\mathcal{U}^{T})), \max(min(\mathcal{S}^{I},\mathcal{U}^{I})), \min(max(\mathcal{S}^{F},\mathcal{U}^{F}))\}$.

Definition 2.5 [6]

Let $\mathcal{S}$ be a Neutrosophic Fuzzy Matrix of order l. If there exist a matrix $\mathcal{V} \in (NFMs)_{l}$, such that $\mathcal{SV}\,\mathcal{S}^{k} = \mathcal{S}^{k}$, for some positive integer k, the matrix is said to be right k – regular. Then $\mathcal{V}$ is called left k – g - inverse of $\mathcal{S}$. We write $\mathcal{S}_{l}\{1^{k}\} = \{\mathcal{V} | \mathcal{SV}\,\mathcal{S}^{k} = \mathcal{S}^{k}\}$.

Definition 2.6 [6]

Let $\mathcal{S}$ be a Neutrosophic Fuzzy Matrices of order l. If there exist the matrix $\mathcal{U} \in (NFMs)_{l}$, such that $\mathcal{S}^{k}\mathcal{US} = \mathcal{S}^{k}$ for some positive integer k, the matrix is said to be right k – regular. Then $\mathcal{U}$ is called right k – g - inverse of. We write $\mathcal{S}_{r}\{1^{k}\} = \{\mathcal{U} | \mathcal{S}^{k}\,\mathcal{U}\,\mathcal{S} = \mathcal{S}^{k}\}$.

Proposition 2.7 [6]

Let $\mathcal{S},\mathcal{U} \in (NFMs)_{l}$ for some positive integer k then,

(i) If $\mathcal{S}$ is a right k – regular and $\mathcal{R}(\mathcal{U}) \subseteq \mathcal{R}(\mathcal{S}^{k})$ then $\mathcal{U} = \mathcal{U}X\mathcal{S}$ for each right k – g – inverse X of $\mathcal{S}$.

(ii) If $\mathcal{S}$ is a right k – regular and $\mathcal{C}(\mathcal{U}) \subseteq \mathcal{C}(\mathcal{S}^{k})$ then $\mathcal{U} = \mathcal{S}Y\mathcal{U}$for each right k – g – inverse Y of $\mathcal{S}$.

Proposition 2.8 [6]

Let $\mathcal{S},\mathcal{U}$ in Neutrosophic Fuzzy Matrices of order l× z then $\mathcal{R}(\mathcal{SU}) \subseteq \mathcal{R}(\mathcal{S})$, $\mathcal{C}(\mathcal{SU}) \subseteq \mathcal{C}(\mathcal{S})$.

Proposition 2.9 [6]

Let $\mathcal{S} \in (NFMs)_{l\times m}$ and $\mathcal{U} \in (NFMs)_{l\times z}$ $\mathcal{R}(\mathcal{U}) \subseteq \mathcal{R}(\mathcal{S})$ iff $\mathcal{U} = X\mathcal{S}$ for some $X \in (NFMs)_l$, $\mathcal{C}(\mathcal{S}) \subseteq \mathcal{C}(\mathcal{S})$ iff $\mathcal{U} = \mathcal{S}Y$ for some $Y \in (NFMs)_z$.

Proposition 2.10 [6]

If $\mathcal{S} \in (NFMs)_l$ and k be a positive integer, then X $\in \mathcal{S}_r\{1^k\}$ iff $X^T \in \mathcal{S}_l^k\,\{1^k\}$.

Remark 2.11 [6]

Every member of the set $\mathcal{S}\{1^k\}$ is reffered to a k – g –inverse of $\mathcal{S}$. Then for any positive integer p ≥ k if $\mathcal{S}$ k – regular then $\mathcal{S}$ is p – regular. For k = 1, $\mathcal{S}\{1^k\}$ reduces to the set of all g – inverse of a regular matrix $\mathcal{S}$.

Definition 2.12 [7]

Let $\mathcal{S}$ be a Neutrosophic Fuzzy Matrices of order l, is said to be $\{3^k\}$ inverse if there exist a matrix X $\in (NFMs)_l$ such that $(\mathcal{S}^kX)^T = \mathcal{S}^kX$ for some positive integer k. Then X is called $\{3^k\}$ inverse is $\mathcal{S}$.we can write $\mathcal{S}\,\{3^k\} = \left\{ X \big/ (\mathcal{S}^kX)^T = \mathcal{S}^kX \right\}$.

Definition 2.13[7]

Let $\mathcal{S}$ be a Neutrosophic Fuzzy Matrices of order l, is said to be $\{4^k\}$ inverse if there exist a matrix Y $\in (NFMs)_l$ such that $(\mathcal{S}^kY)^T = \mathcal{S}^kX$ for some positive integer k. Then Y is called $\{3^k\}$ inverse is $\mathcal{S}$.we can write $\mathcal{S}\,\{4^k\} = \left\{ Y \big/ (\mathcal{S}^kY)^T = \mathcal{S}^kY \right\}$.

Definition 2.14[14]

Each row and each column should have exactly one (1,1,0) and all the other entries are (0,0,1) of a square Neutrosophic Fuzzy Matrices is called the Neutrosophic Fuzzy Permutation matrices.

Theorem 2.15 [7]

For $\mathcal{S} \in NFMs_l$ and $\mathcal{E} = \mathcal{F}\mathcal{S}\mathcal{G}$

(i) If $\mathcal{F}, \mathcal{G} \in \mathcal{S}_r\{1^k\}$ then $\mathcal{E} \in \mathcal{S}_r\{1^k\}$
(ii) If $\mathcal{F}, \mathcal{G} \in \mathcal{S}_l\{1^k\}$ then $\mathcal{E} \in \mathcal{S}_l\{1^k\}$
(iii) If $\mathcal{F} \in \mathcal{S}_r\{1^k\}$ then $\mathcal{G} \in \mathcal{S}_r\{3^k\}$ then $\mathcal{E} \in \mathcal{S}\{3^k\}$
(iv) If $\mathcal{G} \in \mathcal{S}_l\{1^k\}$ then $\mathcal{G} \in \mathcal{S}_r\{4^k\}$ then $\mathcal{E} \in \mathcal{S}\{4^k\}$.

Proposition 2.16 [7]

If $\mathcal{S}, \mathcal{V} \in (NFMs)_{l\times z}$,$\mathcal{R}(\mathcal{V}) \subseteq \mathcal{R}(\mathcal{S})$ iff $\mathcal{V} = X\mathcal{S}$ for some X$\in (NFMs)_l$, $\mathcal{C}(\mathcal{V}) \subseteq \mathcal{C}(\mathcal{S})$ iff $\mathcal{V} = \mathcal{S}Y$ for some Y$\in (NFMs)_z$.

Proposition 2.17 [7]

Let $\mathcal{S}$ Neutrosophic Fuzzy Matrix in order l and k be a positive integer, then$X \in \mathcal{S}_r\{1^k\}$ iff $X^T \in\in \mathcal{S}_l^T\{1^k\}$.

Proposition 2.18 [7]

Let $\mathcal{S}$ and $\mathcal{V}$ are two Neutrosophic Fuzzy Matrices in order $l \times z$, $\mathcal{R}(\mathcal{S}\mathcal{V}) \subseteq \mathcal{R}(\mathcal{S})$, $\mathcal{C}(\mathcal{S}\mathcal{V}) \subseteq \mathcal{C}(\mathcal{S})$.

Remark 2.19 [7]

If every element of the set $\mathcal{S}\{\,1^k\}$ is called a k – g- inverse of $\mathcal{S}$. If $\mathcal{S}$ is k –regular then $\mathcal{S}$ is g- regular for all integers p ≥ k. For k = 1, $\mathcal{S}\{\,1^k\}$ reduces to the set of all g- inverse of a regular matrix $\mathcal{S}$.

Remark 2.20 [7]

If $\mathcal{S}$ is Neutrosophic Fuzzy Matrix of row rank $\rho_r(\mathcal{S})$, has number of independent rows, which generates the row space $\mathcal{R}(\mathcal{S})$ .If $\mathcal{S}$ Neutrosophic fuzzy is a matrix of column rank $\rho_{\mathcal{C}}(\mathcal{S})$ of has numbers columns, which generated the column space $\mathcal{C}(\mathcal{S})$.

3. k – minus Inverse of Neutrosophic Fuzzy Matrices

Definition 3.1

If $\mathcal{S}, \mathcal{V}$ in Neutrosophic Fuzzy Matrices of minus orderings which is denoted as $\mathcal{S} \ <_k^- \mathcal{V}$ and is defined as $\mathcal{S} \ <_k^- \mathcal{V}$ iff $\mathcal{S}^k$X = $V^k X$ for some $X \ \in \mathcal{S}\{1^k\}$ and $Y\mathcal{S}^k$ = YV^k for some $Y \in \mathcal{S}\{1^k\}$.

Example

$\mathcal{S}$ = $\begin{bmatrix}(0.5,0.2,0.1) & (0.2,0.3,0.4)\\(0.3,0.2,0.1) & (0.2,0.5,0.4)\end{bmatrix}$ $\mathcal{S}^2 = \begin{bmatrix}(0.5,0.2,0.1) & (0.2,0.3,0.4)\\(0.3,0.2,0.1) & (0.2,0.5,0.4)\end{bmatrix}$

$\mathcal{S} = \mathcal{S}^2$

$\wp_1 = \begin{bmatrix}(1,1,0) & (0,0,1)\\(0,0,1) & (1,1,0)\end{bmatrix}$ and $\wp_2 = \begin{bmatrix}(0,0,1) & (1,1,0)\\(1,1,0) & (0,0,1)\end{bmatrix}$

$\mathcal{S}\wp_1\mathcal{S} = \mathcal{S}$

$\mathcal{S}\wp_1$= $\begin{bmatrix}(0.5,0.2,0.1) & (0.2,0.3,0.4)\\(0.3,0.2,0.1) & (0.2,0.5,0.4)\end{bmatrix}$

$\mathcal{S}\wp_1\mathcal{S} = \mathcal{S}, \mathcal{S}\wp_1\mathcal{S} \neq \mathcal{S}$, Hence $\mathcal{S}$ is not regular

For, X = $\begin{bmatrix}(0.5,0.2,0.1) & (0.1,0.3,0.4)\\(0.2,0.2,0.1) & (0.2,0.5,0.4)\end{bmatrix}$

$\mathcal{S}^2X$ = $\begin{bmatrix}(0.5,0.2,0.1) & (0.2,0.3,0.4)\\(0.3,0.2,0.1) & (0.2,0.5,0.4)\end{bmatrix}$

$\mathcal{S}^2 \, X\mathcal{S} = \mathcal{S}^2$

Hence $\mathcal{S}$is 2 – regular and $\mathcal{V}$ is a 2 – g inverse

For, $\mathcal{V} = \begin{bmatrix}(0.6,0.2,0.1) & (0.2,0.3,0.4)\\(0.3,0.1,0.1) & (0.2,0.5,0.4)\end{bmatrix}$, $\mathcal{V}^2 = \begin{bmatrix}(0.6,0.2,0.1) & (0.2,0.3,0.4)\\(0.3,0.1,0.1) & (0.2,0.5,0.4)\end{bmatrix}$

$\mathcal{V} = \mathcal{V}^2$

$\mathcal{V}^2X$ = $\begin{bmatrix}(0.5,0.2,0.1) & (0.2,0.3,0.4)\\(0.3,0.2,0.1) & (0.2,0.5,0.4)\end{bmatrix}$

$\mathcal{V}^2X$ = $\mathcal{S}^2X = \mathcal{S}$

Y $\mathcal{V}^2$ = $\begin{bmatrix}(0.5,0.2,0.1) & (0.2,0.3,0.4)\\(0.3,0.2,0.1) & (0.2,0.5,0.4)\end{bmatrix}$

Y = $\begin{bmatrix}(0.5,0.2,0.1) & (0.2,0.3,0.4)\\(0.2,0.2,0.1) & (0.2,0.5,0.4)\end{bmatrix}$

$\mathcal{S}Y$ = $\begin{bmatrix}(0.5,0.2,0.1) & (0.2,0.3,0.4)\\(0.3,0.2,0.1) & (0.2,0.5,0.4)\end{bmatrix}$

$Y\mathcal{S}^2 = \begin{bmatrix} (0.5,0.2,0.1) & (0.2,0.3,0.4) \\ (0.3,0.2,0.1) & (0.2,0.5,0.4) \end{bmatrix}$

$Y\mathcal{S}^2 = \mathcal{S} = Y\mathcal{V}^2$.

Proposition 3.2

For $\mathcal{S}, \mathcal{V} \in (NFM)_r^-$ the following statement are,

(i) If $\mathcal{V}$ is right k – right and $\mathcal{R}(\mathcal{S}^k) \subseteq \mathcal{R}(V^k)$ then $\mathcal{S}^k = \mathcal{S}^k X\mathcal{V}$ for every right k – g-inverse X of $\mathcal{V}$.

(ii) If $\mathcal{V}$ is left k – right and $\mathcal{C}(\mathcal{S}^k) \subseteq \mathcal{C}(V^k)$ then $\mathcal{S}^k = \mathcal{V}Y\mathcal{S}^k$ for every left k – g-inverse Y of $\mathcal{V}$.

Proof

(i) Since $\mathcal{R}(\mathcal{S}^k) \subseteq \mathcal{R}(V^k)$ there exist regular matrix $\mathcal{Z}$ such that
$\mathcal{S}^k = \mathcal{Z}V^k = \mathcal{Z}V^k X\mathcal{V}$
$= \mathcal{S}^k X\mathcal{V}$

(ii) Since $\mathcal{C}(\mathcal{S}^k) \subseteq \mathcal{C}(V^k)$ there exist a regular matrix $\mathcal{Z}$ such that

$\mathcal{S}^k = V^k\mathcal{Z} = VY^k\,\mathcal{Z}$

$\mathcal{S}^k = \mathcal{V}Y\mathcal{S}^k$.

Theorem 3.3

If $\mathcal{S}\ and\ \mathcal{V}$ are neutrosophic fuzzy matrices of order l, then the following

Conditions holds,

(i) $\mathcal{S} <_k^- \mathcal{V}$

(ii) $\mathcal{S}^k = V^k X\,\mathcal{S} = \mathcal{S}YV^k$ for some X,Y $\in \mathcal{S}\{1^k\}$.

Proof

(i) $\Rightarrow$ (ii)

$\mathcal{S} <_k^- \mathcal{V} \Rightarrow \mathcal{S}^k\text{X} = V^k Y, \quad \text{X} \in \mathcal{S}\{1_r^k\}$

$\text{Y}\,\mathcal{S}^k = YV^k, \quad Y \in \mathcal{S}\{1_l^k\}$

$\mathcal{S}^k = \mathcal{S}^k\text{X}\,\mathcal{S} = (\mathcal{S}^k\text{X})\,\mathcal{S} = V^k\text{X}\,\mathcal{S}$

$\mathcal{S}^k = V^k\text{X}\,\mathcal{S} = \mathcal{S}Y\,V^k$

(ii) $\Rightarrow$(i)

Let $\mathcal{C} = X\mathcal{S}X, \quad X \in \mathcal{S}\{1_r^k\}$

$\mathcal{S}^k\,\mathcal{C}\,\mathcal{S} = \mathcal{S}^k(X\mathcal{S}X)\mathcal{S} = (\mathcal{S}^k X\mathcal{S})X\mathcal{S}$

$= \mathcal{S}^k X\mathcal{S}$

$= \mathcal{S}^k$

Since $\mathcal{C} \in \mathcal{S}\{1_r^k\}$

Similarly,

$\mathcal{S}\mathcal{D}\mathcal{S}^k = \mathcal{S}^k, \mathcal{D} = Y\mathcal{S}Y$ for Y $\in \mathcal{S}\{1_l^k\}$

$\mathcal{S}^k\ \mathcal{C} = \mathcal{S}^k(X\mathcal{S}X)$

$= (\mathcal{S}^k X\mathcal{S})X$

$= \mathcal{S}^k X$

$= (V^k X \mathcal{S}) X$

$= V^k (X \mathcal{S} X)$

$= V^k \mathcal{C}$

Here, $\mathcal{S}^k\mathcal{C} = V^k\mathcal{C}$ for some $\mathcal{C} \in \mathcal{S}\{1_r^k\}$

$\mathcal{D}\mathcal{S}^k = \mathcal{D}\, V^k$ for some $\mathcal{D} \in \mathcal{S}\{1_l^k\}$

Hence the proof.

Theorem 3.4

For $\mathcal{S}$ *and* $\mathcal{V}$ are Neutrosophic Fuzzy Matrices in minus ordering of order l, if $\mathcal{S} <_k^- \mathcal{V}$ then $\mathcal{R}(\mathcal{S}^k) \subseteq \mathcal{R}(V^k)$, $\mathcal{C}(\mathcal{S}^k) \subseteq \mathcal{C}(V^k)$ and $\mathcal{S}^k X \mathcal{S} = \mathcal{S}^k = \mathcal{V}\, Y\mathcal{S}^k$ for each X $\in \mathcal{V}\{1_r^k\}$ and for each Y $\in \mathcal{V}\{1_l^k\}$.

Proof

By theorem(3.3), $\mathcal{S}^k = \mathcal{S}\ Y\ V^k = V^k X \mathcal{S}$

By proposition (3.2), $\mathcal{R}(\mathcal{S}^k) = \mathcal{R}(\ \mathcal{S}\ Y\ V^k) \subseteq \mathcal{R}(V^k)$

$\mathcal{C}(\mathcal{S}^k) = \mathcal{C}(V^k X \mathcal{S}) \subseteq \mathcal{C}(V^k)$.

Theorem 3.5

Let $\mathcal{S}$, $\mathcal{V}$ are two minus ordering neutrosophic fuzzy matrices of order l, then the following are equivalent,

(i) $\mathcal{S} <_k^- \mathcal{S}$.
(ii) $\mathcal{S} <_k^- \mathcal{V}$ and $\mathcal{V} <_k^- \mathcal{S}$, then $\mathcal{S}^k = V^k$.
(iii) $\mathcal{S} <_k^- \mathcal{V}$ and $\mathcal{V} <_k^- \mathcal{U}$, then $\mathcal{S} <_k^- \mathcal{U}$.

Proof

(i) $\mathcal{S} <_k^- \mathcal{S}$ is obvious
Hence $<_k^-$ is reflexive.
(ii) By proposition (3.2)

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$\mathcal{S} <_k^- \mathcal{V} \Rightarrow \mathcal{S}^k = V^k X\mathcal{S}$ for some X ∈ $\mathcal{V}\{1_r^k\}$ and

$\mathcal{V} <_k^- \mathcal{S} \Rightarrow V^k = \mathcal{V}Y\mathcal{S}^k$ for some Y ∈ $\mathcal{V}\{1_l^k\}$

Now, $\mathcal{S}^k = V^k X\mathcal{S}$

$= (\mathcal{V}Y\mathcal{S}^k)\mathrm{X}\,\mathcal{S}$

$= (\mathcal{V}Y\mathcal{S}^k)$

$= \mathcal{S}^k$

(iii) From theorem (3.2)
$\mathcal{S} <_k^- \mathcal{V} \Rightarrow \mathcal{S}^k\mathcal{V}^-\mathcal{V} = V\mathcal{V}^-\mathcal{S}^k$ for $\mathcal{V}^- \in \mathcal{V}\{1^k\}$
$\mathcal{V} <_k^- \mathcal{U} \Rightarrow V^k = \mathcal{U}^k\mathcal{V}^-\mathcal{V} = V\mathcal{V}^-\mathcal{U}^k$ for $\mathcal{V}^- \in \mathcal{S}\{1^k\}$

Let $X^- = \mathcal{V}^-\mathcal{V}\mathcal{K}$ for some $\mathcal{V}^- \in \mathcal{V}\{1_r^k\}$, $\mathcal{A} \in \mathcal{S}\{1_r^k\}$

$\mathcal{S}^k X'\mathcal{S} = \mathcal{S}^k(\mathcal{V}^-\mathcal{V}\mathcal{A})\mathcal{S}$

$= (\mathcal{S}^k\mathcal{V}^-\mathcal{V})\mathcal{A}\mathcal{S}$

$= \mathcal{S}^k\,\mathcal{U}\mathcal{S} = \mathcal{S}^k$

Therefore $X' \in \mathcal{S}\{1_r^k\}$

Let $Y' = B\mathcal{V}\mathcal{V}^-$ for $\mathcal{V}^- \in \mathcal{V}\{1_r^k\}$ and $B \in \mathcal{S}\{1_l^k\}$

Then, $\mathcal{S}Y'\mathcal{S}^k = \mathcal{S}(B\mathcal{V}\mathcal{V}^-)\mathcal{S}^k$

$= \mathcal{S}\mathcal{V}(V\mathcal{V}^-\mathcal{S}^k)$

$= \mathcal{S}B\mathcal{S}^k = \mathcal{S}^k$

Therefore, $Y' \in \mathcal{S}\{1_l^k\}$

$\mathcal{S}^k\mathcal{A} = \mathcal{S}^k(\mathcal{V}^-V^k)$

$= \mathcal{S}^k(\mathcal{V}^-\mathcal{V})\,\mathcal{A}$

$= \mathcal{S}^k\mathcal{A}$

$\mathcal{S}^k\mathcal{A} = \mathcal{V}^{\,k}\mathcal{A}$

$= \mathcal{U}^k(\mathcal{V}^-\mathcal{V}\,\mathcal{A})$

$= \mathcal{U}^k\mathcal{A}$ for some $X' \in \mathcal{S}\{1_r^k\}$

$X'\,\mathcal{S}^k = (\mathcal{B}\mathcal{V}\mathcal{V}^-)\,\mathcal{S}^k$

$= \mathcal{B}(\mathcal{V}\mathcal{V}^-\,\mathcal{S}^k)$

$= \mathcal{B}\,\mathcal{S}^k = \mathcal{S}\mathcal{V}^{\,k}$

$= \mathcal{B}(\mathcal{V}\mathcal{V}^-\mathcal{U}^k)$

= $(\mathcal{B}\mathcal{V}\mathcal{V}^{-})\mathcal{U}^{k}$

= $Y'\mathcal{U}^{k}$ for some $Y' \in \mathcal{S}\{1_l^k\}$

Therefore, $\mathcal{S} <_k^- \mathcal{V}$ and $\mathcal{V} <_k^- \mathcal{U}$, then $\mathcal{S} <_k^- \mathcal{U}$

Hence $<_k^-$ is transitive.

Theorem 3.6

If $\mathcal{S}, \mathcal{V} \in (NFMs)_l^-$, then the following conditions are holds.

(i) $\mathcal{R}(\mathcal{V}) \subseteq \mathcal{R}(\mathcal{S}^k) \Rightarrow \mathcal{C}(V^T) \subseteq \mathcal{C}(\mathcal{S}^T)$
(ii) $\mathcal{C}(\mathcal{V}) \subseteq \mathcal{C}(\mathcal{S}^k) \Rightarrow \mathcal{R}(V^T) \subseteq \mathcal{R}(\mathcal{S}^T)$

Proof

(i) By Proposition (2.17)
If $\mathcal{V}$ is a right k – regular and $\mathcal{R}(\mathcal{V}) \subseteq \mathcal{R}(\mathcal{S}^k)$, then
$\mathcal{V} = \mathcal{V}X\mathcal{S} \Rightarrow V^T = (\mathcal{V}X\mathcal{S})^T$

$= \mathcal{S}^T X^T V^T$

By Proposition (2.17),$\mathcal{C}(V^T) \subseteq \mathcal{C}(\mathcal{S}^T)$.

(ii) By Definition (2.11)
If $\mathcal{S}$is left k – regular and $\mathcal{C}(\mathcal{V}) \subseteq \mathcal{C}(\mathcal{S}^k)$
$\mathcal{V} = \mathcal{S}Y\mathcal{V} \Rightarrow V^T = (\mathcal{S}Y\mathcal{V})^T$

$= V^T Y^T \mathcal{S}^T$

By Definition (2.11), $\mathcal{R}(V^T) \subseteq \mathcal{R}(\mathcal{S}^T)$.

Theorem 3.7

For any two minus ordering Neutrosophic fuzzy matrices of order l, if $\mathcal{S} <_k^- \mathcal{V}$ with V^k is idempotent then $\mathcal{S}^k$is idempotent.

Proof

By Proposition (2.7) $\mathcal{S} <_k^- \mathcal{V} \Rightarrow \mathcal{S}^k = \mathcal{S}YV^k = V^kX\mathcal{S}$ for X,Y $\in \mathcal{S}\{1^k\}$

$\mathcal{S}^{2k} = \mathcal{S}^k\mathcal{S}^k$

$= (\mathcal{S}YV^k)(V^kX\mathcal{S})$

$= \mathcal{S}Y(V^{2k})X\mathcal{S}$

$= \mathcal{S}YV^kX\mathcal{S}$

$= \mathcal{S}^kX\mathcal{S}$

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$$\mathcal{S}^{2k} = \mathcal{S}^k$$

Hence the proof.

Example

$\mathcal{S} = \begin{bmatrix} (0.7,0.6,0.2) & (0.2,0.2,0.2) \\ (0.2,0.2,0.2) & (0.6,0.5,0.4) \end{bmatrix} = \mathcal{S}^2$, is idempotent

$\mathcal{V} = \begin{bmatrix} (0.7,0.6,0.2) & (0.2,0.2,0.2) \\ (0.2,0.2,0.2) & (0.6,0.5,0.2) \end{bmatrix} = V^2$, is idempotent

Let $\mathcal{A} = \begin{bmatrix} (0.7,0.6,0.2) & (0.2,0.2,0.2) \\ (0.2,0.2,0.1) & (0.6,0.5,0.2) \end{bmatrix}$

$$\mathcal{S}^2\mathcal{A} = V^2\mathcal{A} = \begin{bmatrix} (0.7,0.6,0.2) & (0.2,0.2,0.2) \\ (0.2,0.2,0.2) & (0.6,0.5,0.2) \end{bmatrix}$$

Let $\mathcal{B} = \begin{bmatrix} (0.8,0.7,0.2) & (0.2,0.2,0.2) \\ (0.2,0.2,0.2) & (0.9,0.6,0.2) \end{bmatrix} = \mathcal{B}^2$

$$\mathcal{B}\,\mathcal{S}^2 = \mathcal{B}V^2 = \begin{bmatrix} (0.7,0.6,0.2) & (0.2,0.2,0.2) \\ (0.2,0.2,0.2) & (0.6,0.5,0.2) \end{bmatrix}.$$

Conclusion

In this article, we discuss about k – minus and k - regular Neutrosophic fuzzy matrices. Also we extend the intuitionistic k – minus and k - regular fuzzy matrices to Neutrosophic fuzzy matrices.

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