LRS BIANCHI TYPE-I INHOMOGENEOUS ANISOTROPIC COSMOLOGICAL MODELS WITH PERFECT FLUID

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Abstract: The present study deals with inhomogeneous anisotropic locally rotationally symmetric (LRS) Bianchi Type-I space time with perfect fluid. It is discussed that when the metric potential B be a separable function of x and t, then the field equation and conservation equations are solvable. Some physical properties of the solutions are also discussed.

Key Words: LRS Bianchi Type-I models, general relativity, cosmological model.

DOI: 10.48047/ecb/2023.12.Si12.146

Introduction

It is most important to study the problems related to inhomogeneous and anisotropic space-time to know about the formation of galaxies and the process of homonization and isotropization of the universe. Here we consider locally rotationally symmetric (LRS) Bianchi Type-I space time to study inhomogeneity. The process of isotropization of the universe can be studied through the Bianchi type cosmological models which are homogeneous and anisotropic. Anisotropic universe has more generality than isotropic models. Hence these models are suitable models of our universe, because of the simplicity of field equations.

In this note, we discuss about LRS Bianchi type-I cosmological space times in general relativity. Mazumdar [1] has obtained cosmological solutions for LRS Bianchi type-I space-time filled with a perfect fluid with arbitrary cosmic scale functions and studied kinematic properties of the particular form of the solution. Mohanti [2,3] also obtained some solutions for the same field equations by using solution generation technique with the matter perfect fluid. Her we have taken an attempt to solve the field equation to obtain general solutions of this problem, when the space-time is inhomogeneous and anisotropic. Also studied some physical and geometrical properties of the solutions.

1. Einstein Field Equations

The metric of inhomogeneous and anisotropic space time is

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 + dz^2) \quad (1)$$

Where A & B are the functions of x and t only.

The energy momentum tensor for a perfect fluid is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{2}$$

Together with commoving co-ordinates $u^i u_i = 1$ (3)

Where u_i is four velocity vectors of fluid, p and ρ are proper pressure and density of the distribution respectively.

The Einstein field equations can be written as

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi T_{ij} \tag{4}$$

Equation (4) for metric (1) is commoving co-ordinate system i.e. $u_i = (0, 0, 0, 1)$ gives

$$\frac{2B_{44}}{B} + \frac{{B_4}^2}{B^2} - \frac{{B_1}^2}{A^2 B^2} = -8\pi p \tag{5}$$

$$B_{41} - \frac{B_1 A_4}{A} = 0 \tag{6}$$

$$\frac{B_{44}}{B} + \frac{A_{44}}{A} + \frac{A_1 B_1}{A^3 B} + \frac{A_4 B_4}{AB} - \frac{B_{11}}{A^2 B} = -8\pi p \tag{7}$$

$$\frac{2B_{11}}{A^2 B} + \frac{{B_1}^2}{A^2 B^2} - \frac{2A_1 B_1}{A^3 B} - \frac{2A_4 B_4}{AB} - \frac{{B_4}^2}{B^2} = -8\pi\rho \tag{8}$$

Here and afterwards the suffix 1 and 4 represents partial differentiation with respect to x and t respectively.

The energy conservation equation is $\quad {T_{;j}}^{ij} = 0$

For $i = 1$ gives $\quad p_1 = 0$ (9)

This gives $\quad p = p(t)$ (10)

The equations of motion for $i = 2, 3$ yields zero and for $i = 4$ gives

$$\rho_4 + (p + \rho)\left[\frac{A_4}{A} + \frac{2B_4}{B}\right] = 0 \tag{11}$$

2. Solutions of the field equations

After integrating equation (6), we get

$$\frac{B_1}{A} = K(x) \tag{12}$$

The field equations are nonlinear; it is difficult to find the explicit solution of the field equations. Assume that B is separable function of x and t i.e.

$$B = g(x)h(t) \tag{13}$$

From equation (5) and (7) we get

$$\frac{B_{11}}{A^2B} - \frac{{B_1}^2}{A^2B^2} - \frac{A_1B_1}{A^3B} - \frac{A_4B_4}{AB} + \frac{{B_4}^2}{B^2} + \frac{B_{44}}{B} - \frac{A_{44}}{A} = 0 \tag{14}$$

By using equation (12) and (13), equation (14) gives

$$h(t)k(x) = L \tag{15}$$

Here L is the constant of integration. Hence the metric potentials are

$$A = \frac{Lg_1h}{g} \; and \; B = gh \tag{16}$$

Where L is a non-zero arbitrary constant. Using above potentials, metric (1) becomes

$$ds^2 = dt^2 - \frac{h^2}{g^2}[L^2{g_1}^2dx^2 - g^4(dy^2 + dz^2)] \tag{17}$$

The energy conservation equation becomes

$$\rho_4 = -\frac{h_4}{h}(p + \rho) \tag{18}$$

Since $h(t) \neq 0$, hence the matter density depends on t only. The scalar expansion θ and shear σ for the model (17) are

$$\theta = \frac{3h_4}{h}, \quad \sigma = 0 \tag{19}$$

Hence we get isotropic nature of the space-time because $\frac{\sigma}{\theta} = 0$.

And the rotation and acceleration become zero.

3. Conclusion

In this paper, we have discussed about LRS Bianchi type-I cosmological space times in general relativity. And obtained cosmological solutions for LRS Bianchi type-I space-time filled with a perfect fluid with arbitrary cosmic scale functions and studied kinematic properties of the particular form of the solution. We have derived Einstein's field equations and conservation equations for LRS Bianchi type-I cosmological space times. The matter density depends on t only and we get isotropic nature of the space-time.

4. References

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