Section A-Research paper

ROOT CUBE MEAN CORDIAL LABELING OF SUBDIVISION OF SOME GRAPHS

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Abstract : Let G =(V,E)be a graph and f be a mapping from $V(G) \to \{0,1,2\}$. For each edge uv of G assign the label $\left\lfloor \sqrt{\frac{f(u)^3+f(v)^3}{2}} \right\rfloor$, ,f is called a root cube mean cordial labeling if $|v_f(i)-v_f(j)| \leq 1$ and $|e_f(i)-e_f(j)| \leq 1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with $x, x\in\{0,1,2\}$ respectively. A graph with a root cube mean cordial labeling is called root cube mean cordial graph. In this paper, we investigate about the existence of root cube mean cordiality of subdivision of some graphs such as path, cycle, star, bistar, H- graph, $H\odot K_1$Graph, $P_n\odot K_1$, $C_n\odot K_1$ Graphs.

Keywords: path, cycle, star, bistar, H- graph, $H\odot K_1$Graph, $P_n\odot K_1$, $C_n\odot K_1$ Graphs, root cube mean cordial labeling, root cube mean cordial graphs.

I. INTRODUCTION

The graphs considered here are finite, undirected and simple. The vertex set and edge set of a graph G are denoted by V(G) and E(G) respectively. Labeled graphs are useful models for a broad range of applications such as coding theory, X- ray crystallography, astronomy, circuit design etc. The concept of cordial labeling was introduced by Cahit in the year 1987. After the introduction of cordial labeling various types of cordial labeling has been studied. Motivated by the works of many researchers in the area of cordial labeling, we introduced a new type of labeling called root cube mean cordial labeling. In this paper we have discussed about the cordiality of subdivision of some graphs.

Definition 1.1: **Subdivision of a Graph**

A subdivision of a graph *G* denoted by *S(G)* is a graph resulting from the subdivision of edges in *G*. The subdivision of some edge *e* with endpoints *u*, *v* yields a graph containing one new vertex *w*, and with an edge set replacing *e* by two new edges *uw* and *wv*.

Example 1.2:

Consider the following graph

Here the edge *e* is subdivided into two new edges *uw* and *wv*.

Result 1.3:

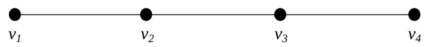
Consider the graph path P_n . The subdivision of path P_n , $S(P_n)$ is P_{2n-1} , which is a path.

Already we have proved that path is a root cube mean cordial graph [6].

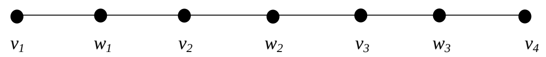
Hence subdivision of path is also a root cube mean cordial graph.

Example 1.4:

Consider the path P_4



Now $S(P_4)$



Here $S(P_4) = P_{2(4)-1} = P_7$, is a path.

Clearly, P_7 is a root cube mean cordial graph.

Result 1.5:

Consider the graph cycle C_n .

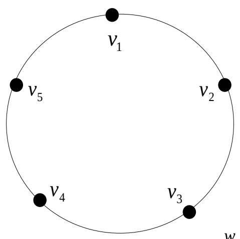
The subdivision of C_n, $S(C_n)$ is C_{2n}, which is also a cycle.

Already we have proved that cycle is root cube mean cordial iff $n \equiv 1,2(\bmod 3)$.[6]

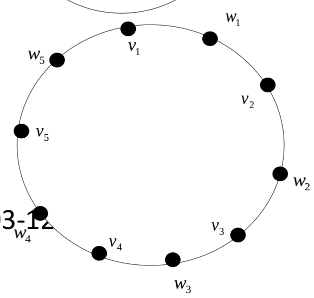
Hence subdivision of cycle is also a root cube mean cordial graph.

Example 1.6:

Consider the cycle C_5



Now $S(C_5)$



Here $S(C_5) = C_{2(5)} = C_{10}$

Clearly C_{10} is root cube mean cordial graph.

2. Main Results

Theorem 2.1:

$S(K_{1,n})$ is root cube mean cordial graph.

Proof:

Let *u* be the central vertex.

Let $v_1, v_2, ..., v_n$ be the pendent vertices of $K_{1,n}$.

Let $u_1, u_2, ..., u_n$ be the subdivisional vertices of $S(K_{1,n})$.

Now $S(K_{1,n})$ has $2n+1$ vertices and $2n$ edges.

Case (i):

Let $n = 3t$

$n \equiv 0 (\bmod 3)$

Define $f(u) = 1$

$f(u_i) = 0, \ 1 \le i \le t$

$f(u_{t+i}) = 1, \ 1 \le i \le 2t$

$f(v_i) = 0, \ 1 \le i \le t$

$f(v_{t+i}) = 2, \ 1 \le i \le 2t$

Then $v_f(0) = 2t$, $v_f(1) = 2t+1$, $v_f(2) = 2t$

Let the central vertex u be labeled by 1. Now assign the label *0* for t subdivisional vertices $u_1, u_2, ..., u_t$ and the label *0* for t vertices that are adjacent to t end vertices $v_1, v_2, ..., v_t$ respectively.

Also the central vertex u which is labeled as 1 is adjacent to *2t* subdivisional vertices $u_{t+1}, u_{t+2}, ..., u_{3t}$ are labeled with 1.

Now these *2t* vertices adjacent to *2t* end vertices $v_{t+1}, v_{t+2}, ..., v_{3t}$ are labeled with 2.

ie, $e_f(0) = 2t$, $e_f(1) = 2t$, $e_f(2) = 2t$

Clearly $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1 \ \forall \ i, j \in \{0,1,2\}$

Case (ii):

Let $n = 3t+1$

$n \equiv 1 \pmod 3$

Let the central vertex u be labeled by 1.

Define $f(u) = 1$

$f(u_i) = 0, \ 1 \le i \le t$

$f(u_{t+i}) = 1, \ 1 \le i \le 2t$

$f(u_n) = 0$

$f(v_i) = 0, \ 1 \le i \le t$

$f(v_{t+i}) = 2, \ 1 \le i \le t$

$f(v_n) = 2$

Then $v_f(0) = 2t+1$, $v_f(1) = 2t+1$, $v_f(2) = 2t+1$

$e_f(0) = 2t+1$, $e_f(1) = 2t$, $e_f(2) = 2t+1$

Clearly $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1 \ \forall \ i, j \in \{0,1,2\}$

Case (iii):

Let $n = 3t + 2$

$n \equiv 2 (\text{mod}\, 3)$

Let the central vertex u be labeled by 1.

Define $f(u) = 1$

$f(u_i) = 0,\ 1 \le i \le t+1$

$f(u_{t+1+i}) = 1,\ 1 \le i \le 2t$

$f(u_n) = 1$

$f(v_i) = 0,\ 1 \le i \le t+1$

$f(v_{t+1+i}) = 2,\ 1 \le i \le 2t$

$f(v_n) = 2$

Then $v_f(0) = 2t + 2$, $v_f(1) = 2t + 2$, $v_f(2) = 2t + 1$

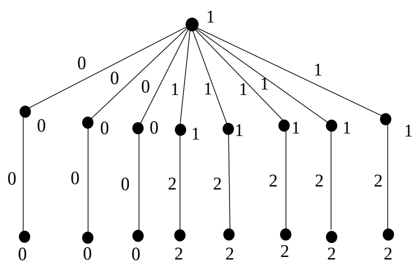
$e_f(0) = 2t + 2$, $e_f(1) = 2t + 1$, $e_f(2) = 2t + 1$

Clearly $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1 \ \forall\ i, j \in \{0,1,2\}$

Hence from all the three cases, $S(K_{1,n})$ is root cube mean cordial graph.

Example 2.2:

Consider the graph $S(K_{1,8})$



Theorem 2.3:

Subdivision of a Bistar graph, $S(B_{n,n})$ admits root cube mean cordial labeling

Proof:

Let $V(S(B_{n,n})) = \{(u, v, w, u_i, v_i, x_i, y_i) : i$ varies from 1 to *n*}, and

$E(S(B_{n,n})) = \{[(ux_i) : i$ varies from 1 to $n] \cup [(x_i u_i) : i$ varies from 1 to $n] \cup$

$[(uw)] \cup [(wv)] \cup [(vy_i) : i$ varies from 1 to $n] \cup$
$[(y_i v_i) : i$ varies from 1 to $n]\}$.

where *u*, *v* are the apex vertices; u_i, v_i are the pendent vertices; w, x_i, y_i are the subdivisional vertices of uv_i, vv_i and uv respectively.

Define $f : V(G) \rightarrow \{0,1,2\}$ as follows.

Case (i):

Let $n = 3t$

$n \equiv 0(\text{mod } 3)$

Let us label the apex vertices and the subdivision of apex vertices as 1.

Now $f(u) = 1$

$f(v) = 1$

$f(w) = 1$

$f(x_i) = 0, \ 1 \le i \le t$

$f(x_{t+i}) = 1, \ 1 \le i \le 2t$

$f(u_i) = 0, \ 1 \le i \le t$

$f(u_{t+i}) = 2, \ 1 \le i \le 2t$

$f(y_i) = 0, 1 \le i \le t+1$

$f(y_{t+1+i}) = 2, \ 1 \le i \le t$

$f(y_{2t+1+i}) = 1, \ 1 \le i \le t$

$f(v_i) = 0, \ 1 \le i \le t$

$f(v_{t+i}) = 2,\ 1 \le i \le t+1$

$f(v_{2t+1+i}) = 1,\ 1 \le i \le t$

Then $v_f(0) = 4t+1,\ v_f(1) = 4t+1,\ v_f(2) = 4t+1$

$e_f(0) = 4t+1,\ e_f(1) = 4t,\ e_f(2) = 4t+1$

Clearly $\left|v_f(i) - v_f(j)\right| \le 1$ and $\left|e_f(i) - e_f(j)\right| \le 1\ \forall\ i, j \in \{0,1,2\}$

Case (ii):

Let $n = 3t+1$

$n \equiv 1 \pmod 3$

Let's label the apex vertices and the subdivision of apex vertices as 1.

Now $f(u) = 1$

$f(v) = 1$

$f(w) = 1$

$f(x_i) = 0,\ 1 \le i \le t+1$

$f(x_{t+1+i}) = 1,\ 1 \le i \le t$

$f(x_{2t+1+i}) = 2,\ 1 \le i \le t$

$f(u_i) = 0,\ 1 \le i \le t+1$

$f(u_{t+1+i}) = 1,\ 1 \le i \le t$

$f(u_{2t+1+i}) = 2,\ 1 \le i \le t$

$f(y_i) = 0,\ 1 \le i \le t$

$f(y_{t+i}) = 1,\ 1 \le i \le t$

$f(y_{2t+i}) = 2,\ 1 \le i \le t+1$

$f(v_i) = 0, \ 1 \le i \le t$

$f(v_{t+i}) = 1, \ 1 \le i \le t$

$f(v_{2t+i}) = 2, 1 \le i \le t+1$

Then $v_f(0) = 4t+2$, $v_f(1) = 4t+3$, $v_f(2) = 4t+2$

$e_f(0) = 4t+2$, $e_f(1) = 4t+2$, $e_f(2) = 4t+2$

Clearly $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1 \ \forall \ i, j \in \{0,1,2\}$

Case (iii):

Let $n = 3t+2$

$n \equiv 2 (\bmod 3)$

Let the apex vertices *u* and *v* be 1 and let the subdivision of *u* and *v* are the new vertex *w* and is labelled as 1.

Now $f(u) = 1$

$f(v) = 1$

$f(w) = 1$

$f(x_i) = 0, \ 1 \le i \le t+1$

$f(x_{t+1+i}) = 1, \ 1 \le i \le t$

$f(x_{2t+1+i}) = 2, \ 1 \le i \le t+1$

$f(u_i) = 0, \ 1 \le i \le t+1$

$f(u_{t+1+i}) = 1, \ 1 \le i \le t$

$f(u_{2t+1+i}) = 2, \ 1 \le i \le t+1$

$f(y_i) = 0, 1 \le i \le t+1$

$f(y_{t+1+i}) = 1, \ 1 \le i \le t$

$f(y_{2t+1+i})=2,\ 1\le i\le t$

$f(y_n)=1$

$f(v_i)=0,\ 1\le i\le t+1$

$f(v_{t+1+i})=1,\ 1\le i\le t$

$f(v_{2t+1+i})=2,\ 1\le i\le t+1$

Then $v_f(0)=4(t+1)$, $v_f(1)=4(t+1)$, $v_f(2)=4(t+1)-1$

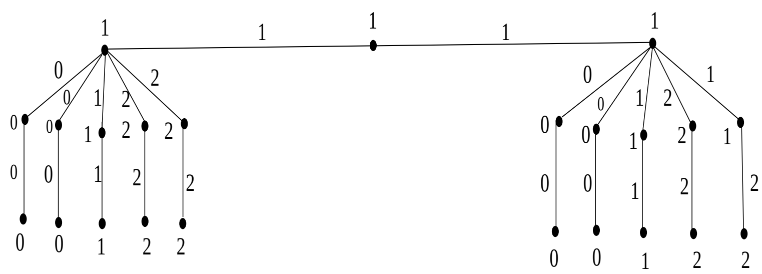
$e_f(0)=4t+1$, $e_f(1)=e_f(2)=4(t+1)-1$

Clearly $|v_f(i)-v_f(j)|\le 1$ and $|e_f(i)-e_f(j)|\le 1\ \forall\ i,j\in\{0,1,2\}$

From all the three cases, it is clear that subdivision of Bistar $S(B_{n,n})$ is root cube mean cordial graph.

Example 2.4:

Consider the subdivision of graph $B_{5,5}$



Theorem 2.5:

Subdivision of $P_n \odot K_1$, $S(P_n \odot K_1)$ is root cube mean cordial graph.

Proof:

Let P_n be the path $u_1,u_2,.........,u_n$ and v_i be the pendent vertices adjacent to $u_i (1\le i\le n)$.

Let x_i be the vertex which subdivided the edge $u_iu_{i+1}(1\le i\le n-1)$ and y_i be the vertex which subdivided the edge $u_iv_i (1\le i\le n)$.

Case (i):

Let $n = 3t$

$n \equiv 0(\bmod 3)$

Define $f : V(G) \to \{0,1,2\}$

Let $f(u_i) = 0,\ 1 \le i \le t$

$f(u_{t+i}) = 1,\ 1 \le i \le 2t$

$f(u_{2t+i}) = 2, 1 \le i \le t$

$f(x_i) = 0, 1 \le i \le t$

$f(x_{t+i}) = 1,\ 1 \le i \le t$

$f(x_{2t+i}) = 2, 1 \le i \le t$

$f(v_i) = 0,\ 1 \le i \le t$

$f(v_{t+i}) = 1,\ 1 \le i \le t$

$f(v_{2t+i}) = 2,\ 1 \le i \le t$

$f(y_i) = 0, 1 \le i \le t$

$f(y_{t+i}) = 1,\ 1 \le i \le t$

$f(y_{2t+i}) = 2,\ 1 \le i \le t$

Then $v_f(0) = 4t$, $v_f(1) = 4t,\ v_f(2) = 4t - 1$

$e_f(0) = 4t$, $e_f(1) = 4t - 1,\ e_f(2) = 4t - 1$

Clearly $\left|v_f(i) - v_f(j)\right| \le 1$ and $\left|e_f(i) - e_f(j)\right| \le 1 \forall i, j \in \{0,1,2\}$

Case (ii):

Let $n = 3t$

$n \equiv 1 \pmod 3$

Let $f(u_i) = 0,\ 1 \le i \le t$

$f(u_{t+i}) = 1,\ 1 \le i \le t+1$

$f(u_{2t+1+i}) = 2,\ 1 \le i \le t$

$f(x_i) = 0,\ 1 \le i \le t$

$f(x_{t+i}) = 1,\ 1 \le i \le t$

$f(x_{2t+i}) = 2,\ 1 \le i \le t$

$f(v_i) = 0,\ 1 \le i \le t$

$f(v_{t+i}) = 1,\ 1 \le i \le t$

$f(v_{2t+i}) = 2,\ 1 \le i \le t+1$

$f(y_i) = 0,\ 1 \le i \le t+1$

$f(y_{t+1+i}) = 1,\ 1 \le i \le t$

$f(y_{2t+1+i}) = 2,\ 1 \le i \le t$

Then $v_f(0) = 4t+1,\ v_f(1) = v_f(2) = 4t+1$

$e_f(0) = 4t+1,\ e_f(1) = 4t,\ e_f(2) = 4t+1$

Clearly $\left|v_f(i) - v_f(j)\right| \le 1$ and $\left|e_f(i) - e_f(j)\right| \le 1\ \ \forall\ i, j \in \{0,1,2\}$

Case (iii):

Let $n = 3t+2$

$n \equiv 2 \pmod 3$

$f(u_i) = 0,\ 1 \le i \le t$

$f(u_{t+i}) = 1,\ 1 \le i \le t+2$

$f(u_{2t+2+i}) = 2,\ 1 \le i \le t$

$f(x_i) = 0,\ 1 \le i \le t$

$f(x_{t+i}) = 1,\ 1 \le i \le t+1$

$f(x_{2t+1+i}) = 2,\ 1 \le i \le t$

$f(v_i) = 0,\ 1 \le i \le t+1$

$f(v_{t+1+i}) = 1,\ 1 \le i \le t+1$

$f(v_{2t+2+i}) = 2,\ 1 \le i \le t$

$f(y_i) = 0,\ 1 \le i \le t+1$

$f(y_{t+1+i}) = 1,\ 1 \le i \le t$

$f(y_{2t+1+i}) = 2,\ 1 \le i \le t+1$

Then $v_f(0) = 4t+2$, $v_f(1) = 4t+3$, $v_f(2) = 4t+2$

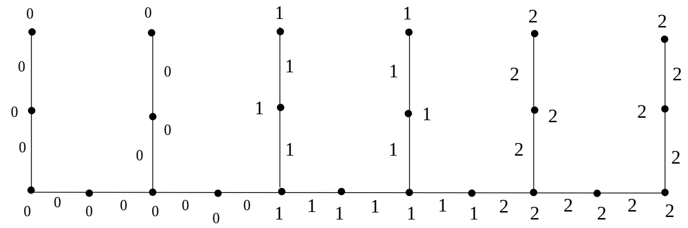
$e_f(0) = 4t+2$, $e_f(1) = 4t+2$, $e_f(2) = 4t+2$

Clearly $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1\ \ \forall\ i, j \in \{0,1,2\}$

From all the above cases $S(P_n \odot K_1)$ is root cube mean cordial graph

Example 2.6:

Consider the graph $S(P_6 \odot K_1)$



Theorem 2.7:

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The subdivision of $C_n \odot K_1$, $S(C_n \odot K_1)$ is root cube mean cordial graph for $n \equiv 1,2 (\bmod 3)$

Proof :

Let C_n be the cycle $u_1, u_2, ..., u_n$ and v_i be the pendent vertices adjacent to $u_i (1 \le i \le n)$.

Let x_i be the vertex which subdivides the edge $u_i u_{i+1} (1 \le i \le n)$ and y_i be the vertex which subdivides the edge $u_i v_i (1 \le i \le n)$.

Case (i):

Let $n = 3t + 1$

$n \equiv 1 (\bmod 3)$

Define $f : V(G) \to \{0,1,2\}$

Let $f(u_i) = 0,\ 1 \le i \le t$

$f(u_{t+i}) = 1,\ 1 \le i \le t+1$

$f(u_{2t+1+i}) = 2,\ 1 \le i \le t$

$f(x_i) = 0,\ 1 \le i \le t$

$f(x_{t+i}) = 1,\ 1 \le i \le t$

$f(x_{2t+i}) = 2,\ 1 \le i \le t+1$

$f(v_i) = 0,\ 1 \le i \le t+1$

$f(v_{t+1+i}) = 1,\ 1 \le i \le t$

$f(v_{2t+1+i}) = 2,\ 1 \le i \le t$

$f(y_i) = 0,\ 1 \le i \le t$

$f(y_{t+i}) = 1,\ 1 \le i \le t+1$

$f(y_{2t+1+i}) = 2,\ 1 \le i \le t$

Then $v_f(0) = 4t + 1$, $v_f(1) = 4t + 2$, $v_f(2) = 4t + 1$

$e_f(0) = 4t+1$, $e_f(1) = 4t+1$, $e_f(2) = 4t+2$

Clearly $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1 \;\forall\; i, j \in \{0,1,2\}$

Case (ii):

Let $n = 3t+2$

$n \equiv 2(\bmod 3)$

Let $f(u_i) = 0,\; 1 \leq i \leq t$

$f(u_{t+i}) = 1,\; 1 \leq i \leq t+1$

$f(u_{2t+1+i}) = 2,\; 1 \leq i \leq t+1$

$f(x_i) = 0,\; 1 \leq i \leq t$

$f(x_{t+i}) = 1,\; 1 \leq i \leq t+1$

$f(x_{2t+1+i}) = 2,\; 1 \leq i \leq t+1$

$f(v_i) = 0,\; 1 \leq i \leq t+1$

$f(v_{t+1+i}) = 1,\; 1 \leq i \leq t$

$f(v_{2t+1+i}) = 2,\; 1 \leq i \leq t+1$

$f(y_i) = 0,\; 1 \leq i \leq t+1$

$f(y_{t+1+i}) = 1,\; 1 \leq i \leq t$

$f(y_{2t+1+i}) = 2,\; 1 \leq i \leq t+1$

Then $v_f(0) = 4t+3$, $v_f(1) = 4t+3$, $v_f(2) = 4t+2$

$e_f(0) = 4t+3$, $e_f(1) = 4t+2$, $e_f(2) = 4t+3$

Clearly $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1 \;\forall\; i, j \in \{0,1,2\}$

Case (iii):

Let $n = 3t$

$n \equiv 0 (\bmod 3)$

Define $f : V(G) \rightarrow \{0,1,2\}$

$f(u_i) = 0,\ 1 \le i \le t$

$f(u_{t+i}) = 1,\ 1 \le i \le t$

$f(u_{2t+i}) = 2, 1 \le i \le t$

$f(x_i) = 0,\ 1 \le i \le t$

$f(x_{t+i}) = 1,\ 1 \le i \le t$

$f(x_{2t+i}) = 2, 1 \le i \le t$

$f(v_i) = 0, 1 \le i \le t$

$f(v_{t+i}) = 1, 1 \le i \le t$

$f(v_{2t+i}) = 2, 1 \le i \le t$

$f(y_i) = 0, 1 \le i \le t$

$f(y_{t+i}) = 1,\ 1 \le i \le t$

$f(y_{2t+i}) = 2,\ 1 \le i \le t$

Then $v_f(0) = 4t$, $v_f(1) = 4t,\ v_f(2) = 4t$

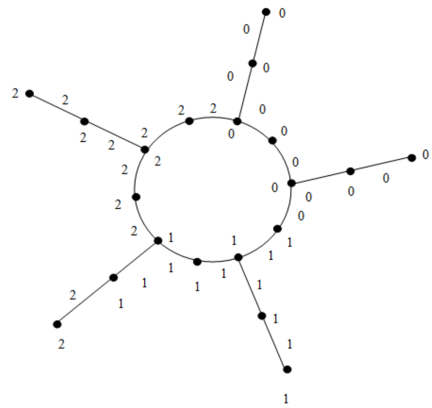
$e_f(0) = 4t$, $e_f(1) = 4t - 1,\ e_f(2) = 4t + 1$

Clearly $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1\ \ \forall\ \ i, j \in \{0,1,2\}$

Hence $S(C_n \odot K_1)$ is root cube mean cordial for $n \equiv 0 (\bmod 3)$

Example 2.8:

Consider $S(C_5 \odot K_1)$



Theorem 2.9:

The subdivision of H graph of path P_n is root cube mean cordial.

Proof:

Let G be a subdivision of H graph of path P_n , P_n be the path with vertices $u_1, u_2, \ldots\ldots\ldots, u_n$, we can obtain H-graph by considering two copies of P_n . Let the other set of vertices of path P_n be $v_1, v_2, \ldots\ldots\ldots, v_n$.

Let x_i be the vertices which subdivides the edge $u_i u_{i+1} (1 \le i \le n-1)$ and y_i be the vertices subdivides the edge $v_i v_{i+1} (1 \le i \le n-1)$ and let z be the vertex subdivide the edge $u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}$, when n is odd and $u_{\frac{n}{2}} v_{\frac{n}{2}+1}$, when n is even.

Define $f : V(G) \to \{0,1,2\}$ as follows.

Case (i):

Let $n = 3t$

$n \equiv 0 (\text{mod } 3)$

Define $f(z) = 1$

$f(u_i) = 0, \ 1 \le i \le t$

$f(u_{t+i}) = 1, \ 1 \le i \le t$

$f(u_{2t+i}) = 2, \ 1 \le i \le t$

$f(x_i)=0,\ 1\le i\le t$

$f(x_{t+i})=1,\ 1\le i\le t-1$

$f(x_{2t-1+i})=2,\ 1\le i\le t$

$f(v_i)=0,\ 1\le i\le t$

$f(v_{t+i})=1,\ 1\le i\le t$

$f(v_{2t+1})=2,\ 1\le i\le t$

$f(y_i)=0, 1\le i\le t$

$f(y_{t+i})=1,\ 1\le i\le t$

$f(y_{2t+i})=2, 1\le i\le t$

Then $v_f(0)=4t$, $v_f(1)=4t,\ v_f(2)=4t-1$

$e_f(0)=4t$, $e_f(1)=4t-1,\ e_f(2)=4t-1$

Here $\left|v_f(i)-v_f(j)\right|\le 1$ and $\left|e_f(i)-e_f(j)\right|\le 1\ \forall\ \ i, j\in\{0,1,2\}$

Case (ii):

Let $n=3t+1$

$n\equiv 1(\bmod 3)$

Define $f(z)=1$

$f(u_i)=0,\ 1\le i\le t$

$f(u_{t+i})=1,\ 1\le i\le t$

$f(u_{2t+i})=2,\ 1\le i\le t+1$

$f(x_i)=0,\ 1\le i\le t$

$f(x_{t+i}) = 1,\ 1 \le i \le t$

$f(x_{2t+i}) = 2,\ 1 \le i \le t$

$f(v_i) = 0,\ 1 \le i \le t+1$

$f(v_{t+1+i}) = 1,\ 1 \le i \le t$

$f(v_{2t+1+i}) = 2,\ 1 \le i \le t$

$f(y_i) = 0, 1 \le i \le t$

$f(y_{t+i}) = 1,\ 1 \le i \le t$

$f(y_{2t+i}) = 2, 1 \le i \le t$

Then $v_f(0) = 4t+1$, $v_f(1) = 4t+1$, $v_f(2) = 4t+1$

$e_f(0) = 4t+1$, $e_f(1) = 4t$, $e_f(2) = 4t+1$

Here $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1\ \forall\ i, j \in \{0,1,2\}$

Case (iii):

Let $n = 3t+2$

$n \equiv 2(\mathrm{mod}\,3)$

Define $f(z) = 1$

$f(u_i) = 0,\ 1 \le i \le t+1$

$f(u_{t+1+i}) = 1,\ 1 \le i \le t$

$f(u_{2t+1+i}) = 2,\ 1 \le i \le t+1$

$f(x_i) = 0,\ 1 \le i \le t$

$f(x_{t+i}) = 1,\ 1 \le i \le t+1$

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$f(x_{2t+1+i}) = 2,\ 1 \le i \le t$

$f(v_i) = 0,\ 1 \le i \le t+1$

$f(v_{t+1+i}) = 1,\ 1 \le i \le t$

$f(v_{2t+1+i}) = 2,\ 1 \le i \le t+1$

$f(y_i) = 0, 1 \le i \le t$

$f(y_{t+i}) = 1,\ 1 \le i \le t+1$

$f(y_{2t+1+i}) = 2, 1 \le i \le t$

Then $v_f(0) = 4t+2$, $v_f(1) = 4t+3$, $v_f(2) = 4t+2$

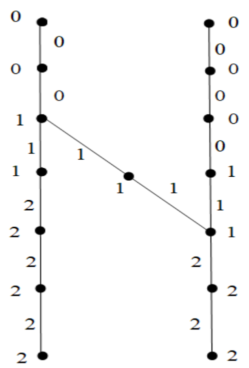
$e_f(0) = 4t+2$, $e_f(1) = 4t+2$, $e_f(2) = 4t+2$

Here $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1\ \forall\ i, j \in \{0,1,2\}$

From all the three cases, the subdivision of H-graph is root cube mean cordial graph.

Example 2.10:

Consider the graph



Theorem 2.11:

The subdivision of $H \odot K_1$, graph of path P_n is root cube mean cordial.

Proof:

Let G be the subdivision of H-graph of path P_n , P_n be the $u_1,u_2,.........,u_n$, we obtain H-graph by considering two copies of P_n . Let the other set of vertices of path be $v_1,v_2,.........,v_n$.

Join each vertices, $u_1,u_2,\dots,u_n$ and $v_1,v_2,\dots,v_n$ by an edge to the vertices $u'_1,u'_2,\dots,u'_n$ and $v'_1,v'_2,\dots,v'_n$ respectively.

Let x_i be the vertices which subdivides the edge $u_iu_{i+1}\,(1\le i\le n-1)$ and x_i' be the vertices which subdivides the edge $u_iu'_{i+1}$, y_i be the vertices which subdivides the edge $v_iv_{i+1}\,(1\le i\le n-1)$ and y_i' be the vertices which subdivides the edge $v_iv'_{i+1}$. Let z be the vertex subdivide the edge $u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}$, where n is odd and $u_{\frac{n}{2}}v_{\frac{n}{2}+1}$, when n is even.

Define $f:V(G)\to\{0,1,2\}$ as follows.

Case (i):

Let $n=3t$

$n\equiv 0(\mathrm{mod}\,3)$

Define $f(z)=1$

$f(u_i)=0,\ 1\le i\le t$

$f(u_{t+i})=1,\ 1\le i\le t$

$f(u_{2t+i})=2,\ 1\le i\le t$

$f(x_i)=0,\ 1\le i\le t$

$f(x_{t+i})=1,\ 1\le i\le t-1$

$f(x_{2t-1+i})=2,\ 1\le i\le t$

$f(v_i)=0,\ 1\le i\le t$

$f(v_{t+i})=1,\ 1\le i\le t$

$f(v_{2t+i})=2,\ 1\le i\le t$

$f(y_i)=0, 1\le i\le t$

$f(y_{t+i})=1,\ 1\le i\le t$

$f(y_{2t+i})=2,\ 1\le i\le t$

$f(u_i^{'})=0,\ 1\le i\le t$

$f(u^{'}_{t+i})=1,\ 1\le i\le t$

$f(u^{'}_{2t+i})=2,\ 1\le i\le t$

$f(v_i^{'})=0,\ 1\le i\le t$

$f(v^{'}_{t+i})=1,\ 1\le i\le t$

$f(v^{'}_{2t+i})=2,\ 1\le i\le t$

$f(x_i^{'})=0,\ 1\le i\le t$

$f(x^{'}_{t+i})=1,\ 1\le i\le t$

$f(x^{'}_{2t+i})=2,\ 1\le i\le t$

$f(y_i^{'})=0,\ 1\le i\le t$

$f(y^{'}_{t+i})=1,\ 1\le i\le t$

$f(y^{'}_{2t+i})=2,\ 1\le i\le t$

Then $v_f(0)=8t$, $v_f(1)=8t,\ v_f(2)=8t-1$

$e_f(0)=8t$, $e_f(1)=8t-1,\ e_f(2)=8t-1$

Here $\left|v_f(i)-v_f(j)\right|\le 1$ and $\left|e_f(i)-e_f(j)\right|\le 1\ \forall\ i,j\in\{0,1,2\}$

Case (ii):

Let $n=3t+1$

$n\equiv 1(\mathrm{mod}\ 3)$

Define $f(z)=1$

$f(u_i)=0,\ 1\le i\le t$

$f(u_{t+i})=1,\ 1\le i\le t$

$f(u_{2t+1+i})=2,\ 1\le i\le t$

$f(x_i)=0,\ 1\le i\le t$

$f(x_{t+i})=1,\ 1\le i\le t$

$f(x_{2t+i})=2,\ 1\le i\le t$

$f(v_i)=0,\ 1\le i\le t$

$f(v_{t+i})=1,\ 1\le i\le t+1$

$f(v_{2t+1+i})=2,\ 1\le i\le t$

$f(y_i)=0, 1\le i\le t$

$f(y_{t+i})=1,\ 1\le i\le t$

$f(y_{2t+i})=2, 1\le i\le t$

$f(u_i^{'})=0,\ 1\le i\le t$

$f(u^{'}_{t+i})=1,\ 1\le i\le t$

$f(u^{'}_{2t+i})=2,\ 1\le i\le t+1$

$f(v_i^{'})=0,\ 1\le i\le t+1$

$f(v^{'}_{t+1+i})=1,\ 1\le i\le t$

$f(v^{'}_{2t+1+i})=2,\ 1\le i\le t$

$f(x_i^{'})=0,\ 1\le i\le t$

$f(x^{'}_{t+i})=1,\ 1\le i\le t$

$f(x'_{2t+i}) = 2,\ 1 \le i \le t+1$

$f(y'_i) = 0,\ 1 \le i \le t+1$

$f(y'_{t+1+i}) = 1,\ 1 \le i \le t$

$f(y'_{2t+1+i}) = 2,\ 1 \le i \le t$

Then $v_f(0) = 8t+2$, $v_f(1) = 8t+3$, $v_f(2) = 8t+2$

$e_f(0) = 8t+2$, $e_f(1) = 8t+2$, $e_f(2) = 8t+2$

Here $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1\ \forall\ i, j \in \{0,1,2\}$

Case (iii):

Let $n = 3t+2$

$n \equiv 2 (\mathrm{mod}\, 3)$

Define $f(z) = 1$

$f(u_i) = 0,\ 1 \le i \le t$

$f(u_{t+i}) = 1,\ 1 \le i \le t+1$

$f(u_{2t+1+i}) = 2,\ 1 \le i \le t+1$

$f(x_i) = 0,\ 1 \le i \le t$

$f(x_{t+i}) = 1,\ 1 \le i \le t+1$

$f(x_{2t+1+i}) = 2,\ 1 \le i \le t$

$f(v_i) = 0,\ 1 \le i \le t+1$

$f(v_{t+1+i}) = 1,\ 1 \le i \le t+1$

$f(v_{2t+2+i}) = 2,\ 1 \le i \le t$

$f(y_i) = 0,\ 1 \le i \le t$

$f(y_{t+i}) = 1,\ 1 \le i \le t+1$

$f(y_{2t+1+i}) = 2, 1 \le i \le t$

$f(u_i^{'}) = 0,\ 1 \le i \le t+1$

$f(u^{'}_{t+1+i}) = 1,\ 1 \le i \le t$

$f(u^{'}_{2t+1+i}) = 2,\ 1 \le i \le t+1$

$f(v_i^{'}) = 0,\ 1 \le i \le t+1$

$f(v^{'}_{t+1+i}) = 1,\ 1 \le i \le t$

$f(v^{'}_{2t+1+i}) = 2,\ 1 \le i \le t+1$

$f(x_i^{'}) = 0,\ 1 \le i \le t+1$

$f(x^{'}_{t+1+i}) = 1,\ 1 \le i \le t$

$f(x^{'}_{2t+1+i}) = 2,\ 1 \le i \le t+1$

$f(y_i^{'}) = 0, 1 \le i \le t+1$

$f(y^{'}_{t+1+i}) = 1,\ 1 \le i \le t$

$f(y^{'}_{2t+1+i}) = 2, 1 \le i \le t$

Then $v_f(0) = 8t+5$, $v_f(1) = 8t+5$, $v_f(2) = 8t+5$

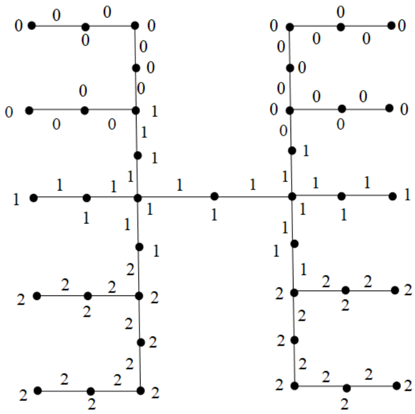
$e_f(0) = 8t+5$, $e_f(1) = 8t+4$, $e_f(2) = 8t+5$

Here $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1\ \forall\ i, j \in \{0,1,2\}$

From all the above cases, the subdivision of $H \odot K_1$ is root cube mean cordial graph.

Example 2.12:

Consider the graph $S(H\odot K_1)$



Conclusion

In this paper root cube mean cordiality of subdivision of some graphs such as path, cycle, star, bistar, H-graph, $H\odot K_1$Graph, $P_n\odot K_1$, $C_n\odot K_1$ Graphs, are studied.

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