A SPACE-TIME FRACTIONAL IS MATHEMATICALLY SOLVED USING A SEMI-LINEAR DIFFUSION EQUATION

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Abstract: The work's main goal is to use an implied technique to address the underlying limit esteem problem for the semi-direct space-time fragmented dissemination condition. The methodology's central idea is to transform the problem into a logarithmic framework, which simplifies the calculations. The strength framework investigation is used to analyses the method's consistency and security. The models of semi-straight fragmented dissemination situations are addressed mathematically. Accurate responses are compared to the learned knowledge. Using appropriate MATLAB models, we examine the error analysis of the indicated limited contrast plan, assembly, and solidity.

Keywords:MATLAB, Constancy, Conjunction

Introduction: Fractional differential equations are essential in the analysis of many biological, chemical, and physical phenomena. As a result, many academics are very interested in the theory, methodology, and applications of fractional differential equations. Therefore, it is essential to investigate reliable and efficient techniques for getting precise or approximate solutions to fractional differential equations. For the approximate solutions of both linear and nonlinear fractional differential equations, the researchers have developed certain numerical techniques. Researchers from diverse fields have examined a range of issues using various terminologies and techniques. Here are a few that must be considered in order to solve the issue at hand. This e-book focuses on Kazuaki's studies on the functional analytic approach to the problem of Markov process building in probability theory. It is well known that the Hille-Yosida theory of semi groups allows one to reduce the complexity of creating Markov processes to the study of boundary value issues for degenerate elliptic integral-differential operators of second order. In their research, Evgeniya et al. (2021) focused on diffusion processes, which are crucial in a number of fields, such as metallurgy, physics, chemistry, and others.This paper investigates and resolves the issue of charged particle diffusion in a semi-infinite thin tube under the influence of an electromagnetic field.

In this chapter, Wang et al. (2020) examine many ordinary differential equation models for the diffusion of innovation and epidemiological models. They examine various ordinary differential equation models for the diffusion of inventions while reviewing the conventional notion of innovation diffusion with an emphasis on online social networks. Khan and Aziz (2018) In this paper, a one-dimensional and two-dimensional hyperbolic partial technique for generating equations is examined, along with a HAAR wavelet-based collocation strategy for numerical solution of diffusion. The numerical results support the accuracy, efficiency, and robustness of the suggested technique.According to Gunvant (2016), the goal of this study is to use a few restricted contrast techniques to obtain a mathematical solution to the underlying limit esteem issue (IBVP) for the semi linear fragmented dissemination condition with variable request. The solidity and union of this process are examined using the Fourier approach. Finally, MATLAB is used to examine and graphically portray the solution to a few numerical instances. According to D'Ambrosio and Paternoster (2014), the objective of this study is to solve partial differential equations that simulate the diffusion problem accurately and quickly using numerical solutions that have been gradually modified. A numerical study demonstrates that using a general-purpose solution is substantially less efficient and accurate when used for both temporal and spatial problems.Bargie (2015) and Tory It is possible to rewrite the nonlinear diffusing equation in a way that leads directly to its stochastic equivalent. By replicating the movements of molecules, the stochastic approach helps us comprehend the physical process better. Our strategy is highly effective in parallel. The hemitrope analysis technique (HAM) is used by Fallahzadeh and Shakibi (2015) to locate solutions to the linear Convection Diffusion (CD) equation series. Gurarslan and Sari's (2011) research provided satisfactory solutions to the problems of direct and nonlinear dissemination. A method with a clear general quadrature approach in space and a flexible soundness protection Runge-Kutta procedure across space was used to address some circumstances.This method may be used to further nonlinear common differential circumstances to produce unquestionably more logical models. According to Griffiths and Schiesser (2012), the one-layered (1D) dispersion condition is a fundamental example of an imperfect differential condition (PDE) that permits voyaging wave arrangements. The numerical solution produced using the technique of lines (MOL) is evaluated using this analytical solution.

This section focuses on determining the numerical solution to the fractional semi linear diffusion problem in space-time. Consider the fractional semi-linear space-time diffusion equation:

$$\frac{\partial^x u}{\partial t^x} = x(a,t)^R D_a^\beta u(a,t) + f(u(a,t),a,t) \quad (1)$$

$$0 < a < l, x(a,t) > 0, 0 < t \leq T, 0 < a \leq 1, 1 < \beta \leq 2$$

$$with initial condition u(a,0) = g(a) (2)$$

$$u(0,t) = 0 = u(l,t) \quad (3)$$

It is known as the spacetime fractional semi-linear diffusion equation's first initial boundary value problem (IBVP). Keep in mind $\frac{\partial^x u}{\partial t^x}$and ${}^R D_a^\beta u(a,t)$are the fractional derivatives of order Caputo and Riemann−Liouville$\alpha(0 < \alpha \leq 1)$ $and \beta(1 < \beta \leq 2)$ $respectively.$

Implicit Finite Difference Scheme: One discretizes the entire first IBVP (1) in this part (3). For each $\beta(0 \leq n - 1 < n)$according to Riemann-Liouville derivative is real & corresponds to the Grunwald-Letnikow derivative. The mathematical estimate of partial request differential conditions is an extra significant result of the connection between the Riemann-Liouville and Grunwald-Letnikov thoughts. This permits the Riemann-Liouville definition to be utilized during issue plan and the Grunwald-Lettikov definition to be utilized during mathematical arrangement.For${}^R D_a^\beta u(a,t)$,At all-time levels, we use the shifted Grunwald formula to approximate the 2nd order space derivative.

$$RD_a^\beta u(a_i, t_{k+1}) = \frac{1}{h^\beta} \sum_{j=0}^{i+1} g_j u(x_i - (j-1)h, t_{k+1}) + O(h) \quad (4)$$

where the Grunwald weights are defined as follows –

$$g_0 = 1, g_j = (-1)^j \frac{(\beta)(\beta-1)(\beta-2)\ldots(\beta-j+1)}{j!}, j = 1,2,3,\ldots \quad (5)$$

Now having equations (3) and (4) in equation (1). we get

$$u(a_i, t_{k+1}) = u(a_i, t_k) + r \sum_{j=0}^{i+1} g_j u(a_{i-j+1}, t_{k+1}) - b_1 u(a_i, t_k) + b_k u(a_i, t_k) +$$

$$\sum_{j=1}^{k-1} (b_j - b_{j+1}) u(a_i, t_{k-j}) + r_1 f(u(a_i, t_k), a_i, t_k) + R_i^{k+1} \quad (6)$$

Where $r = r(i,k) = \frac{x_i^k \tau^x \Gamma(2-x)}{h^\beta}, r_1 = \tau^x \Gamma(2-x)$

$$|R_i^{k+1}| \leq c_1 \tau^x (\tau^{1+x} + h^\beta + \tau) \quad (7)$$

Let u_i^kbe the numerical approximation of$u(a_i, t_k)$and let $f_i^k(u_i^k)$be the closest numerical approximation $f\big(a_i, t_k, u(a_i, t_k)\big)$. As a result, the whole discrete form of the initial IBVP (1)-(3) is obtained.

$$(1+\beta r)u_i^1 - r\sum_{j=0, j\neq 1}^{i+1} g_j u_{i+1-j}^1 = u_i^0 - r_1 f_i^0(u_i^0), k = 0$$

$(1+\beta r)u_i^{k+1} - r\sum_{j=0,j\neq 1}^{i+1} g_j u_{i+1-j}^{k+1} = (1-b_1)u_i^k + r_1 f_i^k\big(u_i^k\big) + \sum_{j=1}^{k-1}\big(b_j - b_{j+1}\big)u_i^{k-j} + b_k u_i^0, k > 1$ (8)

$$initialcondition u_i^0 = g_i, i - 0,1,2, \ldots., m-1 \quad (9)$$

$$u_0^k = 0 = u_m^k, k = 0,1,2, \ldots. n. \quad (10)$$

Put $k = 0, and i = 1,2, \ldots, m-1$We get a group of (m − 1) calculations from the equation (8), which may be represented in the matrix equation as follows.

$$AU^1 = U^0 + r_2 F^0 \quad (11)$$

$$where U^1 = [u_1^1, u_2^1, \ldots., u_{m-1}^1]^T; U^0 = [u_1^0, u_2^0, \ldots., u_{m-1}^0]^T;$$

$F^0 = [f_1^0(u_1^0), f_2^0(u_2^0), \ldots., f_{m-1}^0(u_{m-1}^1)]^T$

where A is a square matrix of order (m - 1) × (m - 1) like that

$$A = \begin{pmatrix} 1+\beta r & -rg_0 & & & & \\ -rg_2 & 1+\beta r & -rg_0 & & & \\ -rg_3 & -rg_2 & 1+\beta r & -rg_0 & & \\ . & . & . & . & . & . \\ & & & & & \\ -rg_{m-1} & -rg_{m-2} & . & . & -rg_2 & 1+\beta r \end{pmatrix}$$

This can be expressed as

$$A_{i,j} = \begin{cases} 0, & \text{when } j > i+1 \\ 1+\beta r, & \text{when } j = i \\ -rg_{i-j+1}, & \text{otherwise} \end{cases}$$

Also for $k = 1, i = 1,2, \ldots, m-1$, the matrix equation is

$$AU^2 = (1-b_1)U^1 + b_1 U^0 + r_1 F^1$$

In general, $k \geq 1, i = 1,2, \ldots, m-1$ we can write as

$$AU^{k+1} = (1-b_1)U^k + \sum_{j=1}^{k-1}(b_j - b_{j+1})\, U_i^{k-j} + b_k U^0 + r_1 F^k \quad (12)$$

Where $F^k = [f(u_1^k), f(u_2^k), \ldots., f(u_{m-1}^k)]^T; U^{k+1} = [u_1^{k+1}, u_2^{k+1}, \ldots., u_{m-1}^{k+1}]^T;$

Stability: In this section, we talk about how stable the implicit finite difference scheme is.Let$\bar{u}_i^k$be the approximation of the implicit finite difference scheme (8)–(100), and let

$f_i^k(\bar{u}_i^k)$be the approximations of$f\left(a_i, t_k, u(a_i, t_k)\right). Setting \epsilon_i^k = u_i^k - \bar{u}_i^k$The round off error equation is obtained.

$$(1+\beta r)\epsilon_i^1 - r\sum_{j=0, j\neq 1}^{i+1} g_j \epsilon_{i+1-j}^1 = \epsilon_i^0 - r_1\left(f_i^0(u_i^0) - f_i^0(\bar{u}_i^0)\right), k = 0$$

$$(1+\beta r)\epsilon_i^{k+1} - r\sum_{j=0, j\neq 1}^{i+1} g_j \epsilon_{i+1-j}^{k+1} = (1-b_1)\epsilon_i^k + r_1\left(f_i^k\left(u_i^k\right) - f_i^k\left(\bar{u}_i^k\right)\right) +$$

$\sum_{j=1}^{k-1}\left(b_j - b_{j+1}\right)\epsilon_i^{k-j} + b_k\epsilon_i^0, k > 1$ (13)

for $i = 1, 2, \ldots, m-1, k = 0, 1, 2, \ldots, n$. Assuming$\|E^k\|_\infty = \max_{1 \le i \le m-1}\left|\epsilon_i^k\right|$

We now examine the stability of the implicit finite difference scheme (8)-(10) using the induction approach. When we enter k = 0 into equation (13), we get ϵ^1

Assume that$|\epsilon_l^1| = max\{|\epsilon_1^1|, |\epsilon_2^1|, \ldots, |\epsilon_{m-1}^1|\}$

$$|\epsilon_l^1| \le (1+\beta r)|\epsilon_l^1| - r\sum_{j=0, j\neq 1}^{i+1} g_j |\epsilon_l^1|$$

$$\le (1+\beta r)|\epsilon_l^1| - r\sum_{j=0, j\neq 1}^{i+1} g_j \left|\epsilon_{l-j+1}^1\right|$$

$$= \left|\epsilon_{l-1}^0 + r_1\left(f_i^0(u_i^0) - f_i^0(\bar{u}_i^0)\right)\right|$$

$$\le (1 + r_1 L)\left|\epsilon_l^0\right|$$

$$\|E^1\|_\infty \le C\|\epsilon^0\|_\infty (\because C = 1 + r_1 L)$$

Let $\|E^{k+1}\|_\infty = \left|\epsilon_l^{k+1}\right| = max\left\{\left|\epsilon_1^{k+1}\right|, \left|\epsilon_2^{k+1}\right|, \ldots, \left|\epsilon_{m-1}^{k+1}\right|\right\}$ and assume that

$\|E^j\|_\infty \le C\|\epsilon^0\|_\infty, j = 1,2, \ldots k$we get

$$\left|\epsilon_l^{k+1}\right| \le (1+\beta r)\left|\epsilon_l^{k+1}\right| - r\sum_{j=0, j\neq 1}^{i+1} g_j \left|\epsilon_l^{k+1}\right|$$

$$\le \left|(1+\beta r)\epsilon_l^{k+1} - r\sum_{j=0, j\neq 1}^{i+1} g_j \epsilon_l^{k+1}\right|$$

$$\sum_{j=1}^{k-1}\left(b_j - b_{j+1}\right)\left|\epsilon_i^{k-j}\right| + b_k\left|\epsilon_i^0\right|$$

$$\le (1-b_1)\left|\epsilon_l^k\right| + r_1 L\left|\epsilon_l^k\right| + (b_1 - b_k)\left|\epsilon_l^k\right| + b_k\left|\epsilon_i^0\right|$$

$$\le (1 + r_1 L)\left|\epsilon_l^0\right|$$

$$\|E^{k+1}\|_\infty \le C_0\|\epsilon^0\|_\infty, \left(\because C_0 = C(1 + r_1 L)\right)$$

As a result, we can prove the following theorem.

Convergence: The convergence of the implicit finite difference scheme is investigated (8) in this section (10). Let$u(a_i, t_k)$be the exact IBVP (1)–(3) solution at mesh point$(a_i, t_k)$and letu_i^kbe the numerical solution of (8)-(10) calculated with the implicit finite difference technique.Define

$e_i^k = u(a_i, t_k) - u_i^kandE^k = \left(e_1^k, e_2^k, \dots, e_{m-1}^k\right)^T$

$$(1+\beta r)e_i^1 - r\sum_{j=0, j\neq 1}^{i+1} g_j e_{i+1-j}^1 = e_i^0 - r_1\left(f_i^0(u_i^0) - f_i^0(\bar{u}_i^0)\right) + R_i^1, k = 0$$

$$(1+\beta r)e_i^{k+1} - r\sum_{j=0, j\neq 1}^{i+1} g_j e_{i+1-j}^{k+1} = (1-b_1)e_i^k + r_1\left(f_i^k(u_i^k) - f_i^k(\bar{u}_i^k)\right) +$$

$$\sum_{j=1}^{k-1}(b_j - b_{j+1})e_i^{k-j} + b_k e_i^0 + R_i^{k+1}, k > 1 \qquad (14)$$

where $i = 1, 2, \dots, m-1, k = 0, 1, 2, \dots, n.$

$$\left|R_l^{k+1}\right| \le c_1\tau^\alpha\left(\tau^{1+\alpha} + h^\beta + \tau\right) \text{ for } i = 1,2, \dots, m-1, k = 0,1,2, \dots, n$$

Following is a proof that the convergence analysis may be established by the use of mathematical induction. In equation (14), when k is equal to zero, we get e^1

Assuming that $\|e^1\|_\infty = |e_l^1| = \max_{1\le i\le m-1}|e_i^1|$

$$|e_l^1| \le (1+\beta r)|e_l^1| - r\sum_{j=0, j\neq 1}^{i+1} g_j|e_l^1|$$

$$\begin{aligned} &\le \left|(1+\beta r)e_l^1 - r\sum_{j=0, j\neq 1}^{i+1} g_j e_{l-j+1}^1\right| \\ &= \left|e_l^1 + r_1\left(f_i^0(u_i^0) - f_i^0(\bar{u}_i^0)\right) + R_l^1\right| \qquad (15) \\ &\le \left|e_l^0\right| + r_1 L\left|e_l^0\right| + |R_l^1| \end{aligned}$$

$$\|E^1\|_\infty \le |R_l^1|$$

$Using e^0 = 0\ and |R_l^1| \le c_1\tau^x(\tau^{1+x} + h^\beta + \tau)$we have

$$\|e^1\|_\infty \le b_0^{-1}c_1\tau^\alpha\left(\tau^{1+\alpha} + h^\beta + \tau\right)$$

Assume that this is the case for $j, \|e^j\|_\infty \le b_{j-1}^{-1}c_1\tau^\alpha\left(\tau^{1+\alpha} + h^\beta + \tau\right)$

$j = 1,2, \dots, k$And $\left|e_l^{k+1}\right| = max\left\{\left|e_1^{k+1}\right|, \left|e_2^{k+1}\right|, \dots, \left|e_{m-1}^{k+1}\right|\right\}$Note that$b_j^{-1} \le b_k^{-1}$

$$\left|e_l^{k+1}\right| \le (1+\beta r)\left|e_l^{k+1}\right| - r\sum_{j=0, j\neq 1}^{i+1} g_j\left|e_l^{k+1}\right|$$

$$\le \left|(1+\beta r)\epsilon_l^{k+1} - r\sum_{j=0, j\neq 1}^{i+1} g_j\epsilon_{l-j+1}^{k+1}\right|$$

$$=|\ (1-b_1)e_i^k + r_1\left(f_i^k(u_i^k) - f_i^k(\bar{u}_i^k)\right) +$$

$$\sum_{j=1}^{k-1}\left(b_j-b_{j+1}\right)e_i^{k-j}+b_k e_i^0+R_l^{k+1}\mid$$

$$\leq(1-b_1)\left|e_i^k\right|+r_1\left|\left(f_i^k\left(u_i^k\right)-f_i^k\left(\bar{u}_i^k\right)\right)\right|+$$

$$\sum_{j=1}^{k-1}\left(b_j-b_{j+1}\right)\left|e_i^{k-j}\right|+\left|R_l^{k+1}\right|$$

$$\leq(1-b_1)\|e^k\|_\infty+r_1L\|e^k\|_\infty+(b_1-b_k)\|e^k\|_\infty+\left|R_l^{k+1}\right|$$

$$\leq b_k^{-1}\{1+r_1L\}c_1\tau^\alpha\left(\tau^{1+\alpha}+h^\beta+\tau\right)$$

$$\|e^{k+1}\|_\infty\leq C_0k^\alpha\tau^\alpha\left(\tau^{1+\alpha}+h^\beta+\tau\right),(\because (1+r_1L)c_1=C_0)$$

If $k\tau\leq T$ is a finite number, then we can prove the following theorem.

Theorem 1: Letu_i^kbe the close approximation of $u(a_i,t_k)$calculated by making use of an implicit finite difference method, & both the basis term and the situation of Lipschitz are met (3.9). If this is the case, there must be a positive constant C_0such that$\left|u_i^k-u(a_i,t_k)\right|\leq C_0(\tau+h)$

- **Test Problem**

Example 1: Take into consideration the fractional semi-linear space-time diffusion equation.

$$\frac{\partial^{0.9}u}{\partial t^{0.9}}=\frac{\partial^{1.8}u}{\partial a^{1.8}}+u^2+f\left(a,t,u(a,t)\right),0<a<\pi,0<t\leq T \qquad (16)$$

$$withinitialconditionu(a,0)=\ sinx \qquad (17)$$

$$u(0,t)=\ 0\ =\ u(\pi,t) \qquad (18)$$

Where $f=t^{0.1}sinaE_{1,1.1}(t)-e^t\sin(a+0.9\pi)-sin^2ae^{2t}$and exact solution is $u(a,t)=e^tsina$

The following is an example of the discrete form of IBVP (16-18):-

$$(1+\beta r)u_i^1-r\sum_{j=0,j\neq1}^{i+1}g_ju_{i+1-j}^1=u_i^0-r_1(u_i^0)^2,k=0$$

$$(1+\beta r)u_i^{k+1}-r\sum_{j=0,j\neq1}^{i+1}g_ju_{i+1-j}^{k+1}=(1-b_1)u_i^k+r_1\left(u_i^k\right)^2+$$

$$\sum_{j=1}^{k-1}\left(b_j-b_{j+1}\right)u_i^{k-j}+b_ku_i^0,k>1$$

$$initialconditionu_i^0=\sin(ih),\ \ i=0,1,2,3,4,5,6.\left(\therefore h=\frac{\pi}{6}\right)$$

$$u_0^k=0=u_m^k,\ \ k=0,1,2,\ldots.,n$$

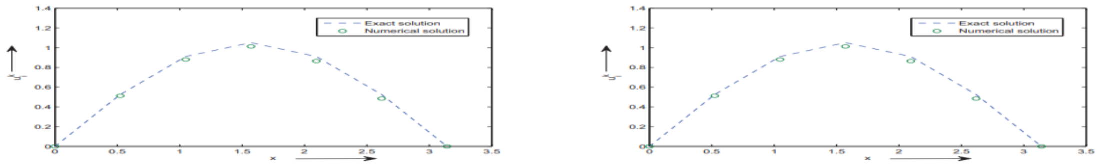

Figure 1: When t equals 0.02 and t equals 0.05, a comparison is made between the exact answer and the numerical solution

Table 1: In the table that follows, a comparison is made between the particularanswer and the numerical answer at the time t = 0.01.

u(a, t)	I.F.D.M.	Particularanswer	Absolute Error	Relative Error	% Error
$u\left(\frac{\pi}{6}, 0.01\right)$	0.5064	0.5050	0.0014	0.0028	0.2772
$u\left(\frac{\pi}{3}, 0.01\right)$	0.8751	0.8747	0.0004	4.573×10^{-4}	0.04578
$u\left(\frac{\pi}{2}, 0.01\right)$	1.0103	1.0101	0.00024	2.376×10^{-4}	0.0238
$u\left(\frac{2\pi}{3}, 0.01\right)$	0.8754	0.8747	0.0007	8.0027×10^{-4}	0.08
$u\left(\frac{5\pi}{6}, 0.01\right)$	0.5064	0.5050	0.0014	0.0028	0.2772

Example 2: Take into consideration the fractional semi-linear space-time diffusion equation.

$$\frac{\partial^a u}{\partial t^a} = \frac{\partial^\beta u}{\partial a^\beta} + sinu, 0 < a < 1, 0 < t \leq T \quad (19)$$

$$withinitialconditionu(a, 0) = x(1 - x) \quad (20)$$

$$u(0, t) = 0 = u(1 - t) \quad (21)$$

The discrete IBVP (19-) (21)-

$$(1 + \beta r)u_i^1 - r \sum_{j=0, j\neq 1}^{i+1} g_j u_{i+1-j}^1 = u_i^0 - r_1(\sin(u_i^0)), k = 0$$

$$\sum_{j=1}^{k-1} (b_j - b_{j+1})u_i^{k-j} + b_k u_i^0, k > 1$$

$$initialconditionu_i^0 = ih(1 - ih), \quad i = 0,1,2,3,4,5. (\because h = 0.2)$$

$$boundaryconditionu_0^k = 0 = (u_m^k), \quad k = 0,1,2, \dots, n.$$

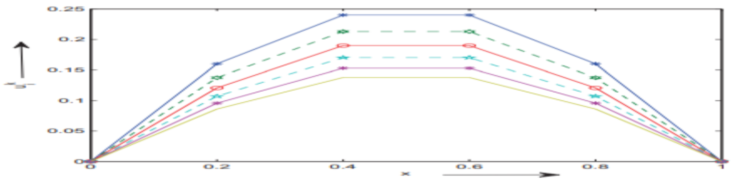

Figure 2: The numerical solution of the function u(a, t) at a variety of time steps for the cases where = 0.9 and = 1.8

1. Conclusions

Using this method, it should be extremely effective to find the mathematical solution for semi-direct fragmentary incomplete differential conditions. The lattice approach aids in stabilizing and preparing for union the understood restricted distinction strategy. Number problems are used to demonstrate how the hypotheses function.

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