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Computation of Multi-Objective Linear Programming Problem by Matrix Inversion Method

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Abstract— The objective of this paper is to solve a Multi-Objective Linear Programming problem to achieve optimal solutions for two objectives. As we solve our problem by using a matrix inversion method and compare the result with existing methods, we are aiming to minimize production and transportation cost. This method is illustrated by solving numerical examples and discussing the results.

Keywords- Linear Programming Problem, Multi-Objective, Echelon method

Introduction:

Numerous research studies have been conducted on the Multi-Objective Problems (MOP) in the past few decades, often involving conflicting and in commensurate objectives [1]. It is often necessary to handle inexact or uncertain input data when dealing with real-world MOP problems. MOP problems with imprecise data have been approached in several ways over the years based on different sources of uncertainty.

The concept of uncertainty may be interpreted as randomness or fuzziness [2]. Stochastic programming approaches are used to deal with randomness in MOP problems [3,4]. MOP problems with fuzziness are dealt with using fuzzy programming techniques [5–7]. Random and fuzzy coefficients are assumed in stochastic and fuzzy programming to have known distributions, respectively. Although it is relatively easy to determine the distribution of random variables and the membership functions of fuzzy numbers, determining their distributions is more difficult. The reason is that sometimes they do not exactly reflect the reality.

Linear programming limitations and mathematical formulation are also explained. Some practical applications of linear programming are discussed. They provide insight on the latest applications of linear programming problem in various fields like sports, lean manufacturing, financial planning and radiotherapy [22]. Fuzzy Linear Programming Problem with Triangular and Trapezoidal fuzzy numbers by using ranking method technique with α-Cut to get optimal solution for solving fuzzy linear programming problem with trapezoidal and triangular fuzzy numbers and also compare the existing methods [23], [27]. Transportation problem is one of the appropriate methods to determine the optimum path for supplier to transport goods from supply centre to demand centre by keeping the minimum transportation cost. This study analyses the identical results while solving the problem by both methods Vogel's method and excel

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solver technique. Here we found that the advantage of excel solver technique is easy to implementation and also get the faster and appropriate solution for large amount of data [24], [25,26].

In MOP problems, interval programming could be considered as an alternative solution. In interval programming, it is assumed that the coefficients perturb independently within the given lower and upper bounds [8]. Actually, uncertain coefficients are modelled by closed intervals in this approach. Since interval programming does not require stringent applicability conditions, therefore, it has been considered a suitable tool for modelling uncertainty in many practical applications [9–20].

This paper focuses on interval programming for dealing with uncertainty in multi-objective linear programming (MOLP) problems. MOLP problems with interval coefficients have been investigated by some authors. Bitran [21] discussed MOLP problems with interval objective function coefficients and introducing two types of solutions. Urli and Nadeau [22] used an interactive method for solving MOLP problems with interval coefficients. Oliveira and Antunes [23] provided an overview of MOLP problems with interval coefficients. Also, Oliveira and Antunes [24] presented an interactive method to solve such problems.

Multi-Objective Linear Programming Problem (MOLPP)

In real life situations, all the linear programming problems are not single objective. The linear programming which are characterized by multiple objective functions are considered here. Special types of linear programming problem in which constraints are equality type and all the objectives are conflicting with each other are called MOLPP. Similar to a typical linear programming problem, in a MOLPP problem. A mathematical model of MOLPP with *r* objectives can be written as:

Mathematically, the problem can be stated as

Minimize or maximize $Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r}.x_{ij} \qquad r = 1,2,3,\dots\dots\dots k$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\dots.,m$$
$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\dots.,n$$
$$x_{ij} \geq 0 \quad \forall\, i,j$$

Matrix Inversion Method:

Step 1: Construct Linear Programming Problem**:**

Minimize or Maximize $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij}$

Subject to Constraints

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\dots.,m$$
$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\dots.,n$$
$$x_{ij} \geq 0 \quad \forall\, i,j$$

Step 2: Subject to constraints considered as system of linear equations:

$$a_{11}x_1 + b_{12}x_2 + c_{13}x_3 = d_1$$
$$a_{21}x_1 + b_{22}x_2 + c_{23}x_3 = d_2$$
$$a_{31}x_1 + b_{32}x_2 + c_{33}x_3 = d_3$$

Step 3: System to equation can be change into the matrix form

Coefficient Matrix $A = \begin{bmatrix} a_{11} & b_{12} & c_{12} \\ a_{21} & b_{22} & c_{22} \\ a_{31} & b_{32} & c_{33} \end{bmatrix}$ Constant Matrix $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Variable Matrix $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Step 4: If Coefficient Matrix A is non-singular matrix than we proceed to further solution, otherwise stop here.

Step 5: Now coefficient matrix A change into inverse matrix i.e. A^{-1}

Step 6: To find the value of basic variables, **t**he system of equation can be written as i.e. $X = A^{-1}B$.

Step 7: To get the final optimum solution of problem, put the value of basic variables in objective functions.

Numerical Problem: Here take a numerical example of multi objective linear programming problem with two objective functions to illustrate the proposed method

$$Max\ Z_1 = 7x_1 + 5x_2 + 10x_3$$
$$Min\ Z_2 = 6x_1 + 7x_2 + 11x_3$$

Subject to Constraints

$$\left.\begin{array}{c} 3x_1 - 2x_2 + 3x_3 \leq 8 \\ 2x_1 + x_2 - x_3 \leq 1 \\ 4x_1 - 3x_2 + 2x_3 \leq 4 \\ x_1, x_2, x_3 \geq 0 \end{array}\right\} \quad (1)$$

Solution by Matrix Inversion Method:

Step 1: LPP Presentation mention as above equation (1).

Step 2: Subject to constraints considered as system of linear equations as follows:

$$3x_1 - 2x_2 + 3x_3 = 8$$
$$2x_1 + x_2 - x_3 = 1$$
$$4x_1 - 3x_2 + 2x_3 = 4$$

Step 3: System to equation can be change into the matrix form

Coefficient Matrix $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$ Constant Matrix $B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

Variable Matrix $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Step 4: Determinant of Coefficient Matrix A is -17, i.e. A is non-singular matrix than we proceed to further solution.

Step 5: Inverse matrix of coefficient matrix A i.e.

$$A^{-1} = \begin{bmatrix} 0.06 & 0.29 & 0.06 \\ 0.47 & 0.35 & -0.53 \\ 0.59 & -0.06 & -0.41 \end{bmatrix}$$

Step 6: After applying the matrix inversion method we get the value of basic variables i.e. $x_1 = 1;\ x_2 = 2$ and $x_3 = 3$, with this values we get the optimum solution of objectives functions are $Z_1 = 47$ and $Z_2 = 53$

Solution by Simplex Method: After applying the simplex method in Tora Software we get the value of basic variables i.e. $x_1 = 1;\ x_2 = 2$ and $x_3 = 3$, with this values we get the optimum solution of objectives functions are $Z_1 = 47$ and $Z_2 = 53$

CONCLUSION: The Matrix inversion method is very simple from the computational point of view and also, simple to understand and apply. By this method, we obtain a sequence of optimal solutions to a MOLPP for a sequence of various time intervals. This method provides a set of optimum solutions to MOLPP which helps the decision makers to take an appropriate decision, depending on his financial position and the decision maker to evaluate the economical activities and make the correct managerial decisions.

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