A comparative study of some integral methods used in a physical system to solve the problems on equation of motion of a particle

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Abstract :
To solve the dynamical system as a motion of a particle with the help of differential equation and some integral transformation. The Elzaki transformation and Laplace transformation methods are used to solve the equation of motion of a particle. Since last few decades the problem on motion of particle had been solved by many mathematical methods. In this paper motion of the problem solved with Elzaki transform and compared the results with Laplace transform. Whenever applied and compared the Laplace and Ezaki methods to motion of particle problem it was confirmed that both methods gave similar results.

Keywords: dynamical system, motion of electron, Elzaki transformation (ET),Laplace transformation (LT),differential equation (DE).

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1. Introduction:

In the mathematical modelling system the integral transforms can be used in various ways. The integral transforms represent good tool to handle many scientific and engineering problems [1-4].The use of integral transforms was very vast to solve the problems as ordinary differential equations, partial differential equations, integral equations etc..[5-9].

In a physical system the differential equations have been solved by a number of integral transforms, including Laplace, Fourier, and others, were utilised.In the literature till now the most used integral transform was Laplace Transform**.** Since last two decades many of researchers tried to introduced a new integral transforms such as Elzaki, Mohand, Sumudu etc….to solve the physical problems.Among all we have used Elzaki transform method to solve the problem. The "Elzaki Transform" is one of the integral transforms that can be utilised in the process of resolving any boundary value problem that manifests itself in the form of a differential equation representing a physical system [10-11].The Elzaki transform is the modified transform of Laplace and Sumudu transform.

In present paper we have solved the differential equations may often be computed by applying the Laplace transform technique as part of the solution process [12-14]. However, the "Elzaki Transform" was used an exception to solve the same problem. In this study, we examined the two different integral transform techniques.

To begin, we explained the basic formulae and attributes shared by both transforms. The aim of this paper is to find the motion of a particle system of ordinary differential equations using the new integral transform and Laplace transform and compared the results.

In this work any physical system problems can be represented in the form of mathematical modelling such as differential equations and such equations can be solved by different transform methods.

2. Methodology

The present problem have been solved by using some definitions and standard results of Elzaki and Laplace transform.

2.1. Definition of Elzaki Transform along with Laplace Transform

Ezaki Transform : for a given function f(t) $t>0$, ET is given by $E\{f(t)\} = v\int_0^\infty e^{-\frac{t}{v}} f(t)\, dt = T(v), k_1 \le v \le k_2.$ E is the Ezaki transform.	Laplace transform : For any function f(t) $t \ge 0$, LT is given by $L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt = \bar{f}(S).$ Where s= parameter. L=Laplace operator.

For both the transform the function f(t)to exist the following conditions:-
Function f(t) is piecewise continuous and is exponential order

2.2 ET and LT of Some standard functions

Sl.no.	f(t)	L[f(t)]	E[f(t)]
1	1	$\frac{1}{s}$	v^2
2	Cosh(at)	$\frac{s}{s^2-a^2}$	$\frac{v^2}{1-a^2v^2}$
3	Sinh(at)	$\frac{a}{s^2-a^2}$	$\frac{av^3}{1-a^2v^2}$
4	Cos(at)	$\frac{s}{s^2+a^2}$	$\frac{v^2}{1+a^2v^2}$
5	e^{at}	$\frac{1}{s-a}$	$\frac{v^2}{1-av}$
6	Sin(at)	$\frac{a}{s^2+a^2}$	$\frac{av^3}{1+a^2v^2}$
7	t	$\frac{1}{s^2}$	v^3

2.3 Definition of Inverse Laplace as well as Elzaki Transform definitions:

Inverse Ezaki Transform : If T(v) is the Ezaki transform of the given function f(t) ,then Inverse ET is given by $f(t)=E^{-1}\{T(v)\}$ is the inverse of T(v) then E^{-1} is the inverse Ezaki operator.	Inverse Laplace transform : If f(s)is the Laplace inverse transform function of the given function , then Inverse of LT is given by $f(t)=L^{-1}\{f(s)\}$ is the inverse of f(s) the L^{-1} is the inverse Laplace transform.

2.4.Inverse ET and inverse LT of some Standard Functions

Sl.no.	f(t)	$L^{-1}[f(t)]$	$E^{-1}[T(v)]$
1	1	$\frac{1}{s}$	v^2
2	Cosh(at)	$\frac{s}{s^2-a^2}$	$\frac{v^2}{1-a^2v^2}$ ”
3	Sinh(at)	$\frac{a}{s^2-a^2}$	$\frac{av^3}{1-a^2v^2}$
4	Cos(at)	$\frac{s}{s^2+a^2}$	$\frac{v^2}{1+a^2v^2}$
5	e^{at}	$\frac{1}{s-a}$	$\frac{v^2}{1-av}$
6	Sin(at)	$\frac{a}{s^2+a^2}$	$\frac{av^3}{1+a^2v^2}$
7	t	$\frac{1}{s^2}$	v^3

3.Results and Discussions

The motion of a particleis governed by the equations

$$\ddot{x}(t)-n\,\dot{y}(t)=0 \text{ ....(1)}$$

and

$$\ddot{y}(t)+n\,\dot{y}(t)=n^{2}a \text{ ....(2)}$$

With the boundary conditions $x(0)=0, y(0)=0$ *and* $\dot{x}(t)=\dot{y}(t)=0$

Solving these equations (1) and by using Ezaki transforms

$$E[\dot{x}(t)]-E[n\,\dot{y}(t)]=E(0)$$

$$\left[\frac{x(v)}{v^{2}}-x(0)-v\,\dot{x}(0)\right]\text{-}\,n\left[\frac{y(v)}{v}-v\,x(0)\right]=0$$

Applying the boundary conditions

$$\frac{x(v)}{v^{2}}-n\frac{y(v)}{v}=0 \text{ ....(3)}$$

Taking equation (2)and applying the Ezaki transform on both sides

$$E[\ddot{y}(t)]+n\,E[\dot{x}(t)]=E[n^{2}a]$$

$$\left[\frac{y(v)}{v^{2}}-y(0)-v\,\dot{y}(0)\right]+n\left[\frac{x(v)}{v}-v\,x(0)\right]=n^{2}a\,v^{2}$$

Applying the boundary conditions

$$\frac{y(v)}{v^{2}}+\frac{n}{v}x(v)=n^{2}a\,v^{2} \text{ .... (4)}$$

Solving equations (3) and (4) we get

$$x(v)=\frac{n^{2}a\,v^{6}}{(1+n^{2}v^{3})}$$

Applying inverse Ezaki transform both the sides

$$E^{-1}[x(v)]=n^{2}a\,E^{-1}\left[\frac{v^{6}}{(1+n^{2}v^{3})}\right]$$

$$x(t)=n^{2}a\,E^{-1}\left[\frac{v^{3}}{n^{2}}-\frac{v^{3}}{n^{2}(1+n^{2}v^{3})}\right]$$

$$x(t)=(nt-\sin nt) \text{ ....(5)}$$

Similarly we get

$$y(t)=a(1-\cos nt) \text{ ....(6)}$$

Now applying Laplace transform for the equations (1) and (2) with the initial conditions

Consider the equation (1)

$$\ddot{x}(t) - n\,\dot{y}(t) = 0$$

$$L[\ddot{x}(t)] - n\,L[\dot{y}(t)] = L[0]$$

$$[s^2 L[x(t)] - s\,x(0) - \dot{x}(0)] - n\left[s\,L[y(t)] - y(0)\right] = 0$$

Applying the initial conditions we get

$$s^2\,L[x(t)] - n\,sL[y(\mathrm{t})] = 0 \text{ ....(7)}$$

Now consider the equation (2) and applying Laplace transform both the sides

$$\ddot{y}(t) + n\,\dot{y}(t) = n^2 a$$

$$L[\ddot{y}(t)] + n\,L[y(t)] = L[n^2 a]$$

$$\left[s^2\,L[y(t)] - s\,y(0) - \ddot{y}(0)\right] + n\left[s\,L[x(t)] - x(0)\right] = \frac{n^2 a}{s}$$

Applying the initial conditions we get

$$s^2\,L[y(t)] + n\,s\,L[x(t)] = \frac{n^2 a}{s} \text{ ....(8)}$$

Solving equations (6) and (7) we get

$$L[x(t)] = \frac{n^3 a}{s(s^2 + n^2)}$$

Now applying inverse transform

$$x(t) = n^3 a\,L^{-1}\left[\frac{1}{s(s^2 + n^2)}\right]$$

By applying partial fractional method

$$x(t) = a(nt - \sin nt) \text{ ....(9)}$$

Similarly we get

$$y(t) = a(1 - \cos nt) \text{ .... (10)}$$

The observation made in the equations of (5) and (6) in the Ezaki transform and the equations (9) and (10) of Laplace transforms are similar. In the previous days the above problem was solved by usually Laplace transform methods but now a days an approach have been changed to many integral methods to solve such problems in a simplified ways. Same problem we have tried by using Ezaki transform which will be useful in solving in many physical system in the science and technology fields.

4. Conclusion:

The study on present system of motion of a particle problem was solved by differential equations using Elzaki transform and Laplace transform and both the results were studied. After applying these methods to this particular system the solution looks similar.The same problem can also be solved by many integral methods.

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